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DEPARTMENT OF ELECTRICAL ENGINEERING

3rd YEAR LMD ELECTRICAL ENGINEERING

COURSE MATERIAL

POWER ELECTRONICS

Prepared by : *Dr. KHESSAM.MEDJDOUB*

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Foreword

This pedagogical handout has been prepared as an educational support for the module “Power Electronics”, taught to third-year Licence students, specialty Electrotechnics, within the framework of the Fundamental Teaching Unit UEF 3.1.1 of the fifth semester. In accordance with the official program, the module represents a total workload of 45 hours (1h30 Lecture and 1h30 Tutorial per week), corresponding to 4 credits and a coefficient of 2.

The purpose of this document is essentially pedagogical. It is designed to accompany the student throughout the semester by providing a structured and progressive learning support, adapted to the level of third-year Electrotechnics students. Its content strictly follows the requirements of the official program and respects the recommended distribution of chapters and volumes.

The methodology adopted in this handout is based on three main principles:

Clarity of presentation, in order to make the concepts easily accessible to students, even those who encounter power electronics for the first time.

Progressive learning, moving gradually from the study of power semiconductor devices to the analysis of complete conversion systems.

Direct link between theory and application, through solved examples, application exercises, and corrected exercises grouped in a final annex.

This document is not intended to replace the lectures given by the teacher, but rather to serve as a complementary tool that helps the student to:

- Review the theoretical concepts presented during lectures,
- Prepare tutorial sessions and exams,
- Develop autonomous learning skills,
- Consolidate the practical understanding of power conversion techniques.

The pedagogical approach adopted in this work is the result of several years of teaching experience in this module. It aims to offer students a reliable and coherent reference that meets the requirements of the LMD system and contributes to strengthening their scientific and technical background in the field of Electrotechnics.

General introduction

General introduction

Power electronics, which are used in all technological fields but particularly in the field of static conversion of electrical energy, are the foundation of today's world. As it can be seen in the third-year curriculum for an electrical engineering degree, which covers all methods for "shaping" the currents and voltages that transfer electrical energy. Nowadays, practically all electrical appliances, whether home or industrial, have electrical circuits. Despite this, only a small number of structures are capable of all conversions and a specific procedure known as "cutting." Generally, there are four possibilities of power electronics conversion (see figure I.1):

- 1- AC to DC converter (rectifier).
- 2- AC to AC converter (dimmer, Cyclo-converter).
- 3- DC-DC converter (chopper).
- 4- Direct current - alternating current converter (inverter).

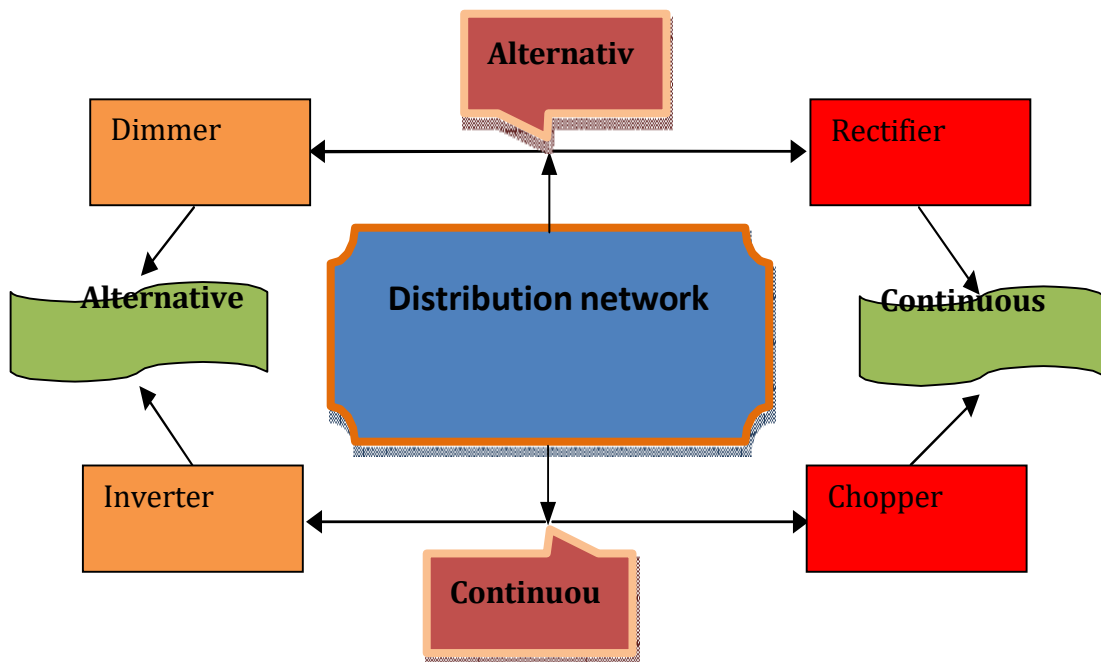


Figure I.1: The different types of converters

The structure of this document consists of five chapters:

Chapter 1: This chapter provides a detailed study of semiconductor power switches, covering their types, characteristics, and operating principles.

Chapter 2: It focuses on rectifiers, both controlled and uncontrolled, with an in-depth analysis of single-phase and three-phase rectifiers.

Chapter 3: This chapter explores AC-AC converters, with special attention to voltage control techniques such as dimmers and their applications.

Chapter 4: It examines DC-DC converters, also known as choppers, analyzing different control methods and their practical implementations.

Chapter 5: The final chapter is dedicated to single-phase inverters, explaining their working principles, types, and applications in converting DC to AC power.

Chapter I: Semi-conductor power switches

I.1 Introduction:

Switches, which are crucial components for charging and adjusting voltages and currents as they make up a static converter, are just one type of semiconductor component used in power electronics to accomplish various switching operations. Various types of switches are available on the market nowadays. The most popular parts of this field are the foundation of this chapter.

The following are the key distinctions:

- 1- Diodes are non-controllable components.
- 2- thyristors and triacs are controllable on closure.
- 3- elements that can be controlled on closing and opening (bipolar transistors, MOSFETs, IGBTs, GTOs).

One or more semiconductor components combine to create a semiconductor switch.

I.2 non-controllable elements

I.2.1 The diode

The diode is a PN junction-based semiconductor component that is frequently employed in power electronics. It is unidirectional, enabling electricity to flow only in one way, and has two electrodes (an anode and a cathode). This extremely high specificity enables it to safeguard a section of an electric circuit, increasing its usefulness.

I.2.2 Symbol

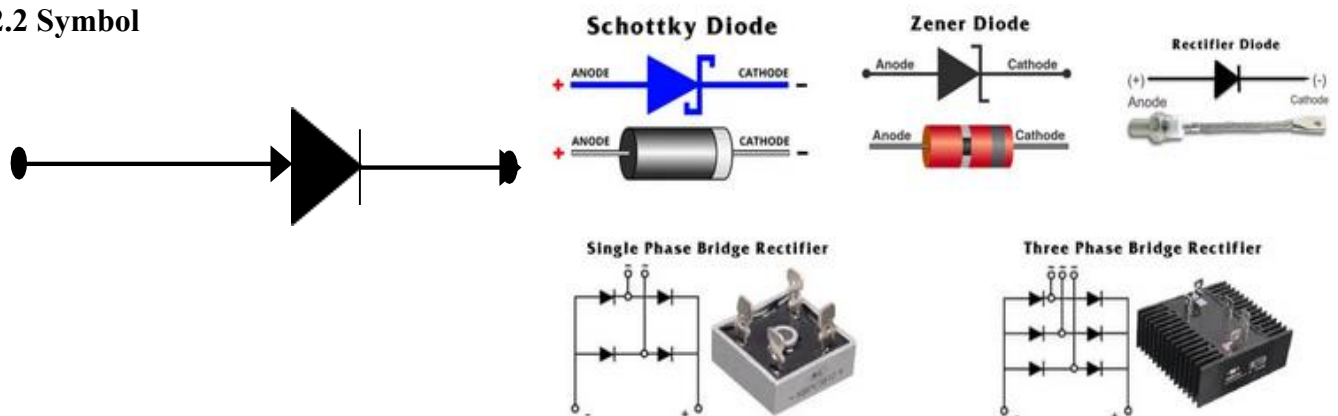
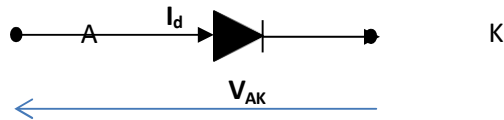


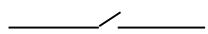
Figure I.2: Electrical representation of a diode

I.2.3 Operation

The diode can be in one of two very distinct states:

- Diode in the state: Passing or conduction.
- Diode in the state: Blocked.



Passing diode 	state of the diode when $V_{AK} \geq V_{seuil}$ which results in $I_d \neq 0$.
Blocked diode 	state of the diode when $V_{AK} < V_{seuil}$ which prevents the passage of current in the diode; $I_d = 0$

I.3 controllable closing elements

I.3.1 The thyristor:

The thyristor is a unidirectional semiconductor electronic switch that is controlled to close, but not to open, as shown in the figure below. It consists of three terminals, the anode (A); the cathode (K) and the gate G

- The anode (A) and cathode(K) are the power access.
- The trigger G is used to control it.

I.3.2 Symbol

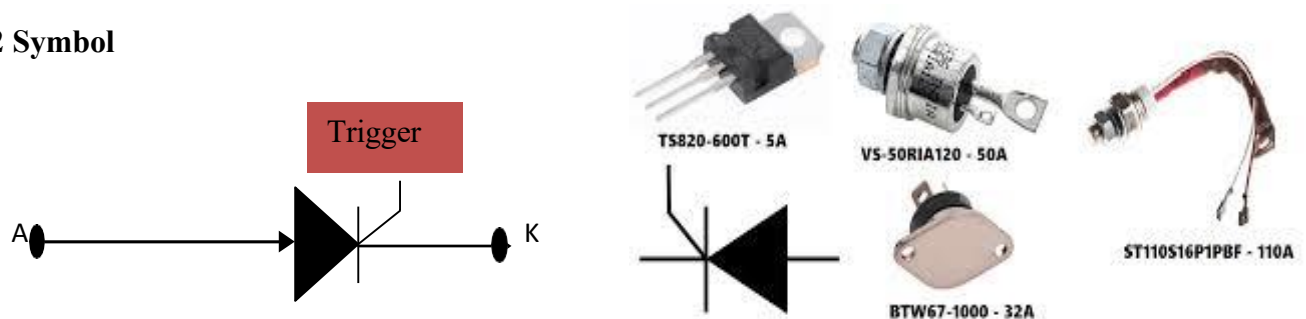
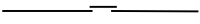
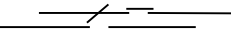


Figure I.3: Electrical representation of a thyristor.

I.3.3 Operation

There are three possible states of the thyristor:

Thyristor passing 	$V_{AK} \geq V_{seuil}$ + a pulse of current $I_{GK} > 0$ was given between the trigger and the cathode
thyristor blocked 	$V_{AK} \geq V_{seuil}$ but $I_{GK} > 0 = 0$.
	$V_{AK} < V_{seuil}$.

I.3.4 Types of thyristors:

Thyristors are a family of semiconductor devices used for switching and controlling high-power applications. They are primarily used in rectifiers, inverters, and other power conversion systems. Here are the main types of thyristors:

1. Silicon-Controlled Rectifier (SCR)

The most common type of thyristor.

Conducts current only when a gate pulse is applied and remains on until the current drops below a certain threshold.

Used in controlled rectifiers, motor drives, and power regulation.

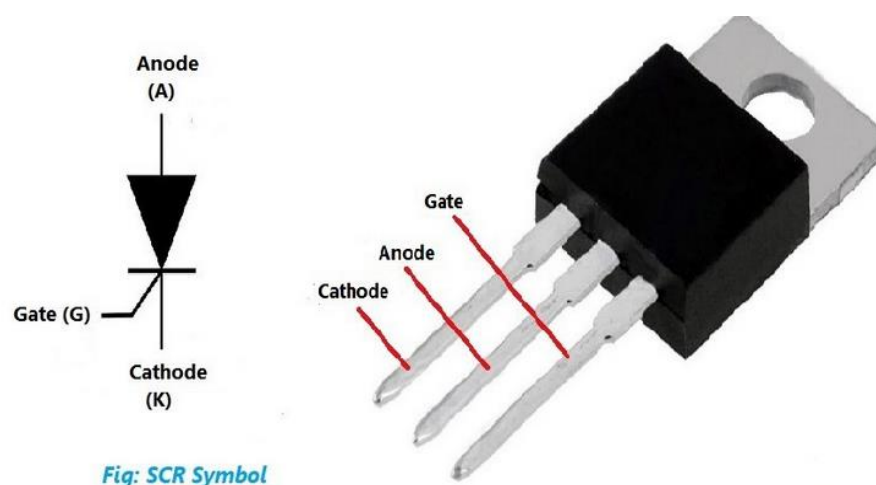


Figure I.4:SCR symbol

2. Gate Turn-Off Thyristor (GTO)

Unlike SCRs, GTOs can be turned off by applying a negative gate current. Used in high-power inverters, DC-DC converters, and traction applications.



Figure I.5: GTO symbol

3. Integrated Gate-Commutated Thyristor (IGCT)

An improved version of GTO with faster switching and lower losses. Used in high-power industrial drives and HVDC transmission.



Figure I.6: IGCT symbol

4. Reverse Conducting Thyristor (RCT)

Has an integrated diode for reverse conduction.

Commonly used in inverter circuits and resonant converters.

5. Light-Triggered Thyristor (LTT)

Turned on using a light signal instead of an electrical gate pulse.

Used in high-voltage DC transmission (HVDC) and pulsed power applications.

- etc.

I.4 Closing and opening controls

I.4.1 Transistor

The transistor is an electronic semiconductor component controlled on and off, open and closed by a control signal, it is found in choppers and inverters.

I.4.2 Symbol

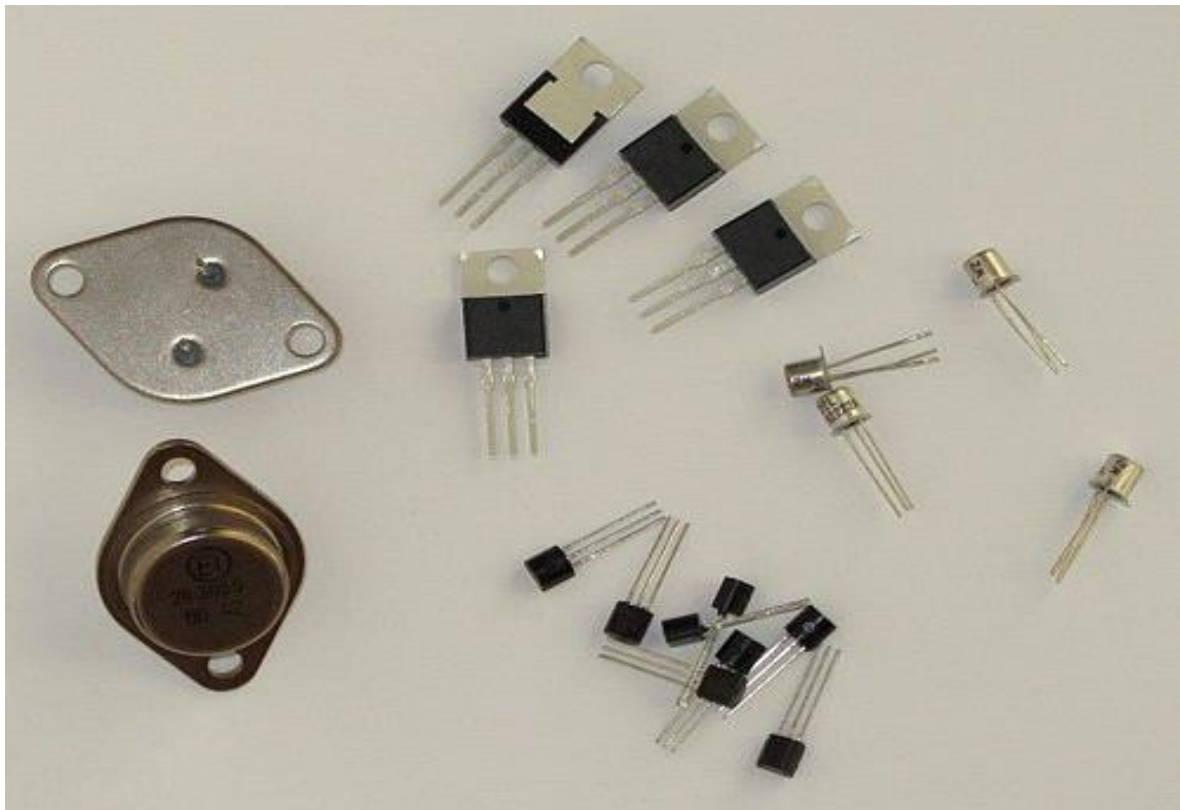
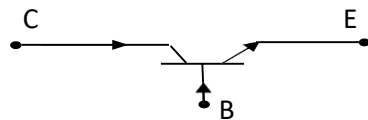


Figure I.7: Electrical symbol of a transistor

I.4.3 Operation

Blocked transistor:

condition reached by canceling the control current i_B current, which results in zero collector current and voltage V_{CE} (source voltage).

This produces the effect of an open switch.

Saturated transistor:

In this case, the current i_B is such that the transistor imposes a voltage of V_{CE} zero while the current i_C reaches a limit value called the saturation value $i_{C_{sat}}$. This gives the equivalent of a closed switch.

I.4.4 Types of Transistors

Transistors are semiconductor devices used for switching and amplification in electronic circuits. They are classified into different types based on their structure and operation.

I.4.4.1. Bipolar Junction Transistor (BJT)

- Operates based on current control.
- Has three terminals: Base (B), Collector (C), and Emitter (E).
- Two types:
 - NPN:** Conducts when a small current flows from base to emitter.
 - PNP:** Conducts when a small current flows from emitter to base.
- Used in amplification, switching circuits, and signal processing.

Bipolar Junction Transistor (BJT)

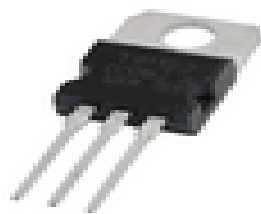


Figure I.8: Bipolar Junction Transistor

I.4.4.2. Field-Effect Transistor (FET)

- Operates based on voltage control.
- High input impedance and low power consumption.
- Main types:

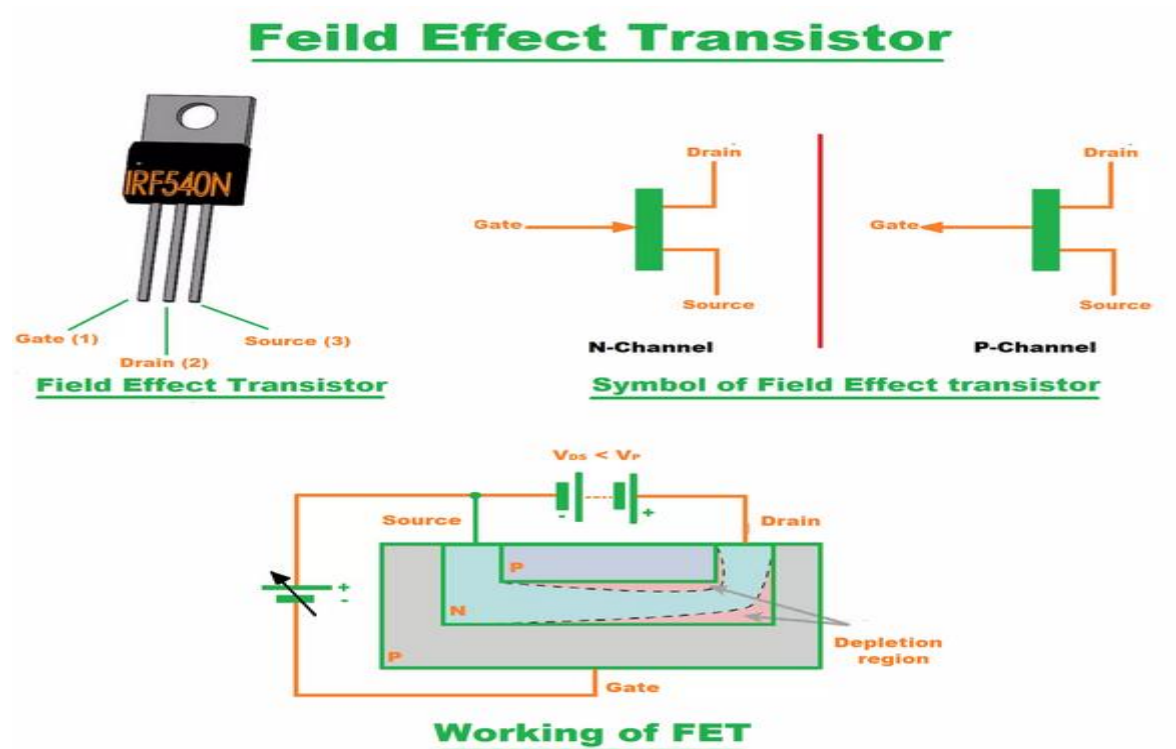


Figure I.9: Field-Effect Transistor

a. Junction Field-Effect Transistor (JFET)

- Has three terminals: Gate (G), Drain (D), and Source (S).
- Controlled by voltage applied to the gate.
- Available as N-channel and P-channel types.
- Used in low-noise amplifiers and analog circuits.

b. Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET)

- Has four terminals: Gate (G), Drain (D), Source (S), and Body (B).
- High-speed switching capability.
- Two main types:
 - Enhancement mode: Normally OFF; turns ON with gate voltage.

- Depletion mode: Normally ON; turns OFF with gate voltage.
- Used in power electronics, switching regulators, and microprocessors.

MOSFET

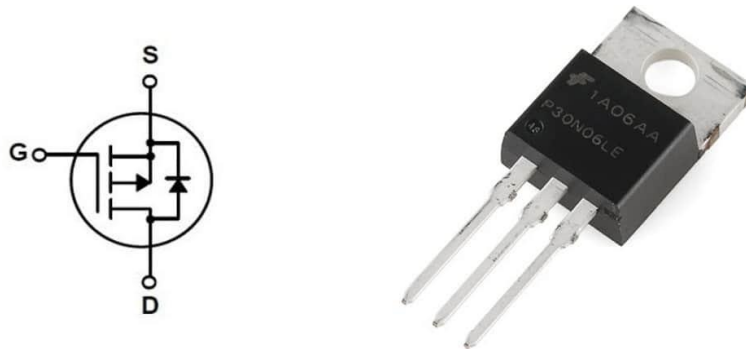


Figure I.10: Power MOSFET Structure

I.4.4.3. Insulated Gate Bipolar Transistor (IGBT)

- Combines BJT and MOSFET advantages.
- Controlled by voltage like a MOSFET but has high current capability like a BJT.
- Used in: choppers, inverters, motor drives, and power converters.



Figure I.11: Structure of a Power IGBT

I.4.4.4. Darlington Transistor

- A pair of BJTs connected to provide high current gain.

- Used in high-power applications and amplifiers.

I.4.4.5. Heterojunction Bipolar Transistor (HBT)

- Uses different semiconductor materials for improved performance.
- Used in RF and microwave applications.

I.4.4.6. Thin-Film Transistor (TFT)

- A special type of FET used in LCD displays and flat-panel screens.

I.5 The different fields of application of power electronics components.

In modern electrical and electronic systems, power electronics has become a fundamental enabling technology. Static converters are no longer limited to laboratory use or specialized industrial systems; they are now present in almost every sector where electrical energy needs to be converted, controlled, or optimized. Their main role is to ensure efficient energy conversion, voltage and current regulation, and precise control of electrical power according to the requirements of the load.

The following sections highlight the most important application fields where power electronics components play a central role:

I.5.1 Systems of Power Supplies

In the event of an outage, Uninterruptible Power Supplies (UPS) guarantee power continuity.

Switched-Mode Power Supplies (SMPS): Offer electronic devices effective voltage conversion.

I.5.2 Systems of Renewable Energy

Renewable energy systems rely heavily on power electronics to interface energy sources with electrical loads or the utility grid.

In photovoltaic systems, solar panels generate DC power. However, electrical grids operate with AC voltage. Therefore, solar inverters convert DC into synchronized AC power while also performing maximum power point tracking (MPPT) to extract the highest possible energy from the panels. Without power electronics, solar energy could not be efficiently integrated into the grid. Similarly, in wind energy systems, the frequency and amplitude of the generated voltage vary depending on wind speed. Power converters regulate and stabilize this output, allowing safe and efficient grid connection. In this context, power electronics ensures energy quality, stability, and compliance with grid standards.

I.5.3 Transportation and Electric Drives

Transportation is one of the fastest-growing fields of power electronics applications.

In electric and hybrid vehicles (EVs and HEVs), DC-DC converters regulate battery voltage to supply auxiliary systems, while inverters convert DC battery power into AC to drive electric motors. These systems must operate with high efficiency, reliability, and compact design.

In railway systems, metro networks, and high-speed trains, traction motors are controlled using high-power inverters. These converters regulate motor speed and torque while optimizing energy consumption and enabling regenerative braking, where kinetic energy is converted back into electrical energy.

Power electronics is therefore a key technology enabling sustainable and efficient transportation.

I.5.4 Automation in Industry

Modern industries rely on precise control of electric machines and processes. Power electronics makes this control possible.

Variable Frequency Drives (VFDs) are widely used to regulate motor speed and torque in pumps, compressors, conveyor belts, and manufacturing systems. By adjusting the frequency and voltage supplied to motors, VFDs significantly reduce energy consumption and mechanical stress.

In robotics and CNC machines, power electronic converters ensure accurate and dynamic motor control. High precision, fast response, and stable operation are achieved thanks to advanced switching techniques and control algorithms.

As a result, power electronics improves productivity, energy efficiency, and process reliability in industrial environments.

I.5.5 Battery chargers for consumer electronics:

found in computers, cell phones, and electric tools.

LED Lighting Systems: Energy-saving lights with effective DC-DC conversion.

I.5.6 Transmission and Distribution of Power

High-Voltage Direct Current (HVDC) systems:

effective transmission of electricity across vast distances.

Flexible AC Transmission Systems, or FACTS, increase electricity efficiency and stability.

The various industries in which power electronics components are used are depicted in the following image.

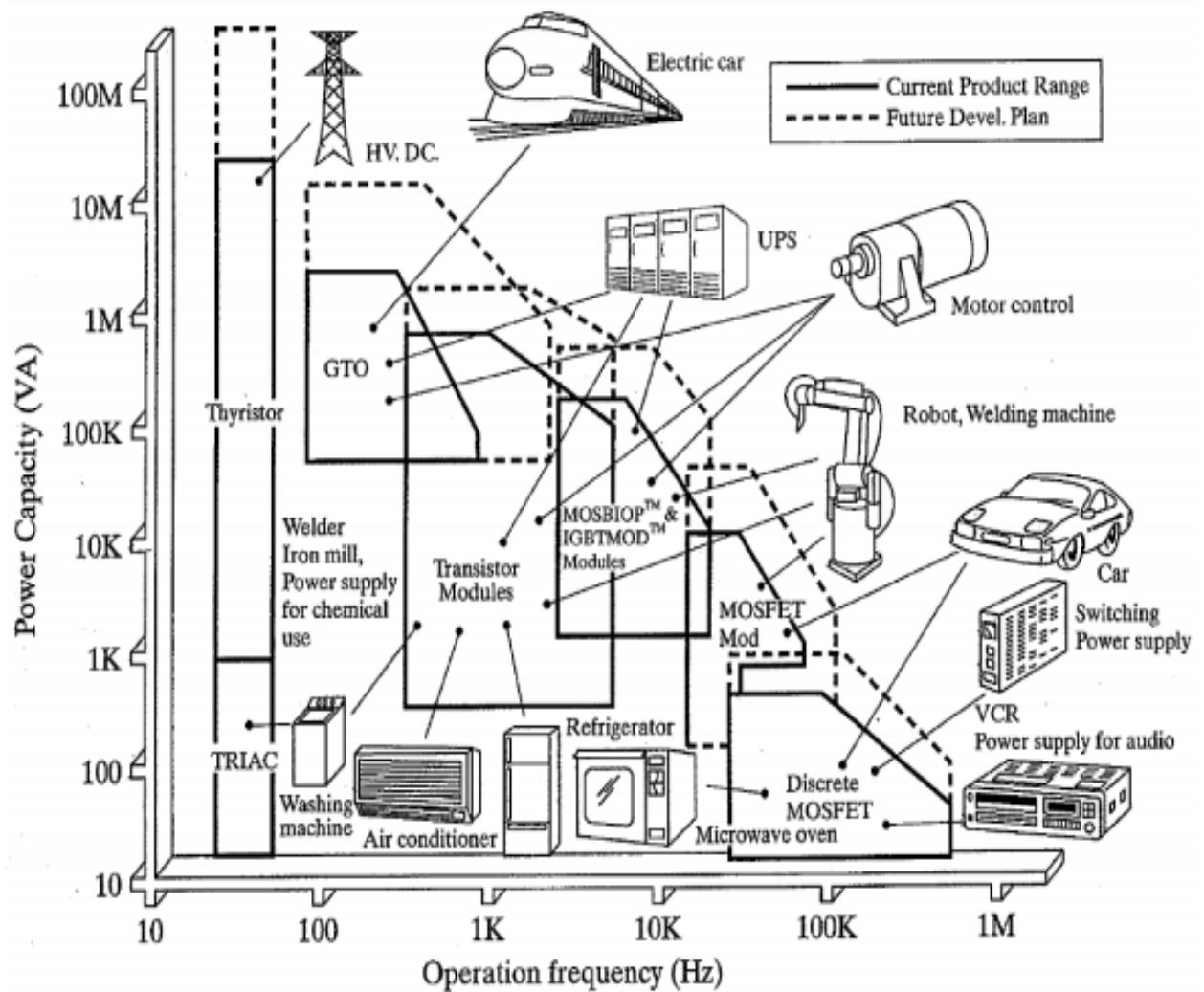


Figure I.12: the various industries in which power electronics components are used.

I.6 Comparison of Different Fully Controllable Switches

In power electronics, fully programmable power switches are essential because they provide accurate voltage and current regulation. Among fully programmable switches, MOSFETs, IGBTs, and Power BJTs are the most often utilized. Their salient features are contrasted in the table below:

Parameter	MOSFET	IGBT	Power BJT
Control Type	Voltage-controlled (low power gate drive)	Voltage-controlled (low power gate drive)	Current-controlled (high base current required)

Switching Speed	Very fast (high frequency operation)	Moderate (lower than MOSFET)	Slow (compared to MOSFET and IGBT)
Current Rating	Moderate	High	Very High
Voltage Rating	Low to medium (up to 900V)	Medium to high (up to 3.3kV)	High (but less efficient at high power)

Exercises on Power Semiconductor Switches

Exercise 1: Power Diode

A power diode conducts a forward current of 25 A with a forward voltage drop of 1.2 V.

Questions:

1. Calculate the conduction power loss in the diode.
2. If the diode operates for 10 hours, calculate the energy dissipated.
3. Explain the difference between an ideal diode and a real power diode.
4. What is reverse recovery time?
5. Why is reverse recovery important in high-frequency applications?

Exercise 2: Thyristor (SCR)

A thyristor is connected to a DC source of 200 V supplying a load of 10 Ω .

Questions:

1. Calculate the load current when the thyristor is ON.
2. Define:
 - 2.1. Latching current
 - 2.2. Holding current
3. Explain how a thyristor is turned ON.
4. Can a thyristor be turned OFF by the gate signal? Justify.
5. Give two practical applications of thyristors.

Exercise 3: Power MOSFET

A MOSFET operates with:

Drain current $I_D=15$ A

$R_{DS(on)}=0.08 \Omega$

Questions:

1. Calculate the conduction losses.
2. Explain why MOSFETs are suitable for high-frequency switching.
3. What is the advantage of voltage-controlled devices?
4. Compare MOSFET and BJT in terms of switching speed.
5. Give two applications of power MOSFETs.

Exercise 4: IGBT vs MOSFET (Comparative Study)

An application requires switching at 5 kHz with voltage 600 V and current 40 A.

Questions:

1. Which device is more suitable: MOSFET or IGBT? Justify.
2. Compare MOSFET and IGBT in terms of:
 - 2.1. Voltage rating
 - 2.2. Current capability
 - 2.3. Switching speed
 - 2.4. Conduction losses
3. Why are IGBTs widely used in motor drives?
4. Draw the symbol of an IGBT and a MOSFET.
5. Explain the main difference between unipolar and bipolar devices.

Chapter II : AC/DC Converters (Rectifiers)

II.1 Introduction:

Although electrical energy is generally generated, transmitted, and distributed in the form of alternating current (AC), a large number of electrical and electronic devices operate using direct current (DC). Industrial power systems supply AC voltage because it is easier to generate and transmit over long distances. However, many applications such as electronic circuits, battery charging systems, DC motor drives, and communication equipment require a stable DC voltage.

For this reason, AC/DC converters, commonly known as rectifiers, are essential components in power electronics. Their main function is to convert an AC input voltage into a DC output voltage suitable for the load. Depending on the type of semiconductor devices used and the required level of control, rectifiers can be classified into two main categories:

Uncontrolled Rectifiers, which use diodes and provide a fixed DC output voltage determined by the AC source.

Controlled Rectifiers, which use thyristors (SCRs) and allow control of the output voltage by adjusting the firing angle.

The study of rectifiers includes the analysis of voltage and current waveforms, calculation of average and RMS values, ripple content, power factor, and performance comparison between different configurations. Understanding these principles is fundamental for designing efficient DC power supply systems and controlled DC drives.

II.1.1 Form Factor:

The form factor is one of the most important parameters used to evaluate the quality of a rectified voltage. It provides a quantitative indication of how close the output waveform of a rectifier is to a pure direct current (DC) signal. In an ideal case, a perfectly smooth DC voltage would have a form factor equal to unity. Therefore, the closer the form factor value is to 1, the better the quality of the rectified voltage and the smaller its ripple content.

This coefficient is particularly useful when comparing different rectifier configurations, such as half-wave and full-wave rectifiers, or single-phase and three-phase systems. A rectifier with a lower form factor produces an output voltage that is more stable and closer to a continuous signal, which is generally desirable in most industrial and electronic applications.

By definition, the form factor is expressed as the ratio between the effective (RMS) value and the average (DC) value of the rectified voltage:

$$F = \frac{\text{Effective value of rectified voltage}}{\text{Average value of rectified voltage}}$$

A form factor close to 1 indicates a high-quality DC output with low ripple, while a higher value reflects a waveform that still contains a significant AC component. For this reason, the form factor is considered a key criterion in the design and performance evaluation of rectifier circuits.

II.2 Uncontrolled rectification (DIODES)

II.2.1 Single diode rectification (single-ended)

* Flow on a resistive load

This relates to rectifying a single-phase sinusoidal signal, as the title implies. We are aware that a sinusoidal signal is made up of both positive and negative alternations.

Since we only utilize one diode in this kind of rectification, the diode will be forward-biased during one alternation and reverse-biased during the other, according to the idea that was previously presented. Let's examine in more depth how this behavior is verified in real-world scenarios.

The circuit schematic may look like this (Figure II.1):

The diode D is taken as perfect or ideal.

The input voltage $V_e(t)$ of the generator is sinusoidal and given by the following expression:

$$V_e(t) = V_m \sin(\omega t),$$

Where V_m is the maximum voltage

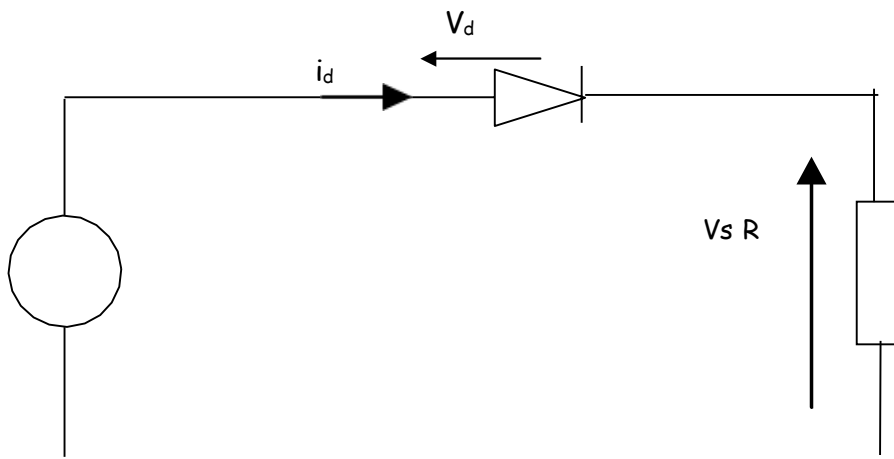


Figure II.1: Single-phase rectifier arrangement feeding a purely resistive load

Analysis of operation

Regardless of the diode's state, we have:

$$V_e = V_d + V_s$$

Verser : $0 < \alpha < \pi$

Then the diode becomes conductive:

- $V_e(t) > 0$ D pass
 $V_d = 0V$
 $V_e = V_s$ and $i_s = \frac{V_e}{R}$

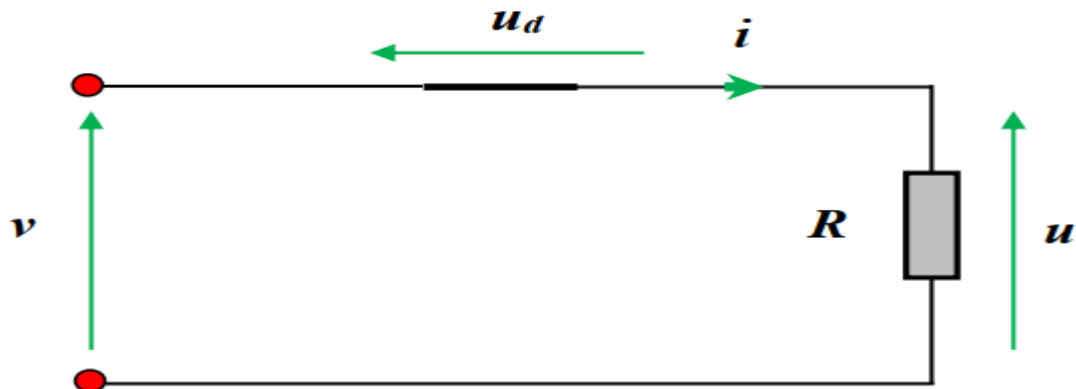


Figure II.2: Equivalent circuit of a single-phase half-wave rectifier with a resistive load using a diode.

Verser : $\pi < \alpha < 2\pi$

- $V_e(t) < 0$, D blocked
 $V_d = V_e$
 $V_s = 0$
and $i_s = 0$

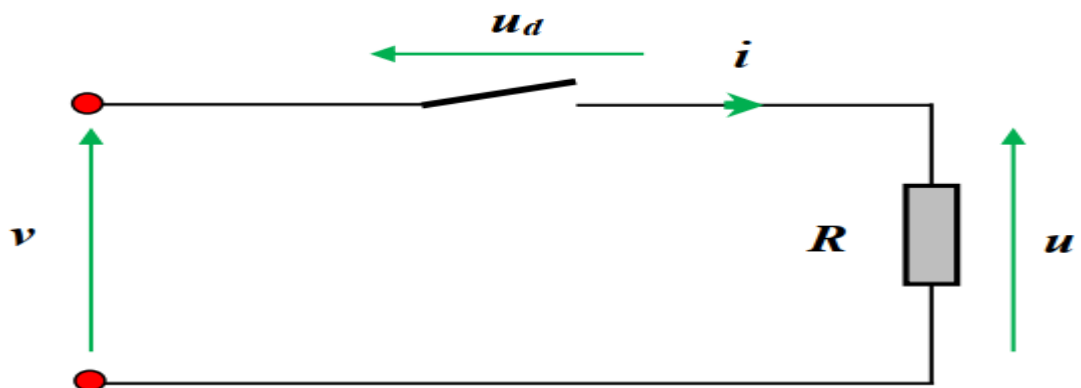
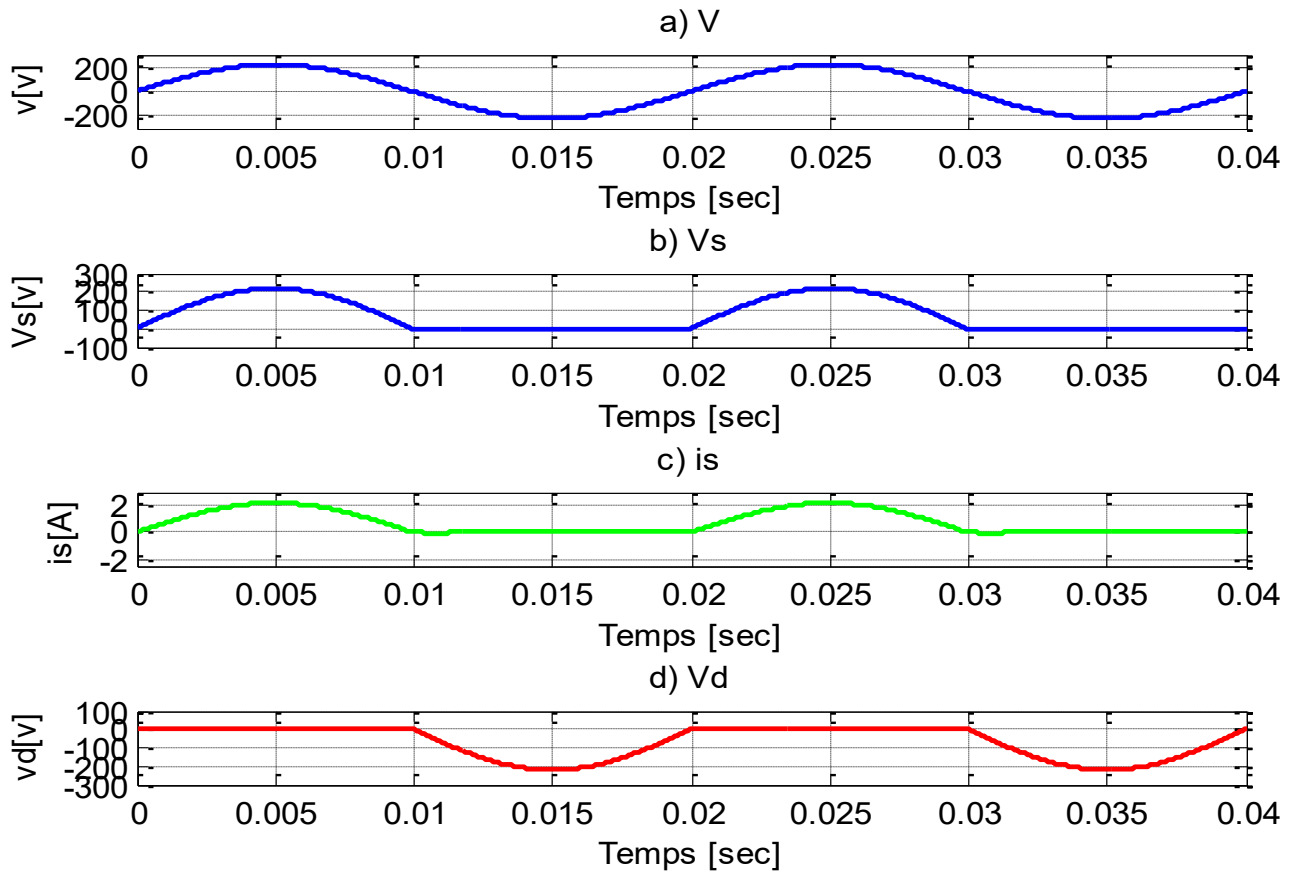


Figure II.3: Representation of the diode as an open switch during the blocking phase

Chronograms



II.4: Voltage and current diagrams for a load R

II.2.1.1 Average values

- The average value of the output voltage is calculated as follows:

$$\begin{aligned}
 V_{s_moy} &= \frac{1}{T} \int_0^T V_s(t) \cdot dt = \frac{1}{T} \int_0^{T/2} V_m \cdot \sin(\omega t) \cdot dt + \int_{T/2}^T 0 = \frac{V_m}{T \cdot \omega} [-\cos(\omega t)]_0^{T/2} \\
 &= \frac{V_m}{T \cdot \frac{2\pi}{T}} * 2
 \end{aligned} \tag{II.1}$$

So:

$$V_{s_moy} = \frac{V_m}{\pi} = \frac{V \cdot \sqrt{2}}{\pi} \approx 0,45 * V$$

V: The input RMS value

- The average value of the output current is calculated as follows:

$$\begin{aligned}
i_{s_{moy}} &= \frac{V_{s_{moy}}}{R} \\
&= \frac{V_m}{\pi.R} = \frac{V.\sqrt{2}}{\pi.R}
\end{aligned} \tag{II.2}$$

$$i_{s_{moy}} \approx 0,45 * \frac{V}{R}$$

- **The rms voltage value across the load**

$$V_{s_{eff}}^2 = \frac{1}{T} \int_0^T V_s^2(t) dt = \frac{1}{T} \int_0^{T/2} (V_m \sin(\omega t))^2 dt \tag{II.3}$$

Let's replace the variables, $\theta = \omega t$ so $T \longrightarrow 2\pi$

$$V_{s_{eff}}^2 = \frac{1}{2\pi} \int_0^\pi (V_m)^2 \cdot \sin^2(\theta) d\theta = \frac{(V_m)^2}{2\pi} \int_0^\pi \frac{1}{2} (1 - \cos(2\theta)) d\theta = \frac{(V_m)^2}{4}$$

$$\text{Therefore: } V_{s_{eff}} = \frac{V_m}{2} \tag{II.4}$$

$$i_{s_{eff}} = \frac{V_{s_{eff}}}{R} = \frac{V_m}{2.R} \tag{II.5}$$

- **Form factor :**

$$F = \frac{V_{s_{eff}}}{V_{s_{moy}}} = \frac{\frac{V_m}{2}}{\frac{V_m}{\pi}} = \frac{\pi}{2} \approx 1.57 \tag{II.6}$$

II.2.2 Energy analysis

Active Power Received by the Load

The active power (PP) received by the load represents the real power that is effectively converted into useful work (such as mechanical energy in motors or heat in resistors). It is given by the formula:

$$P = v_{l_{moy}} * i_{l_{moy}}$$

$v_{l_{moy}}$ is the average rectified voltage across the load.

$i_{l_{moy}}$ is the average rectified current through the load.

Apparent Power Supplied by the Network

When active power (P) and reactive power (Q) are combined, the apparent power (SS) is the overall power provided by the electrical network. It has the following formula and is represented in volt-amperes (VA):

$$S = v_{rms} * i_{rms}$$

S = Apparent power (in volt-amperes, VA)

v_{rms} = RMS voltage supplied by the network (in volts, V)

i_{rms} = RMS current drawn from the network (in amperes, A)

Relation with Active and Reactive Power

Apparent power is related to active power (P) and reactive power (Q) through the power triangle:

$$S^2 = P^2 + Q^2$$

Where:

$$P = S * \cos \varphi \text{ (Active power, in watts, W)}$$

$$Q = S * \sin \varphi \text{ (Reactive power, in volt – amperes reactive, VAR)}$$

φ Phase angle between voltage and current

II.2.2.1 Power Factor

The power factor (PF) is one of the most important parameters used to evaluate the electrical performance of any power system, including rectifiers and other power electronic converters. It provides a clear indication of how efficiently the electrical energy drawn from the source is actually converted into useful work at the load.

In an ideal situation, all the power delivered by the source would be transformed into active power, meaning it would be fully used by the load. However, in real systems, part of the energy circulates between the source and the load without producing any useful effect. This non-productive component is known as reactive power, and it reduces the overall efficiency of the installation. The power factor allows us to quantify this phenomenon.

By definition, the power factor is expressed as the ratio between the active power (P), measured in watts (W), and the apparent power (S), measured in volt-amperes (VA):

$$PF = \cos \varphi = \frac{P}{S}$$

where:

P is the active power effectively consumed by the load,

S is the apparent power supplied by the source,

φ is the phase shift between the voltage and current waveforms.

A power factor close to 1 indicates that the current and voltage are almost in phase, meaning that the electrical energy is used efficiently with minimal losses. On the other hand, a low power factor reflects a large amount of reactive power, which increases current in the network, causes additional heating in cables and transformers, and reduces the overall efficiency of the electrical system.

In power electronics, the concept of power factor becomes even more important because most converters, such as rectifiers and inverters, generate non-sinusoidal currents that contain harmonics. In this case, the power factor is not only determined by the phase shift, but also by the distortion of the current waveform. Therefore, improving the power factor in power electronic systems often requires the use of filtering techniques, PWM control strategies, or specific correction circuits such as Power Factor Correction (PFC) stages.

Overall, maintaining a high-power factor is essential to guarantee efficient energy use, reduce operating costs, and ensure the stability and reliability of the electrical network.

Flow rate on an inductive load

The schematic diagram of the circuit is shown in the figure below.

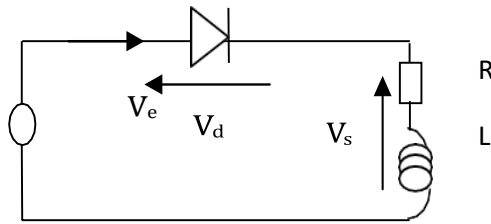


Figure II.5: Single-phase rectifier circuit supplying an

inductive load As long as the diode is conducting, we have $V_s = V_e$,

V_s can therefore become negative. If D is passable then

$$V_s = R \cdot i + L \frac{di}{dt} = V_e \quad \text{so} \quad R \cdot i = V_e - L \frac{di}{dt} \quad i = (V_e - L \frac{di}{dt})/R \quad (\text{II.7})$$

Diode D remains on as long as the current through it remains positive, i.e. as long as

$$V_e > L \frac{di}{dt}$$

The diode blocks at the moment t_2 when the current wants to become negative, i.e. at the moment

when :

$$V_e = L \frac{di}{dt} \quad (\text{II.8})$$

The current is maximum at the time t_1 such that $\frac{di}{dt} = 0$

The current, is the solution of the following differential equation:

$$\frac{L}{R} \frac{di}{dt} + i = \frac{V_m}{R} \cdot \sin(\omega t) \quad (\text{II.9})$$

The solution to this equation is of the form :

$$i = \frac{V_m}{Z} (\sin(\omega t + \varphi) - e^{-\frac{t}{\tau}} \cdot \sin \varphi) \quad (\text{II.10})$$

$$Z = \sqrt{R^2 + (L\omega)^2},$$

$$\varphi = \arctan\left(\frac{L\omega}{R}\right), \tau = \frac{L}{R} \quad (\text{II.11})$$

- **Definition of the average value**

$$\begin{aligned} V_{s_{moy}} &= \frac{1}{2\pi} \int_0^{\theta_{ext}} V_m \sin(\omega t) d\omega t \\ &= \frac{V_m}{2\pi} [-\cos(\omega t)]_0^{\theta_{ext}} \\ &= \frac{V_m}{2\pi} [1 - \cos(\theta_{ext})] \end{aligned} \quad (\text{II.12})$$

RMS voltage across the load.

$$\begin{aligned} V_{s_{eff}}^2 &= \frac{1}{T} \int_0^T V_s^2(t) dt \\ &= \frac{1}{T} \int_0^{\theta_{ext}} (V_m \sin(\omega t))^2 d\omega t \\ &= \frac{V_m^2}{4\pi} [\theta_{ext} - \frac{1}{2} \sin(2 \cdot \theta_{ext})] \end{aligned} \quad (\text{II.13})$$

- **Average current in the load**

$$i_{s_{moy}} = \frac{1}{2\pi} \int_0^{\theta_{ext}} \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t + \varphi) - \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\varphi) e^{-\frac{R}{L\omega} \omega t} d\omega t \quad (\text{II.14})$$

- **RMS current in the load**

$$i_{s_{eff}} = \frac{1}{2\pi} \int_0^{\theta_{ext}} \left[\frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t + \varphi) - \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\varphi) e^{-\frac{R}{L\omega} \omega t} \right]^2 d\omega t \quad (\text{II.15})$$

This operation is illustrated by the following timelines:

Chronograms:

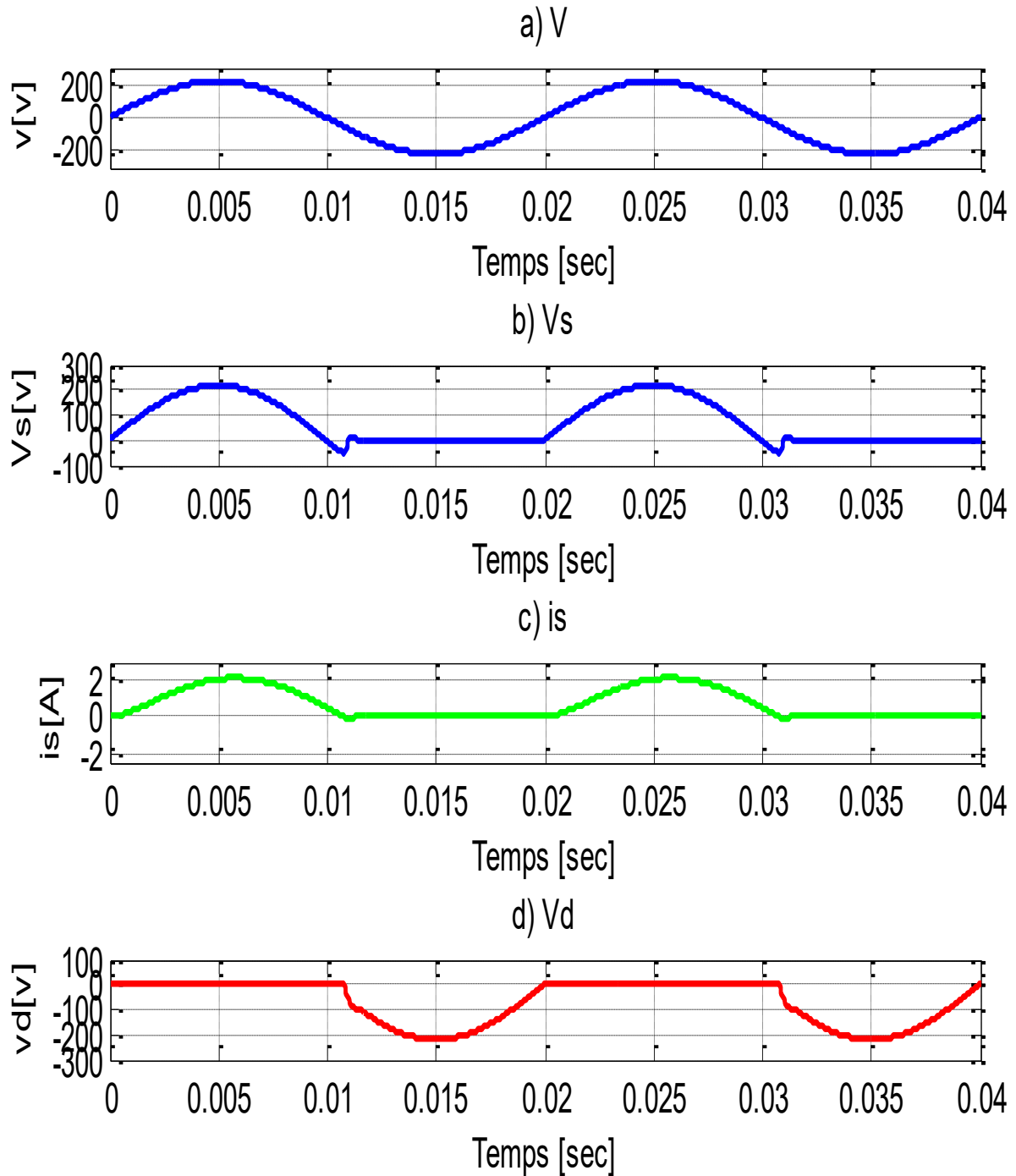


Figure II.6: Voltage and current diagrams for an RL load

II.2.3 Full wave rectifier (PD2)

This type of bridge, also known as the ‘Graetz Bridge’, is the classic structure of

single-phase rectification, consisting of four diodes connected in pairs in reverse, and is supplied with an AC voltage.

- **Flow rate on resistive load:**

The corresponding electrical diagram, still on a resistive load, is shown in Figure II.7, assuming that the four diodes are identical and perfect.

The input voltage $V_e(t)$ is then sinusoidal and given by: $V_e(t) = V_m \sin(\omega t)$

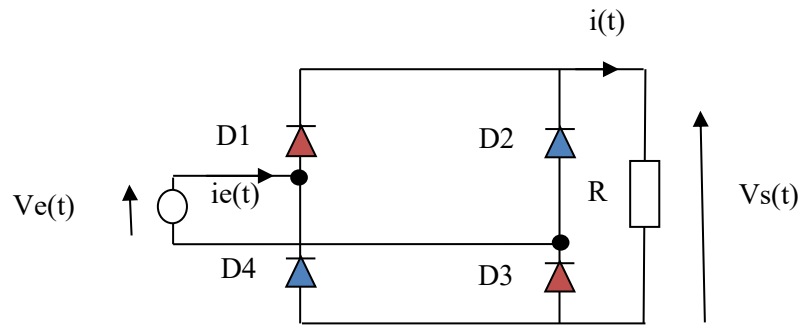


Figure II.7: PD2 rectifier.

When the voltage $V_e(t)$ is positive in the range $[0 \pi]$ diodes D1 and D3 are conducting. However, in the other range $[\pi 2\pi]$ diodes D2 and D4 conduct.

Figure II.8 shows the only two possible configurations of the diode quadrupole and the sign of the resulting output voltage.

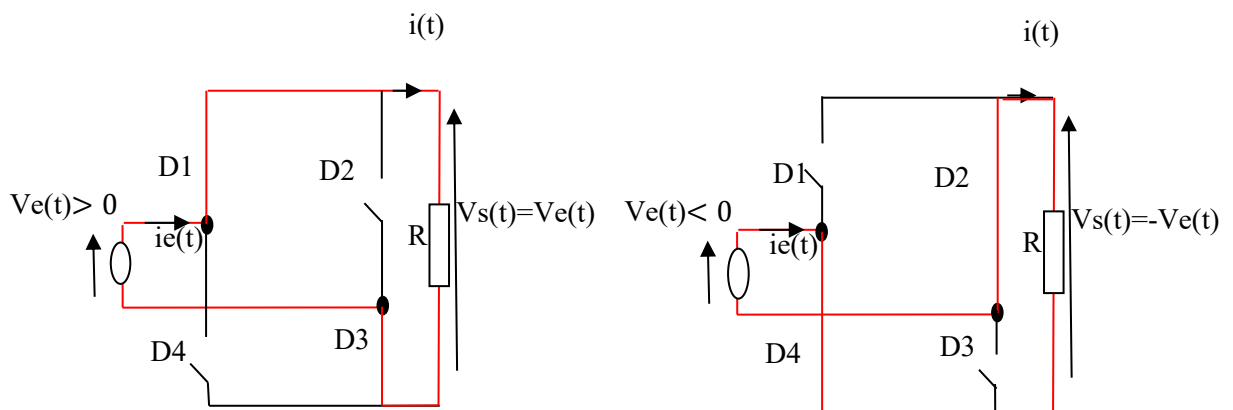


Figure II.8: Diode conduction in the Graetz Bridge

Chronograms:

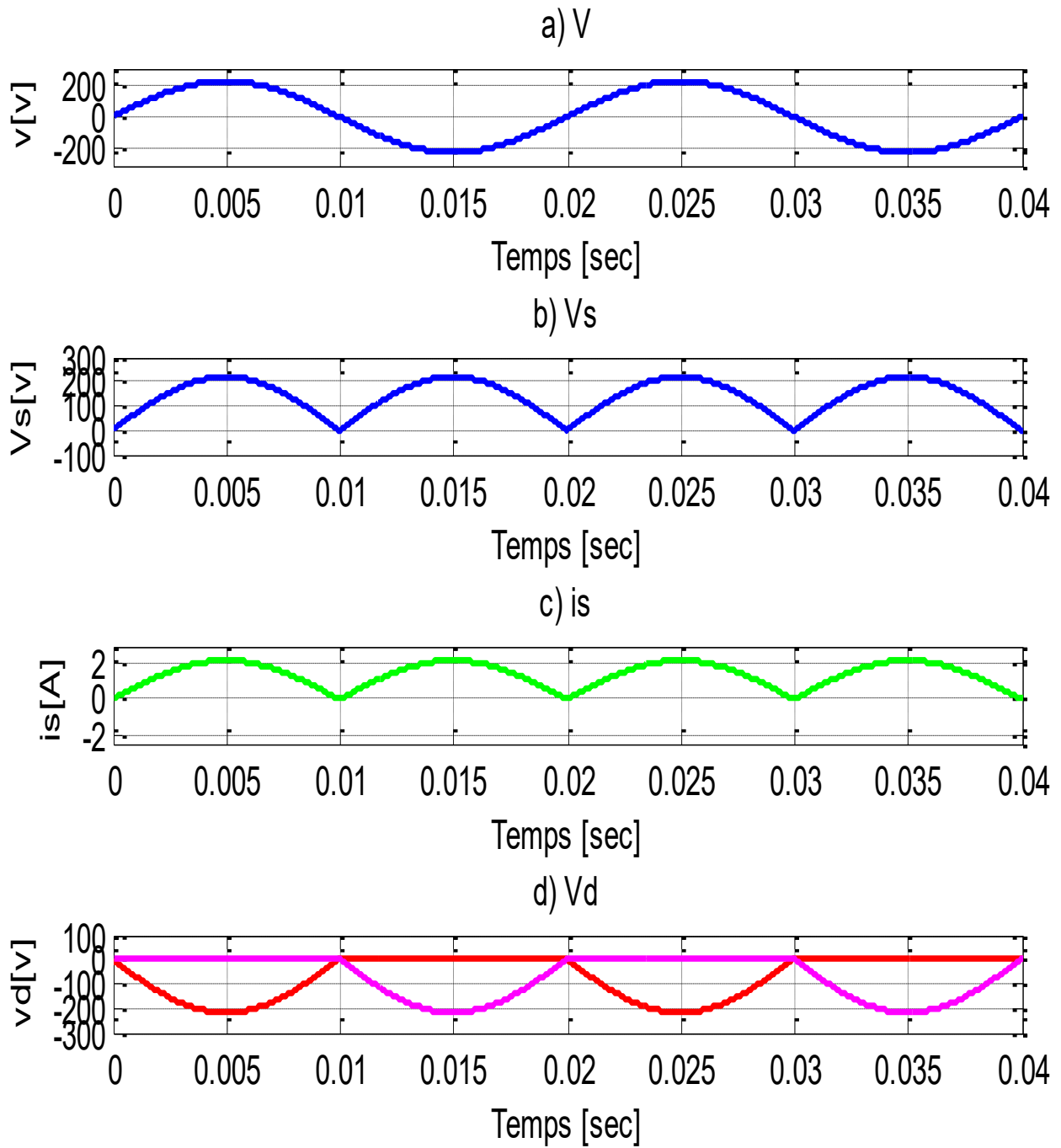


Figure II.9: Voltage and current diagrams for a load R(PD2)

- **Average value of the rectified voltage**

The expression for the average value of the rectified voltage is given by:

$$\begin{aligned}
 V_{s_{moy}} &= \frac{1}{2\pi} \int_0^T V_m \sin(\omega t) d\omega t = \frac{2 \cdot V_m}{2 \cdot \pi} [-\cos(\omega t)]_0^\pi = \frac{V_m}{\pi} [1 + 1] \\
 &= \frac{2 \cdot V_m}{\pi} \\
 V_{s_{moy}} &= \frac{2\sqrt{2}}{\pi} \cdot V \approx 0.9 * V
 \end{aligned} \tag{II.16}$$

- **The average value of the output current :**

$$i_{s_{moy}} = \frac{2 \cdot V_m}{\pi \cdot R} = \frac{2\sqrt{2}}{\pi} \cdot \frac{V}{R} \approx 0.9 * \frac{V}{R} \tag{II.17}$$

- **The rms value of the voltage across the load**

$$\begin{aligned}
 V_{s_{eff}}^2 &= \frac{1}{T} \int_0^T V_s^2(t) dt = \frac{2}{T} \int_0^\pi (V_m \sin(\omega t))^2 dt \\
 V_{s_{eff}}^2 &= \frac{2}{2\pi} \int_0^\pi (V_m)^2 \cdot \sin^2(\theta) d\theta = \frac{2(V_m)^2}{2\pi} \int_0^\pi \frac{1}{2} (1 - \cos(2\theta)) d\theta \\
 &= \frac{(V_m)^2}{2}
 \end{aligned}$$

$$\text{Thus: } V_{s_{eff}} = \frac{V_m}{\sqrt{2}} \tag{II.18}$$

$$i_{s_{eff}} = \frac{V_{s_{eff}}}{R} = \frac{V_m}{\sqrt{2} \cdot R} \tag{II.19}$$

- **Form factor**

$$F = \frac{V_{s_{eff}}}{V_{s_{moy}}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = \frac{\pi}{2\sqrt{2}} \approx 1.11 \tag{II.20}$$

- **Load RL**

The schematic diagram of the full-wave rectifier, with a diode bridge and an RL load, is shown in the figure below.

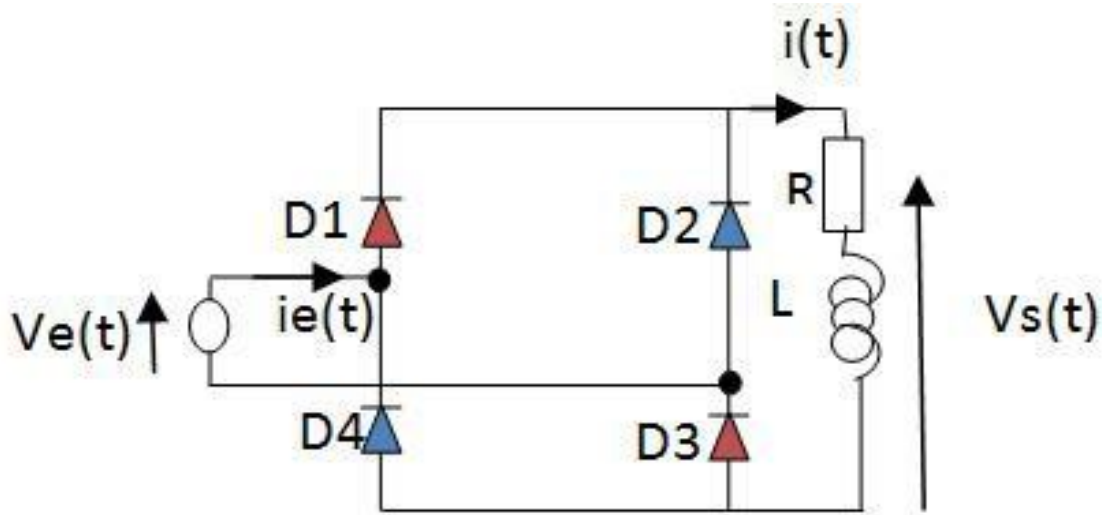


Figure II.10: Full-wave rectifier feeding an RL load.

The current i follows the same relationship as in the previous paragraph, i.e.:

$$i = \frac{V_m}{Z} (\sin(\omega t + \varphi) - e^{-\frac{t}{\tau}} \sin \varphi). \quad (\text{II.21})$$

When the voltage $V_e(t)$ is positive in the range $[0 \pi + \alpha]$ diodes D1 and D3 are conducting. However, in the other range $[\pi + \alpha \pi 2\pi]$ diodes D2 and D4 conduct.

Figure II.10 shows the only two possible configurations of the diode quadrupole and the sign of the resulting output voltage.

Chronograms

$R=100 \text{ Ohms}$, $L=0.4 \text{ H}$.

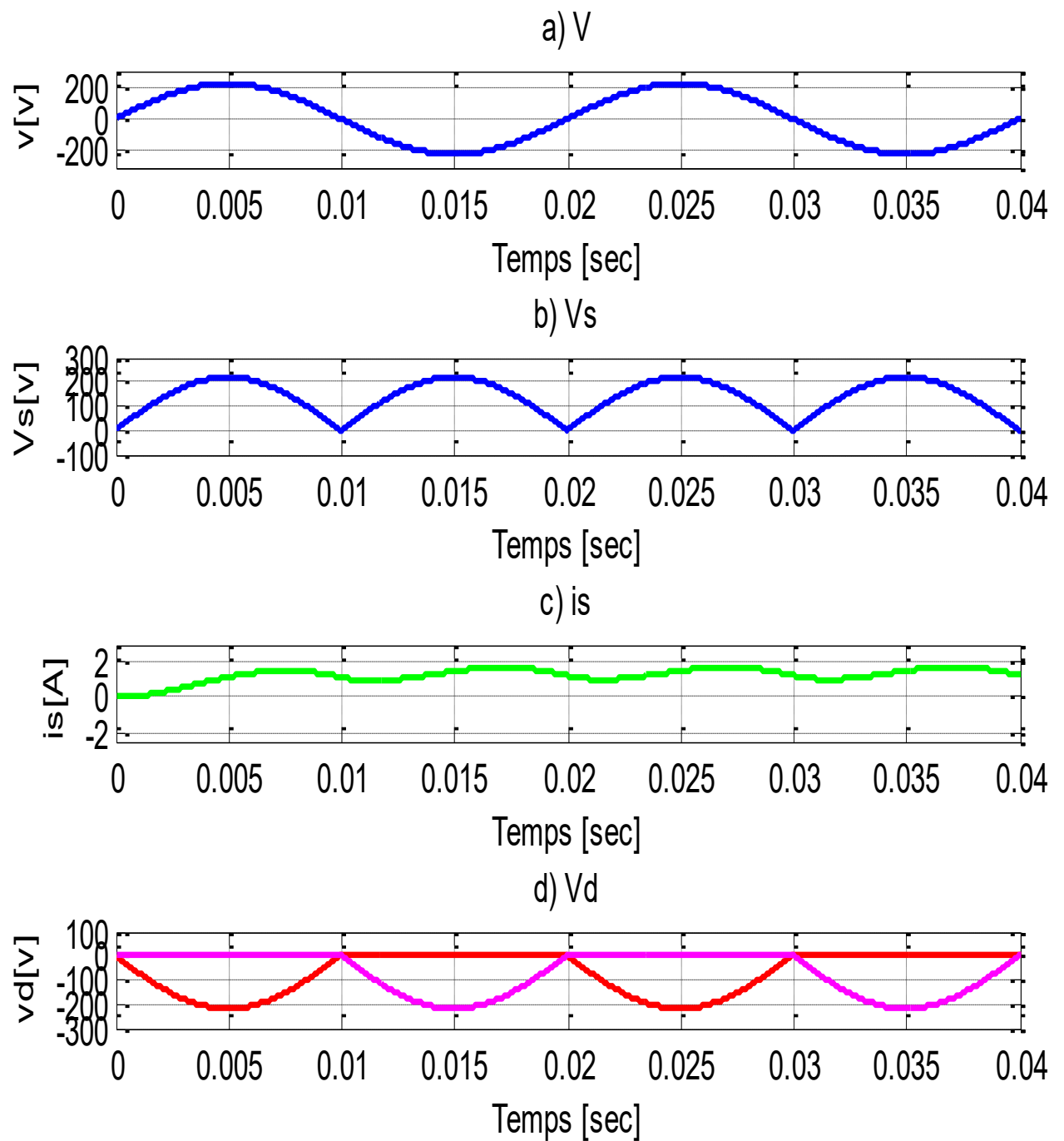


Figure II.11: Voltage and current diagrams for a RL(PD2) load

- **RLE load**

The schematic diagram is shown in the figure below.

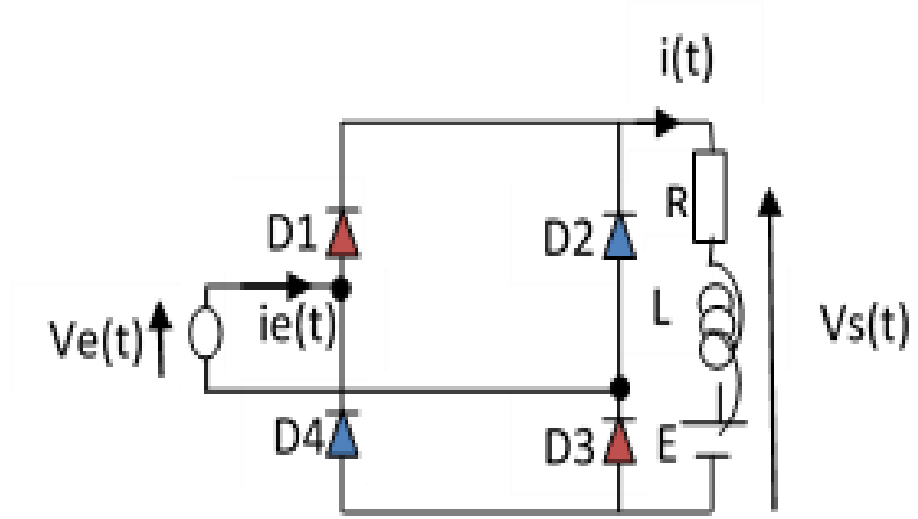


Figure II.12: Single-phase rectifier circuit full-wave feeding an RLE load

E: The back emf is continuous and must satisfy the condition: $E < V_m$.

The various operating phases of the assembly are then described in the following table:

Intervals	V_e	Passing diode	the current	The output voltage V_s
$0 < t < t_1$	$-E < V_e < E$	/	no	$V_s = E$
$t_1 < t < t_2$	$V_e > E$	D_1, D_3	$\frac{L}{R} \frac{di}{dt} + i = \frac{V_m}{R} \sin(\omega t) + \frac{E}{R}$	$V_s = V_e$
$t_2 < t < t_3$	$-E < V_e < E$	/	nul	$V_s = E$
$t_3 < t < t_4$	$V_e < -E$	D_2, D_4	$\frac{L}{R} \frac{di}{dt} + i = \frac{V_m}{R} \sin(\omega t) + \frac{E}{R}$	$V_s = -V_e$

II.2.4 Three-phase full-wave rectification

The three-phase diode rectifier (P3) consists of three diodes connected in a common cathode (called a positive plus switch), each connected to one phase of the secondary of a three-phase transformer.

The load is placed between the cathode and the neutral. The corresponding electrical diagram is shown in Figure II.13.

The three-phase voltage system used must have a neutral point (N) and voltages must comply with the following expressions:

$$\begin{cases} V_1(t) = V \cdot \sqrt{2} \sin(\omega t) \\ V_2(t) = V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3}) \\ V_3(t) = V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3}) \end{cases} \quad (\text{II.22})$$

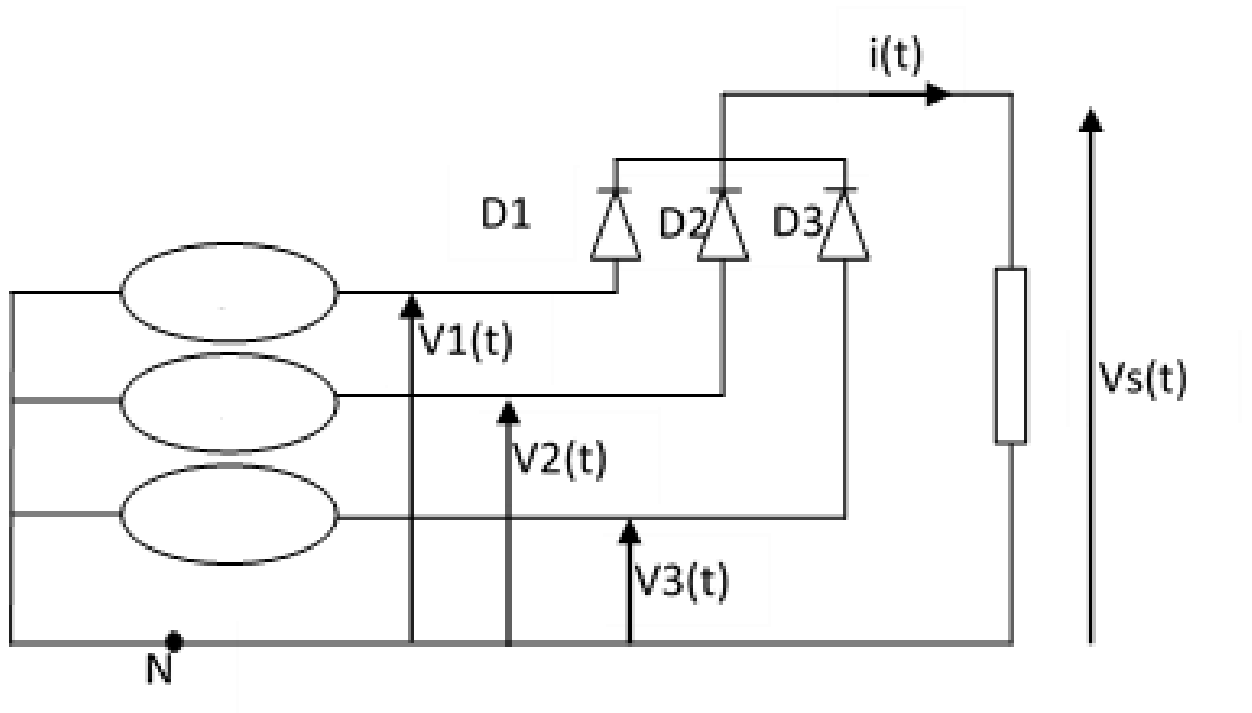


Figure II.13: Three-phase full-wave rectifier (P3)

II.2.4.1 Analysis of the operation

In this case, only the diode with the highest anode potential can conduct, and the others are blocked. The conduction intervals are as follows:

Interval	Conductive diode	Blocked diodes	Output voltage $V_s(t)$	Terminal voltage of D1
$\left[\frac{\pi}{6} \quad \frac{5\pi}{6}\right]$	D_1	D_2, D_3	$V_s(t) = V_1(t) = V \cdot \sqrt{2} \sin(\omega t)$	0
$\left[\frac{5\pi}{6} \quad \frac{3\pi}{2}\right]$	D_2	D_1, D_3	$V_s(t) = V_2(t) = V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$	$V_1(t) - V_2(t)$
$\left[\frac{3\pi}{2} \quad \frac{13\pi}{6}\right]$	D_3	D_1, D_2	$V_s(t) = V_3(t) = V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$	$V_1(t) - V_3(t)$

The advantage here is that 3-phase rectification circuits can power various industrial applications, such as motor control and battery charging, which require more power than a single-phase rectifier can provide.

A 3-phase power supply expands on this concept by combining three AC voltages of the same frequency and amplitude, each referred to as a phase. These three phases are offset by 120 electrical degrees, resulting in a phase rotation of $360^\circ \div 3 = 120^\circ$.

From the waveforms above for a resistive load, we observe that in a half-wave rectifier, each diode conducts for one-third of each cycle. As a result, the output waveform has a frequency three times that of the AC supply. This means that within a single cycle, there are three voltage peaks.

By increasing the number of phases from single-phase to three-phase, the rectification process is improved, leading to a smoother DC output voltage with reduced ripple.

Chronograms

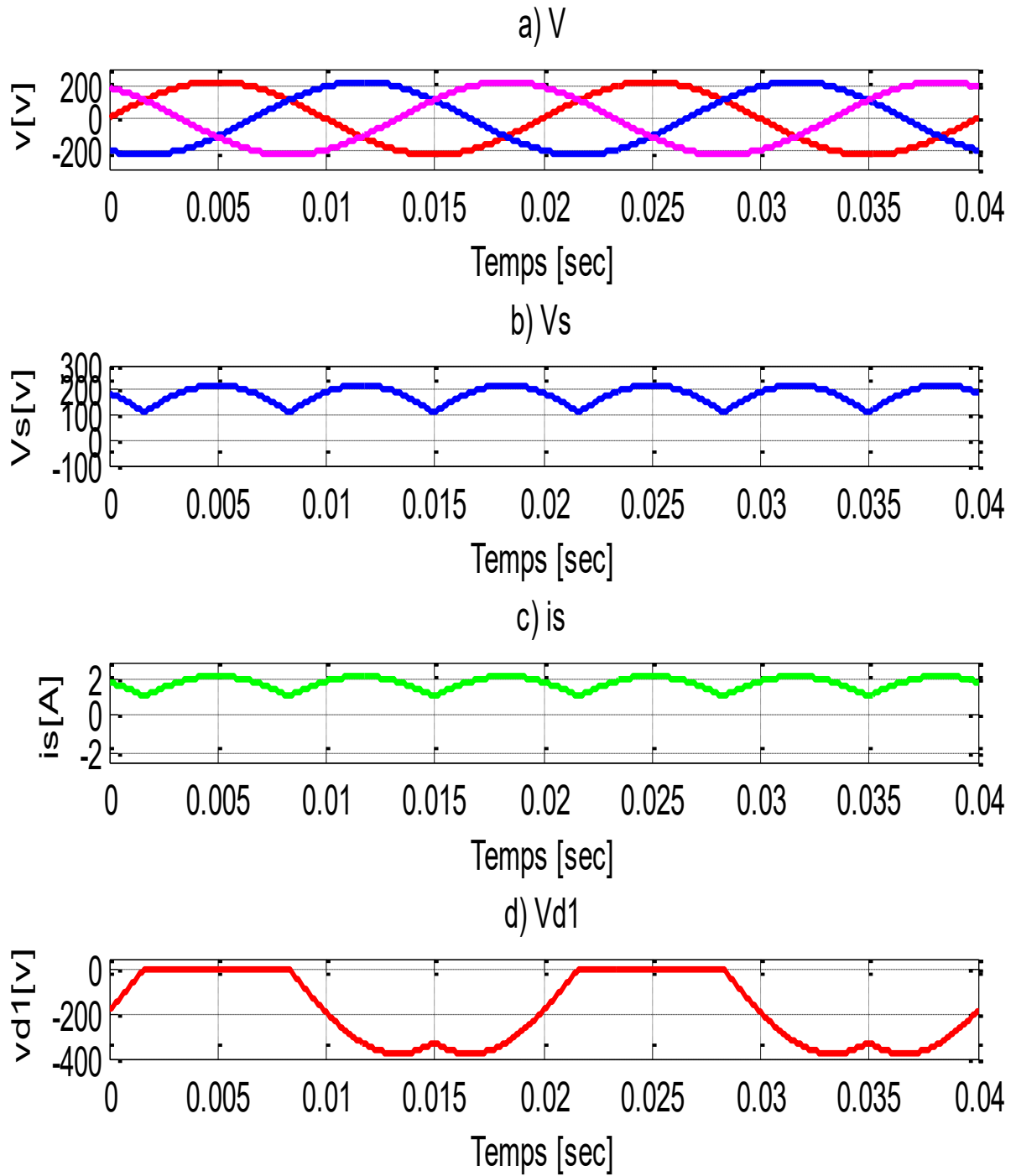


Figure II.14: Voltage and current diagrams for a load R(P3)

- **Average value of the rectified voltage:**

$$V_{s_{moy}} = \frac{1}{T} \int_0^T V_s(t) \cdot dt = \frac{1}{\left(\frac{5\pi}{6} - \frac{\pi}{6}\right)} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} V_m \cdot \sin(\theta) \cdot d\theta = \frac{3V_m}{2\pi} [-\cos(\theta)]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{3V_m}{2\pi} \left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right] = \frac{3\sqrt{3}V_m}{2\pi}$$

$$= \frac{3\sqrt{6}V}{2\pi} \approx 1.17 * V \quad (\text{II.23})$$

- **The average value of the output current :**

The average value of the output current of the purely **resistive** load is calculated by the following equation:

$$i_{s_{moy}} = \frac{3\sqrt{6}V}{2\pi * R} \approx 1.17 * \frac{V}{R} \quad (\text{II.24})$$

- **RMS value of the rectified voltage:**

$$V_{s_{eff}}^2 = \frac{1}{T} \int_0^T V_s^2(t) dt = \frac{3}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (V_m \sin(wt))^2 dt$$

$$V_{s_{eff}}^2 = \frac{3}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (V_m)^2 \cdot \sin^2(\theta) d\theta = \frac{3(V_m)^2}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2} (1 - \cos(2\theta)) d\theta \quad (\text{II.25})$$

$$V_{s_{eff}} = 0.84 * V_m \approx 1.18 * V$$

- **The form factor**

$$F = \frac{V_{s_{eff}}}{V_{s_{moy}}} = \frac{0.84 * V_m}{0.83 * V_m} \approx 1.02 \quad (\text{II.26})$$

II.2.5 Three-phase full wave rectifier PD3

The three-phase full-wave rectifier circuit (PD3) is considered as two three-phase circuits and therefore consists of six diodes, connected two by two, one with a common anode, the other with a common cathode. The corresponding electrical diagram, always on a resistive load, is shown in figure II.15.

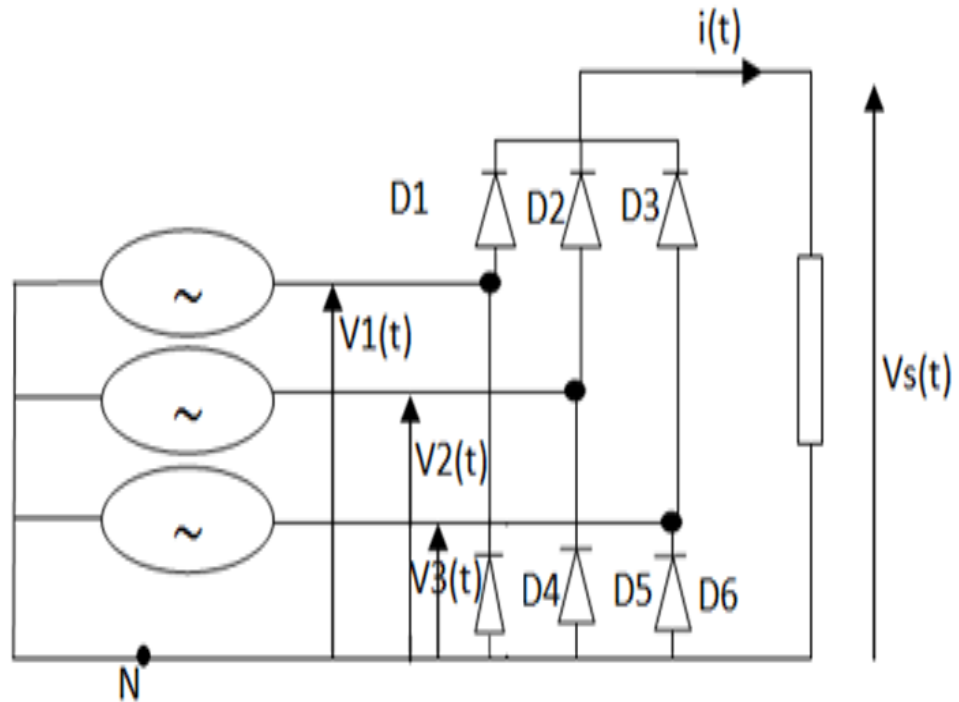


Figure II.15: 3-phase full-wave rectifier Resistive load

The three-phase voltage system used has voltages according to the following expressions:

$$\begin{cases} V_1(t) = V \cdot \sqrt{2} \sin(\omega t) \\ V_2(t) = V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3}) \\ V_3(t) = V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3}) \end{cases} \quad (\text{II.27})$$

Functional analysis

The rule for determining the pass diodes is the same as for the three-phase full wave circuit:

- For the common-cathode switch, the diode with the highest anode potential conducts, hence the name "+ positive";
- For the common anode switch, the diode with the lowest negative cathode potential conducts, hence the name "- negative". Thus, we have:

if:

- $V_1(t) > V_2(t) > V_3(t)$, therefore D_1, D_6 leads and

$$V_s(t) = V_1(t) - V_3(t)$$

$$= V \cdot \sqrt{2} \sin(\omega t) - V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$$
- $V_1(t) > V_3(t) > V_2(t)$, therefore D_1, D_5 leads and

$$V_s(t) = V_1(t) - V_2(t)$$

$$= V \cdot \sqrt{2} \sin(\omega t) - V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$$
- $V_2(t) > V_1(t) > V_3(t)$, therefore D_2, D_6 leads and

$$V_s(t) = V_2(t) - V_3(t)$$

$$= V \cdot \sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$$
- $V_2(t) > V_3(t) > V_1(t)$, therefore D_2, D_4 leads and

$$V_s(t) = V_2(t) - V_1(t)$$

$$= V \cdot \sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t)$$
- $V_3(t) > V_1(t) > V_2(t)$, therefore D_3, D_5 leads and

$$V_s(t) = V_3(t) - V_2(t)$$

$$= V \cdot \sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$$
- $V_3(t) > V_2(t) > V_1(t)$, therefore D_3, D_4 leads and

$$V_s(t) = V_3(t) - V_1(t)$$

$$= V \cdot \sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t)$$

Chronograms

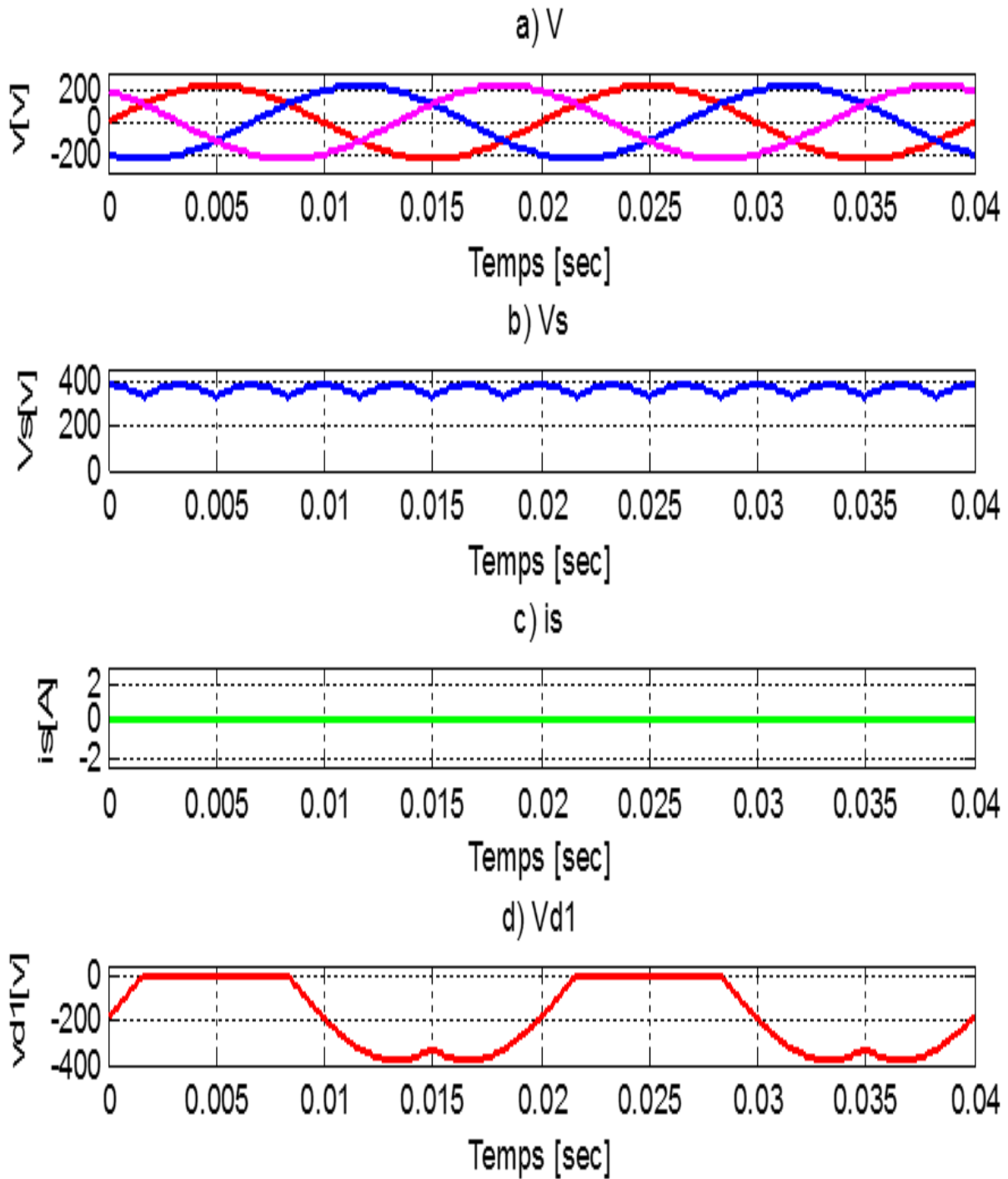


Figure II.16: Voltage and current graphs for a load R(PD3)

- **Average value of the rectified voltage:**

For the expression of the average value of the output voltage, the integral is simply

calculated over the interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$:

- $V_1(t) > V_3(t) > V_2(t)$, therefore D_1, D_5 leads and

$$V_s(t) = V_1(t) - V_2(t)$$

$$= V_m \left[\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{3}\right) \right]$$

$$V_{s_{moy}} = \frac{1}{T} \int_0^T V_s(t) \cdot dt = \frac{1}{\left(\frac{\pi}{2} - \frac{\pi}{6}\right)} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} V_m \cdot \left[\sin(\theta) - \sin\left(\theta - \frac{2\pi}{3}\right) \right] d\theta =$$

$$\frac{6V_m}{2\pi} \left[-\cos(\theta) + \cos\left(\theta - \frac{2\pi}{3}\right) \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{6V_m}{2\pi} \left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right] = \frac{6\sqrt{6}V}{2\pi} \quad (\text{II.28})$$

$$V_{s_{moy}} = \frac{3V\sqrt{6}}{\pi}$$

- **The rms voltage value across the load**

$$\begin{aligned} V_{s_{eff}}^2 &= \frac{1}{T} \int_0^T V_s^2(t) dt \\ &= \frac{6}{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (V_m [\sin(\theta) - \sin(\theta - \frac{2\pi}{3})])^2 d\theta \end{aligned}$$

SO :

$$V_{s_{eff}} = \frac{\sqrt{3}V_m}{\sqrt{2}} \sqrt{1 + \frac{\sin \frac{2\pi}{6}}{\frac{2\pi}{6}}} \approx 1.662 * V_m \quad (\text{II.29})$$

- **The form factor is:**

$$F = \frac{V_{s_{eff}}}{V_{s_{moy}}} = \frac{1.662 * V_m}{\frac{3\sqrt{6}V_m}{\pi}} \approx 1.005 \quad (\text{II.30})$$

II.3 Controlled rectification (THYRISTOR)

Rectification is the process of turning AC electricity into DC power using a regulated rectifier. The output voltage and power can be accurately regulated by varying the semiconductor devices' switching instants. Controlled rectifiers are helpful for DC motor speed control because of this feature.

Furthermore, certain controlled rectifiers have the ability to function in inversion mode, which involves converting DC electricity back into AC. In order to allow energy to be returned back into the supply system, regenerative braking in DC motors requires this functionality.

We note θ_0 the reference-firing angle of a thyristor. The following figure, Figure II.17, shows the controlled rectifiers.

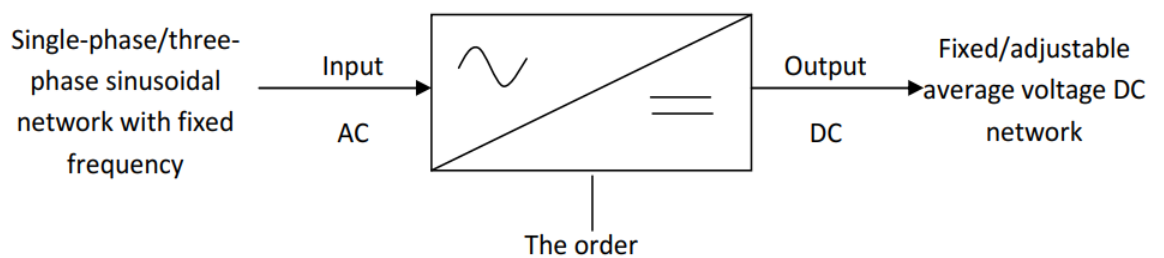


Figure II.17: Representation of controlled rectifiers

II.3.1 Single-phase full-wave rectification

* Flow on a resistible load

The schematic diagram of such a converter is shown in the figure below.

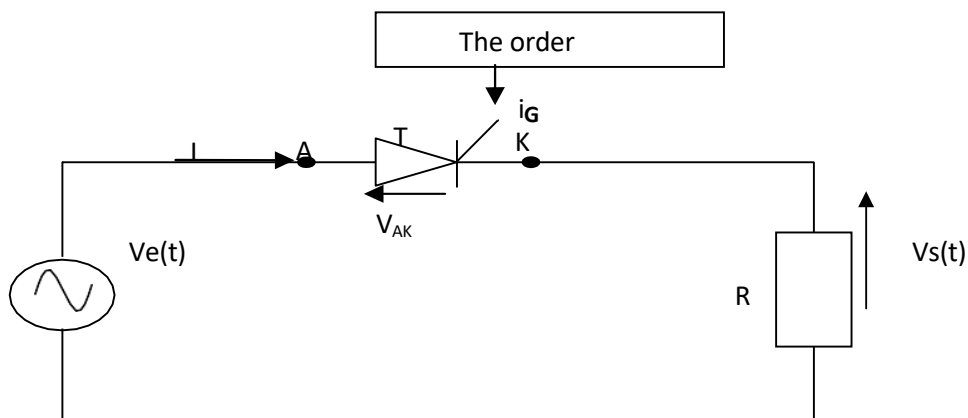


Figure II.18: Controlled half-wave circuit

Chronogram Simple alternation-load rectification R

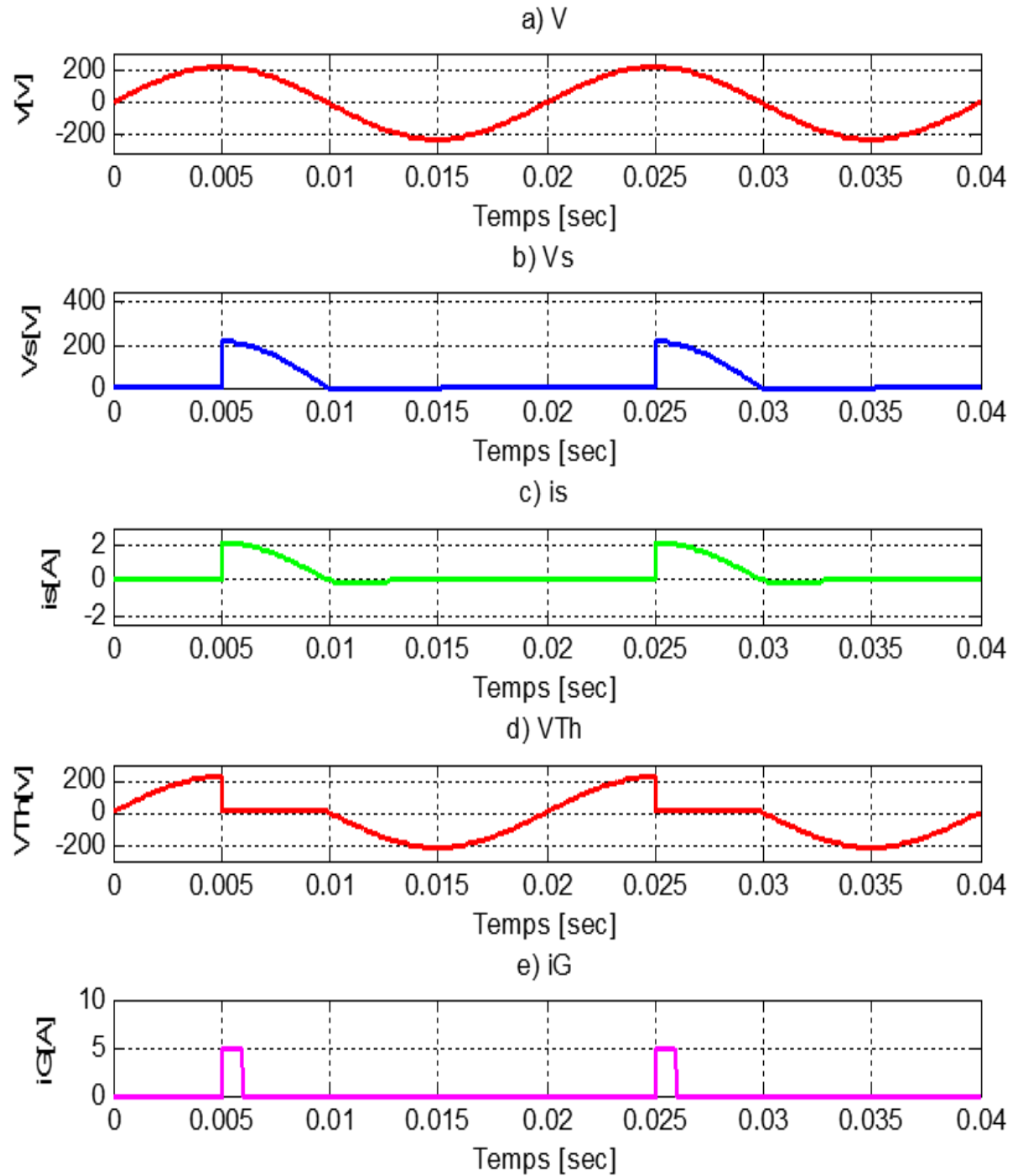


Figure II.19: Voltage and current diagrams

for a load R(P3)

The thyristor leads in the interval $[\theta_0 \pi]$

θ_0 :is called the delay angle.

The supply voltage is:

$$V_e(t) = V \cdot \sqrt{2} \sin(\omega t)$$

If we consider the perfect thyristor, the voltage across R is:

- For:

$$\theta_0 \pm 2k\pi < \omega t < \pi \pm 2k\pi$$

$$k=0, 1, 2, 3, \dots$$

$$V_s(t) = V_e(t) = V \cdot \sqrt{2} * \sin(\omega t)$$

$$V_{AK} = 0$$

- For (ωt) elsewhere

$$V_s(t) = 0$$

$$V_{AK} = V_e(t)$$

$$= V \cdot \sqrt{2} * \sin(\omega t)$$

By varying the angle θ_0 By varying the angle, the mean and rms voltage across the resistor is varied, which allows the current through the resistor to be varied as well.

- **Average value of the rectified voltage:**

$$\begin{aligned} V_{s_moy} &= \frac{1}{T} \int_0^T V_s(t) \cdot dt \\ &= \frac{1}{2\pi} \int_{\theta_0}^{\pi} V_m \cdot \sin(\theta) \cdot d\theta \\ &= \frac{V_m}{2\pi} [-\cos(\theta)]_{\theta_0}^{\pi} \\ &= \frac{V\sqrt{2}}{2\pi} (1 + \cos(\theta_0)) \end{aligned} \tag{II.31}$$

And

$$\begin{aligned} i_{s_moy} &= \frac{V_{s_moy}}{R} \\ &= \frac{V\sqrt{2}}{2\pi R} (1 + \cos(\theta_0)) \end{aligned}$$

- **RMS value**

The RMS voltage is given by:

$$\begin{aligned}
 V_{eff}^2 &= \frac{1}{T} \int_{\theta_0}^{2\pi} V_s(t)^2 dt \\
 V_{eff}^2 &= \frac{1}{2\pi} \int_{\theta_0}^{\pi} V_s(t)^2 d\theta \\
 &= \frac{V_{max}^2}{2\pi} \int_{\theta_0}^{\pi} \sin^2 \theta \cdot d\theta \\
 V_{eff}^2 &= \frac{V_{max}^2}{4\pi} \int_{\theta_0}^{\pi} 1 - \cos 2\theta \cdot d\theta \\
 V_{eff}^2 &= \frac{V_{max}^2}{4\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_{\theta_0}^{\pi} = \frac{V_{max}^2}{4\pi} \cdot \left(\pi - \alpha + \frac{1}{2} \sin 2\theta_0 \right) \\
 V_{eff}^2 &= \frac{V_{max}^2}{4} \left(1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0 \right) \\
 V_{eff} &= \frac{E_{max}}{2} \cdot \sqrt{1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0} \quad (II.32)
 \end{aligned}$$

II.3.2 Full-wave controllable rectifier on a RL load

The schematic diagram of the circuit is shown in the figure below.

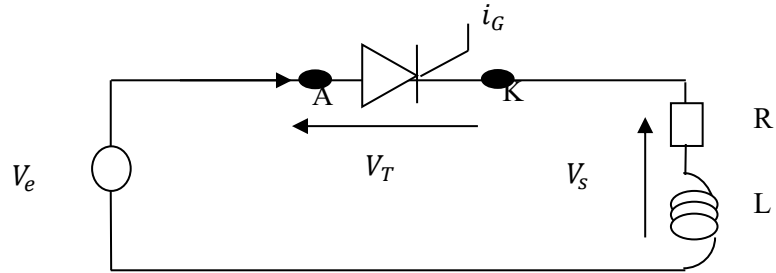


Figure II.20: Single-phase controllable rectifier assembly with RL load

- Before the control pulse is applied, the thyristor is blocked. Therefore $V_s(t) = 0$
- After the thyristor has fired $V_s(t) = V_e(t)$, and $\omega t_1 = \theta_0$ the thyristor conduction continues until θ_1 (blocking time) where the current is cancelled. So over a period of 2π the conduction is from θ_0 to θ_1

- for $\theta_0 < \theta < \theta_1$ Th unlocks $\theta = \theta_0$ we have :

$$L \frac{di_c(\theta)}{dt} + Ri_s(\theta) = V_{max} \sin \theta$$

The general expression

$i_c(\theta)$ is :

$$i_c(\theta) = \frac{V_{max}}{Z} \sin(\theta - \varphi) + K e^{-\frac{\theta}{\tau}}$$

With $\omega t = \theta$

$$Z = \sqrt{R^2 + (L\omega)^2}$$

$$\varphi = \arctan\left(\frac{L\omega}{R}\right)$$

$$\tau = \frac{L\omega}{R}$$

For $\theta = \theta_0$ we have $i_s(\alpha) = 0$

$$= \frac{V_{max}}{Z} \sin(\theta_0 - \varphi) + K e^{-\frac{\theta_0}{\tau}}$$

$$\text{then } K = -\frac{V_{max}}{Z} \sin(\theta_0 - \varphi) e^{-\frac{\theta_0}{\tau}}$$

$$\text{So : } i_s(\theta) = \frac{V_{max}}{Z} \left[\sin(\theta - \varphi) - (\sin(\theta_0 - \varphi) e^{-\left(\frac{\theta - \theta_0}{\tau}\right)}) \right]$$

The thyristor will stop conducting as soon as the current becomes zero. If θ_1 is the extinction angle, we have

$$\sin(\theta_1 - \varphi) - (\sin(\theta_0 - \varphi) e^{-\left(\frac{\theta_1 - \theta_0}{\tau}\right)}) = 0$$

This equation can be solved graphically.

- for $\theta_1 < \theta < 2\pi + \theta_0$ Th blocks because at $\theta = \pi$ the current cancels out: So : $i_s(\theta) = 0$ and $V_s(\theta) = 0$

- **Definition of the average value**

$$V_{smoy} = \frac{1}{2\pi} \int_{\theta_0}^{\theta_1} V_m \sin(\theta) d\theta$$

$$= \frac{V_m}{2\pi} [-\cos(\theta)]_{\theta_0}^{\theta_1}$$

$$= \frac{V_m}{2\pi} [\cos(\theta_0) - \cos(\theta_1)] \quad (\text{II.33})$$

Chronograms

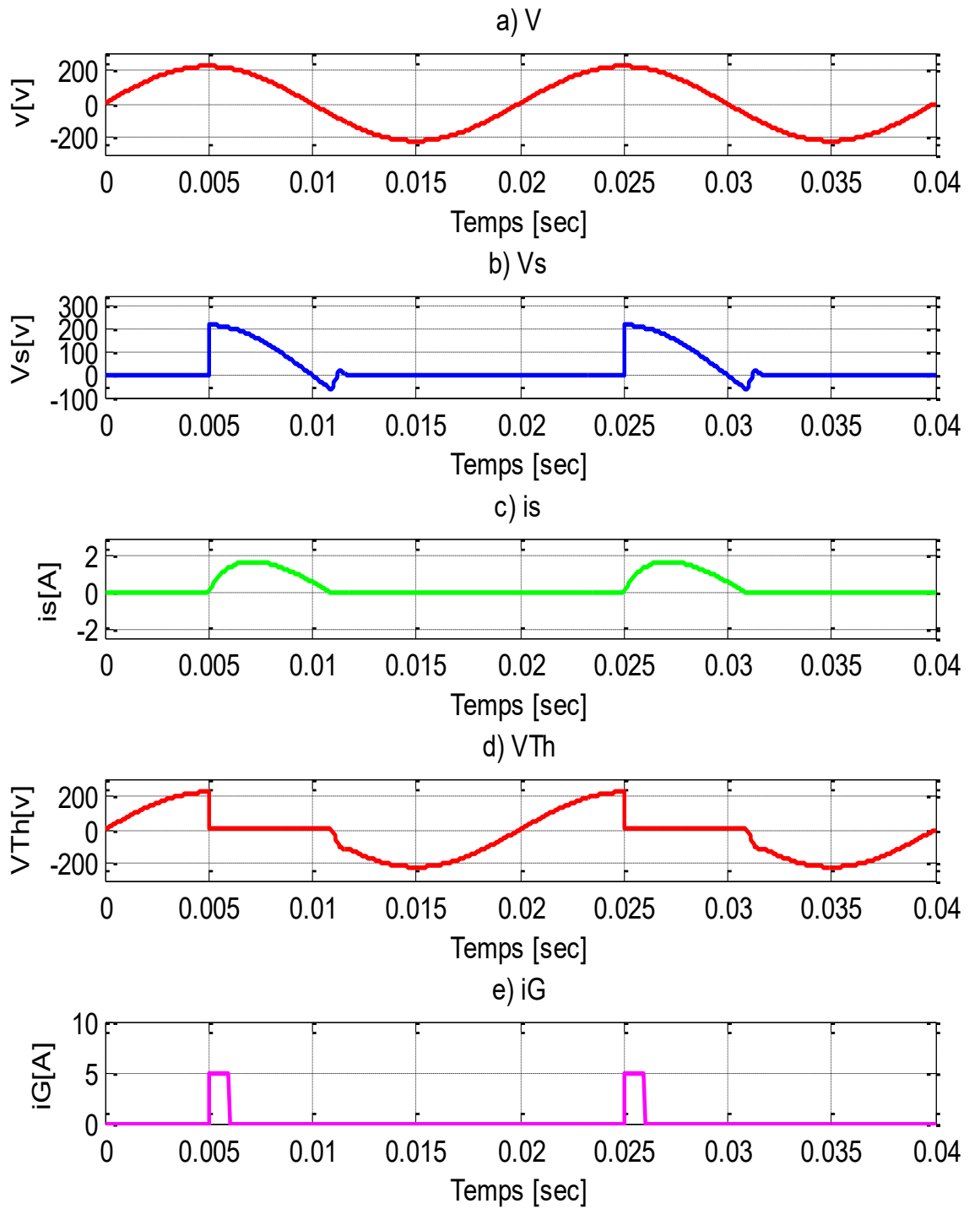


Figure II.21: Voltage and current diagrams for an RL load

II.3.3 Single-phase full-wave rectification

There are three types of full wave single phase rectifiers:

- the all-thyristor Graetz bridge (symmetrical).
- the mixed Graetz bridge (2 thyristors and 2 diodes).
- the midpoint circuit composed of two thyristors and requiring a midpoint transformer.

A full-wave controlled bridge rectifier circuit with a resistive load is depicted in Figure II.22. SCR pairs that are diagonally opposed to one another turn on and off together in this circuit. Similar to the full-wave center-tap circuit covered in Section 1, the circuit operates similarly. By altering the firing angle, the average DC output voltage can be regulated from zero to its maximum value. With a delay angle of α , the SCRs are controlled and fire in pairs. Figure II.23 illustrates how the voltage and current waveforms become full-wave.

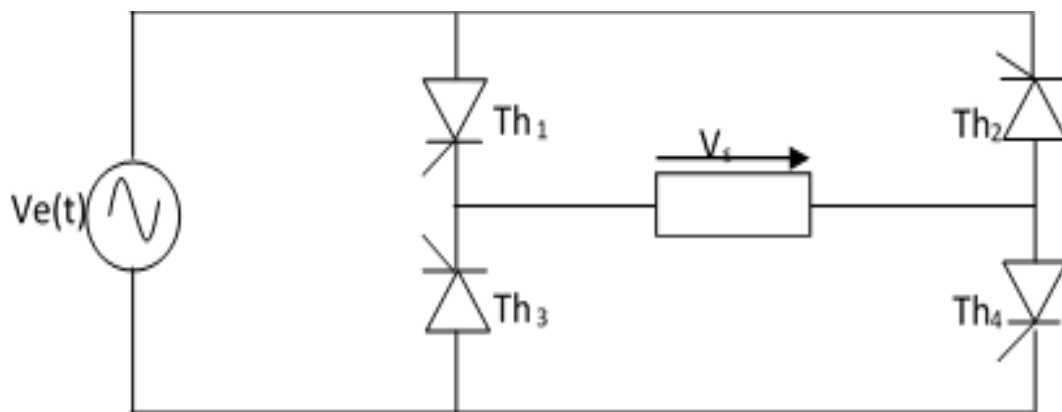


Figure II.22: All-thyristor full-wave rectifier circuit

Chronograms

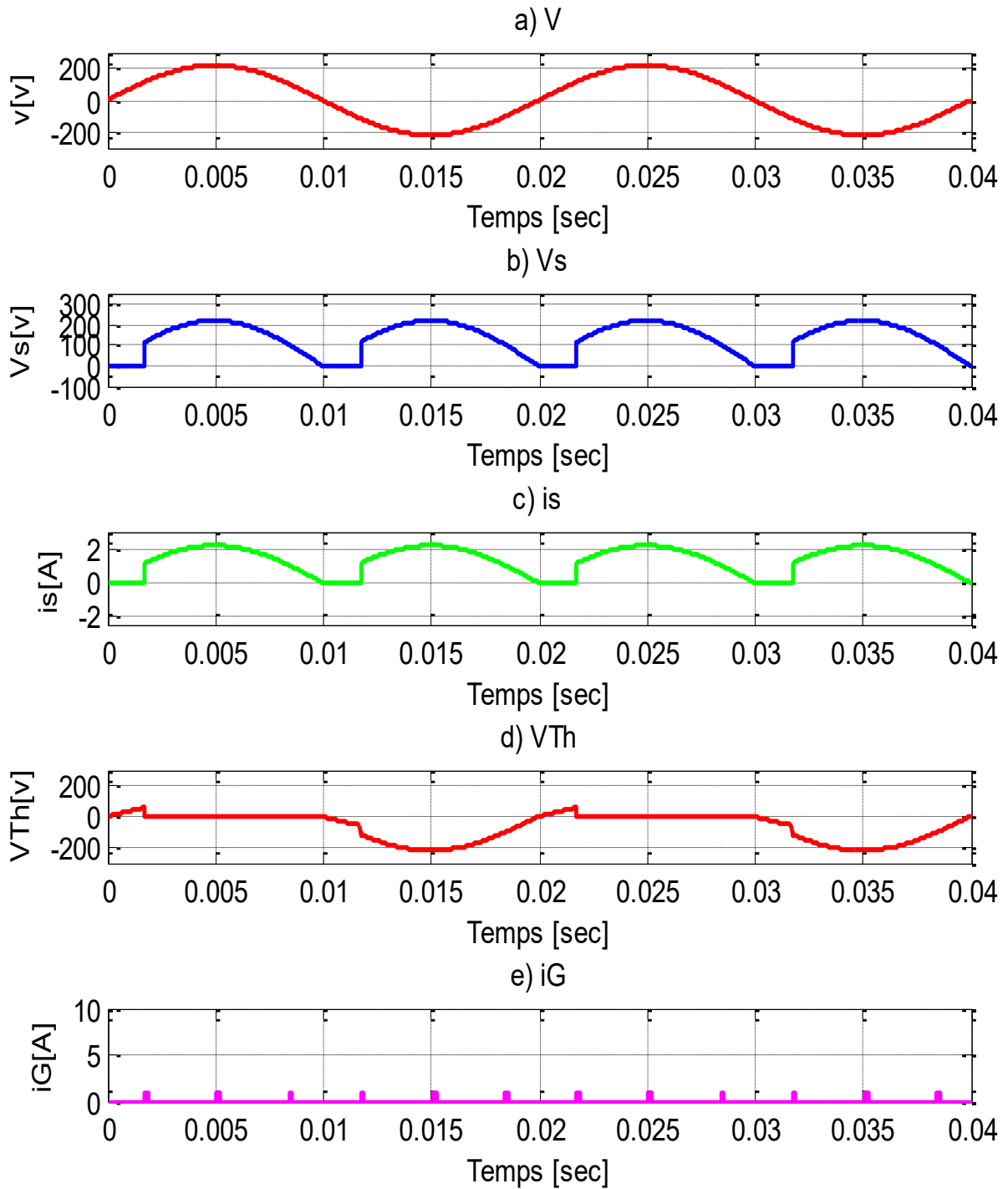


Figure II.23: Voltage and current diagrams for a load R

- **Average value of the rectified voltage**

We have :

$$Ve(t) = V_{max}\sin(\omega t)$$

$$Vs(t) = Ve(t)$$

$$\text{for } \omega t \in [\theta_0, \pi]$$

$$\text{and } Vs(t) = -Ve(t)$$

$$\text{for } \omega t \in [\pi + \theta_0, 2\pi]$$

The result is a rectified voltage of period π .

$$V_{moy} = \frac{2}{T} \int_{\theta_0}^{\pi} Vs(t) \cdot d\theta$$

$$V_{moy} = \frac{1}{\pi} \int_{\theta_0}^{\pi} Ve(t) \cdot d\theta$$

$$= \frac{V_{max}}{\pi} \int_{\theta_0}^{\pi} \sin\theta \cdot d\theta$$

(II.34)

$$V_{moy} = \frac{V_{max}}{\pi} (1 + \cos\theta_0)$$

- **Calculation of the RMS value**

The RMS voltage is given by :

$$V_{eff}^2 = \frac{1}{\pi} \int_{\theta_0}^{\pi} Vs(t)^2 d\theta$$

$$V_{eff}^2 = \frac{1}{\pi} \int_{\theta_0}^{\pi} Ve(t)^2 d\theta$$

$$V_{eff}^2 = \frac{V_{max}^2}{\pi} \int_{\theta_0}^{\pi} \sin^2\theta \cdot d\theta$$

$$V_{eff}^2 = \frac{V_{max}^2}{2\pi} \int_{\theta_0}^{\pi} 1 - \cos 2\theta \cdot d\theta$$

$$V_{eff}^2 = \frac{V_{max}^2}{2\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_{\theta_0}^{\pi}$$

$$V_{eff}^2 = \frac{V_{max}^2}{2\pi} \cdot \left(\pi - \theta_0 + \frac{1}{2} \sin 2\theta_0 \right)$$

(II.35)

$$V_{eff}^2 = \frac{V_{max}^2}{2} \left(1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0 \right)$$

$$V_{eff} = \frac{V_{max}}{\sqrt{2}} \cdot \sqrt{1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0}$$

II.3.4 Three-phase controlled rectification

II.3.4.1 Three-phase full-wave rectifier (P3) controlled:

Figure. II.24 displays the three-phase half-wave regulated rectifier. The load for the half-wave 3-phase uncontrolled diode rectifier is linked between the supply neutral and the converter positive terminal, which is the cathodes of all thyristors. In relation to the neutral, the diode with the highest voltage conducts.

The previously conducting device becomes reverse-biased and naturally commutated when the load current is transferred to the diode with the highest voltage. Figure. II.25 displays the waveforms for the supply voltage, output voltage, and load current.

As with other controlled rectifiers, the diodes are replaced by thyristors.

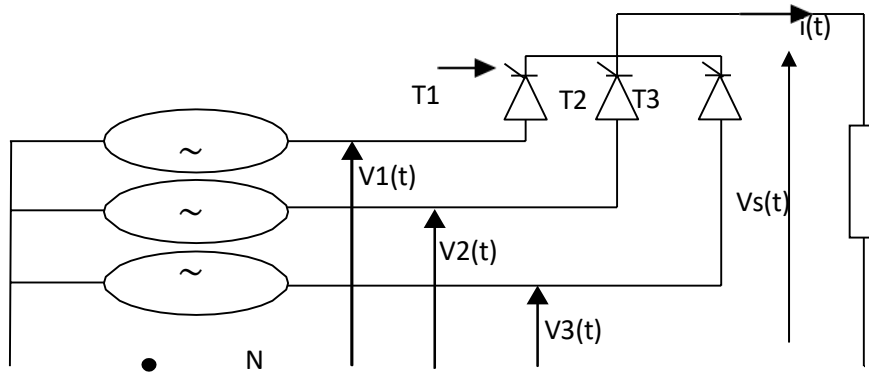


Figure. II.24: Three-phase full-wave rectifiers controlled P3

The three-phase voltage system to be used must have a neutral point (N) and voltages according to the following expressions:

$$\begin{cases} V_1(t) = V \cdot \sqrt{2} \sin(\omega t) \\ V_2(t) = V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3}) \\ V_3(t) = V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3}) \end{cases}$$

The thyristors are turned on with an angle delay of θ_0 , i.e. turn-on pulses are sent to the thyristor gates at the respective corners:

For :

- T1 : $\omega t = \left(\frac{\pi}{6} + \theta_0\right) + 2k\pi$
- T2 : $\omega t = \left(\frac{5\pi}{6} + \theta_0\right) + 2k\pi$

- T3 : $\omega t = \left(\frac{3\pi}{2} + \theta_0\right) + 2k\pi$

II.3.4.2 Functional analysis

We take into account the same circumstances as those that arise for uncontrolled rectifiers in order to make the study simpler.

We suppose that an impulse signal is applied to T1 with a delay ψ in relation to the natural triggering that would occur with diodes ($\pi/6$), in order to examine the process. T1 conducts as a result of this stimulus.

It should be possible to switch on T2 and turn off T1 by applying an impulse to T2 that has a phase shift of $2\pi/3$ with respect to T1.

Likewise, an impulse delivered to T3 that has a phase shift of $2\pi/3$ with respect to T2 ought to guarantee that T3 turns on and T2 turns off.

In this case only the thyristor with the highest anode potential can conduct, and the others are blocked.

The conduction intervals are as follows:

Interval	thyristor in conduction	blocked thyristors	Output voltage $V_s(t)$	Terminal voltage of T1
$\left[\frac{\pi}{6} + \theta_0 \quad \frac{5\pi}{6} + \theta_0\right]$	T_1	T_2, T_3	$V_s(t) = V_1(t)$ $= V \cdot \sqrt{2} \sin(\omega t)$	0
$\left[\frac{5\pi}{6} + \theta_0 \quad \frac{3\pi}{2} + \theta_0\right]$	T_2	T_1, T_3	$V_s(t) = V_2(t)$ $= V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$	$V_1(t) - V_2(t)$
$\left[\frac{3\pi}{2} + \theta_0 \quad \frac{13\pi}{6} + \theta_0\right]$	T_3	T_1, T_2	$V_s(t) = V_3(t)$ $= V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$	$V_1(t) - V_3(t)$

From the table above, the waveform of the rectified voltage is:

Chronograms

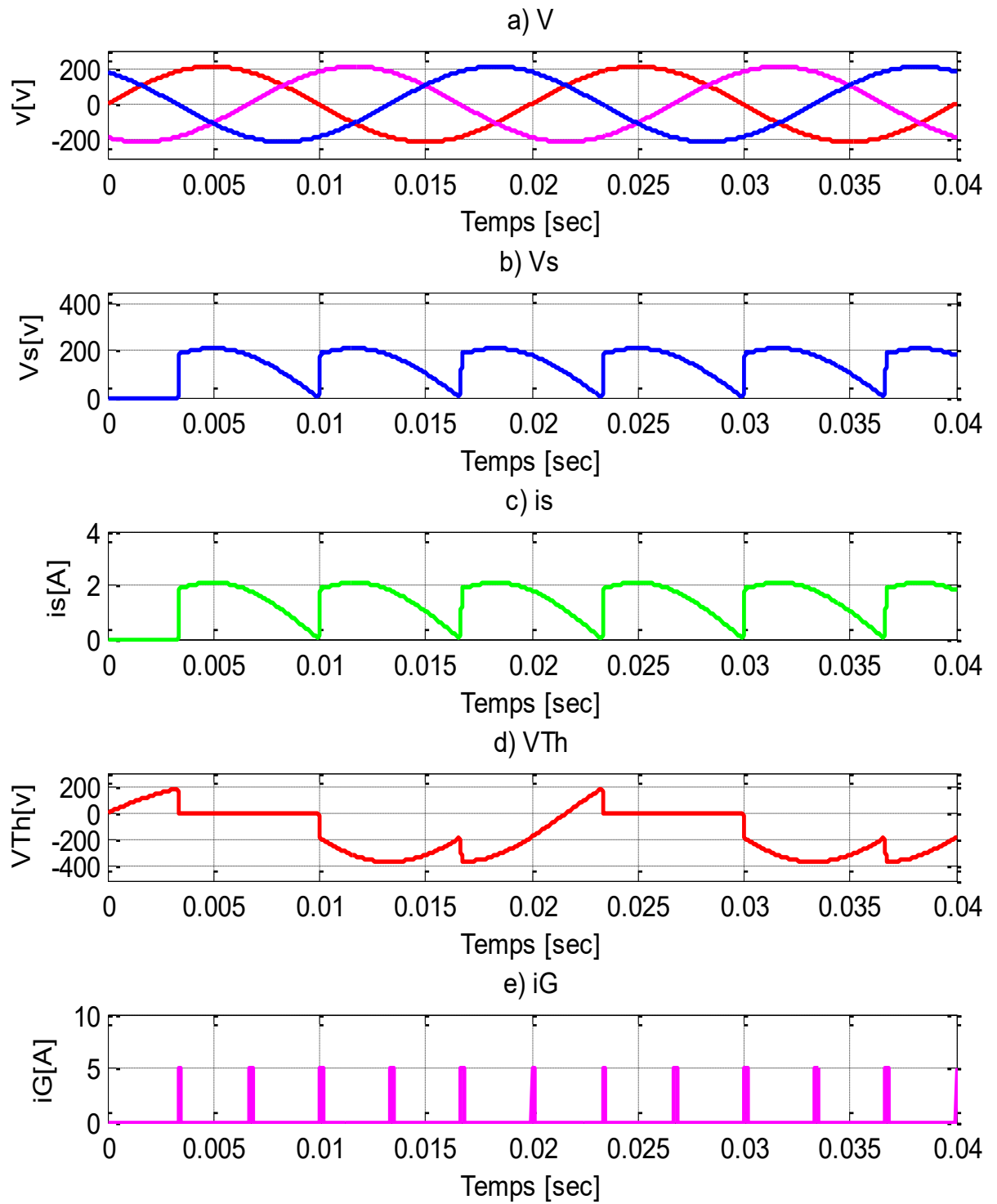


Figure II.25: Voltage and current diagrams for a load R(P3)

- the average value of the rectified voltage

The average value of the rectified voltage is given by :

$$\begin{aligned}
 V_{moy} &= \frac{1}{\frac{5\pi}{6} - \frac{\pi}{6}} \int_{\frac{\pi}{6} + \theta_0}^{\frac{5\pi}{6} + \theta_0} V_1(\theta) \cdot d\theta = \frac{3 \cdot V_{max}}{2\pi} \int_{\frac{\pi}{6} + \theta_0}^{\frac{5\pi}{6} + \theta_0} \sin\theta \cdot d\theta \\
 V_{moy} &= \frac{3 \cdot V_{max}}{2\pi} \cdot (-\cos\theta) \Big|_{\frac{\pi}{6} + \theta_0}^{\frac{5\pi}{6} + \theta_0} \\
 &= \frac{3 \cdot V_{max}}{2\pi} \cdot \left(-\cos\left(\frac{5\pi}{6} + \theta_0\right) + \cos\left(\frac{\pi}{6} + \theta_0\right) \right) \quad (II.36) \\
 &= \frac{3 \cdot V_{max}}{2\pi} \left(\frac{\sqrt{3}}{2} \cos\theta_0 - \frac{1}{2} \sin\theta_0 + \frac{\sqrt{3}}{2} \cos\theta_0 + \frac{1}{2} \sin\theta_0 \right) \\
 V_{moy} &= \frac{3 \cdot \sqrt{3} \cdot V_{max}}{2\pi} \cdot \cos\theta_0
 \end{aligned}$$

$$\text{So: } V_{moy} = \frac{3 \cdot \sqrt{6} \cdot V}{2\pi} \cdot \cos\theta_0$$

For $\theta_0 \leq \frac{\pi}{2}$ $V_{moy} > 0 \implies$ the system operates in **Rectifier**.

For $\theta_0 \geq \frac{\pi}{2}$ $V_{moy} < 0 \implies$ the system operates as an **inverted rectifier**

II.3.4.2 Controlled three-phase full-wave rectification (PD3)

In the same way as in the case of non-drive three-phase full-wave rectifiers, it is possible to replace the diodes (D1, D2 and D3 or D4, D5 and D6) of the full-wave structures by thyristors (T1, T2 and T3 or T4, T5 and T6).

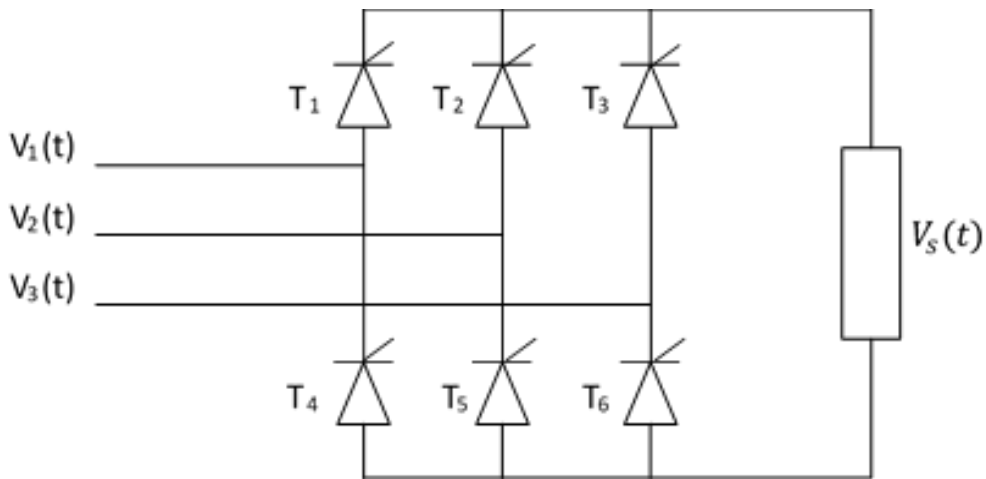


Figure II.26: Full wave rectifier. PD3 controlled circuit

The rule for determining the conducting thyristors is the same as for the three-phase full wave circuit:

- The thyristors T_1, T_2, T_3 are triggered during the **positive** alternation of the voltages $V_1(t), V_2(t), V_3(t)$
- The thyristors T_4, T_5, T_6 are triggered during the **negative** alternation. Thus, we have:

if:

- $V_1(t) > V_2(t) > V_3(t)$, there fore T_1, T_6 leads and

$$V_s(t) = V_1(t) - V_3(t) = V \cdot \sqrt{2} \sin(\omega t) - V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$$

- $V_1(t) > V_3(t) > V_2(t)$, there fore T_1, T_5 leads and

$$V_s(t) = V_1(t) - V_2(t) = V \cdot \sqrt{2} \sin(\omega t) - V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$$

- $V_2(t) > V_1(t) > V_3(t)$, there fore T_2, T_6 leads and

$$V_s(t) = V_2(t) - V_3(t) = V \cdot \sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t + \frac{2\pi}{3})$$

- $V_2(t) > V_3(t) > V_1(t)$, there fore T_2, T_4 leads and

$$V_s(t) = V_2(t) - V_1(t) = V \cdot \sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t)$$

- $V_3(t) > V_1(t) > V_2(t)$, there fore T_3, T_5 leads and

$$V_s(t) = V_3(t) - V_2(t) = V \cdot \sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t - \frac{2\pi}{3})$$

- $V_3(t) > V_2(t) > V_1(t)$, there fore T_3, T_4 leads and

$$V_s(t) = V_3(t) - V_1(t) = V \cdot \sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right) - V \cdot \sqrt{2} \sin(\omega t)$$

II.3.4.3 Chronograms

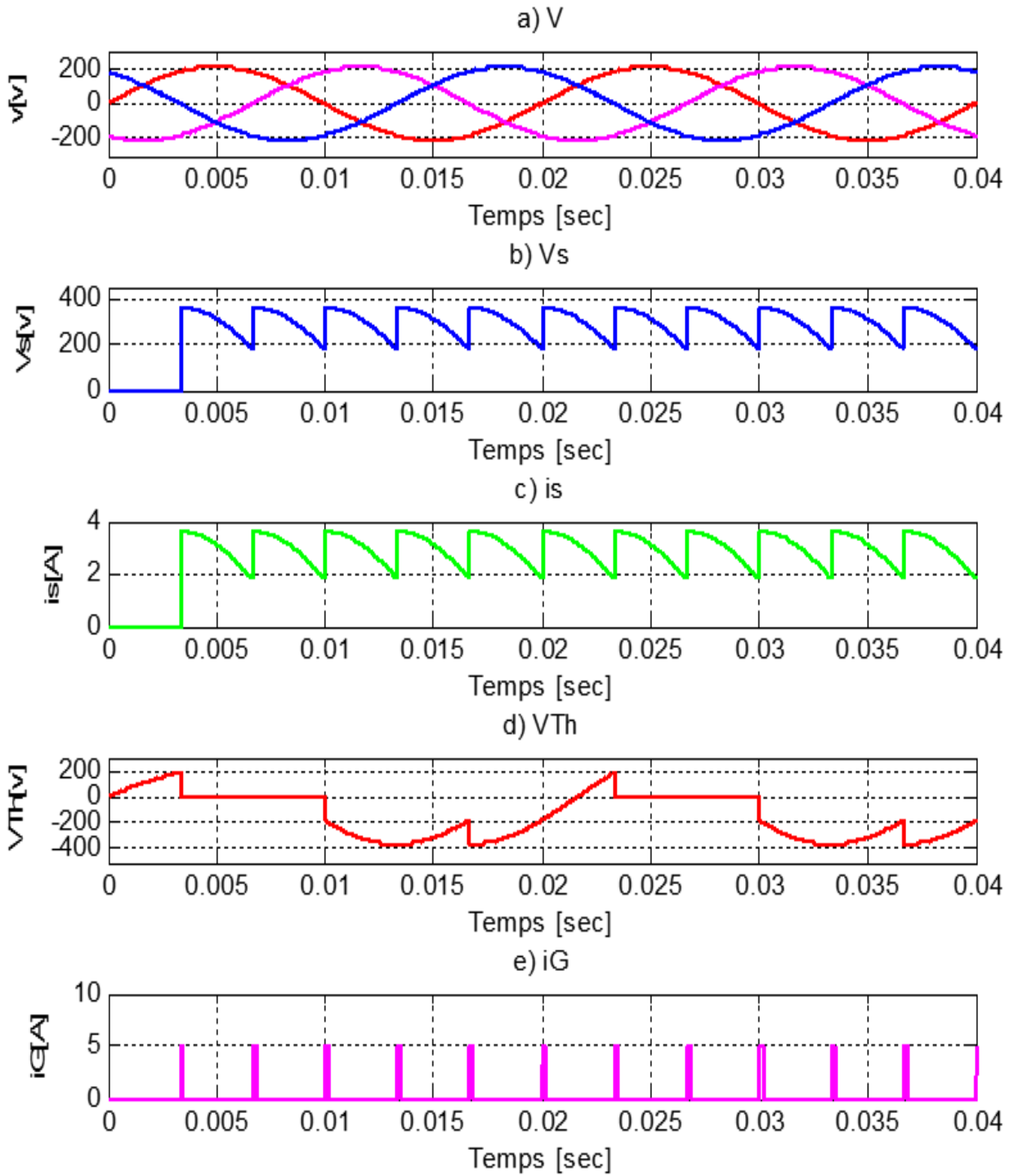


Figure II.27: Voltage and current diagrams for a load R(PD3)

- **Average value of the rectified voltage**

For the expression of the average value of the output voltage, the integral is simply calculated over the interval $\left[\frac{\pi}{6} + \theta_0, \frac{\pi}{2} + \theta_0\right]$:

- $V_1(t) > V_3(t) > V_2(t)$, therefore T_1, T_5 leads and

$$V_s(t) = V_1(t) - V_2(t)$$

$$= V_m \left[\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{3}\right) \right]$$

$$V_{s_{moy}} = \frac{1}{T} \int_0^T V_s(t) \cdot dt$$

$$= \frac{1}{\left(\frac{\pi}{2} + \theta_0 - \left(\frac{\pi}{6} + \theta_0\right)\right)} \int_{\frac{\pi}{6} + \theta_0}^{\frac{\pi}{2} + \theta_0} V_m \cdot \left[\sin(\theta) - \sin\left(\theta - \frac{2\pi}{3}\right) \right] d\theta = \frac{6V_m}{2\pi} \left[-\cos(\theta) + \cos\left(\theta - \frac{2\pi}{3}\right) \right]_{\frac{\pi}{6} + \theta_0}^{\frac{\pi}{2} + \theta_0}$$

$$V_{s_{moy}} = \frac{3V\sqrt{6}}{\pi} \cos(\theta_0) \quad (\text{II.37})$$

Note that :

- $\theta_0 < \frac{\pi}{2}$: $V_{s_{moy}} > 0$.
- $\theta_0 > \frac{\pi}{2}$: $V_{s_{moy}} < 0$.
- $\theta_0 = \frac{\pi}{2}$: $V_{s_{moy}} = 0$.

Exercises on AC/DC Converters (Uncontrolled and Controlled Rectifiers)

Exercise 1: Half-Wave Rectifier (Diode vs Thyristor)

A single-phase source of 220 V (RMS), 50 Hz supplies a resistive load $R=20\ \Omega$.

Case 1: Half-wave diode rectifier

Case 2: Half-wave-controlled rectifier with firing angle $\alpha = 60^\circ$

Questions:

1. Calculate the peak input voltage.
2. Determine the average output voltage for the diode rectifier.
3. Determine the average output voltage for the controlled rectifier.
4. Calculate the average load current in both cases.
5. Compare the output voltage levels and explain the effect of the firing angle.

Exercise 2: Full-Wave Bridge Rectifier Comparison

A 230 V (RMS), 50 Hz source supplies a load $R=25\ \Omega$.

Case 1: Uncontrolled full-wave diode bridge

Case 2: Fully controlled bridge rectifier with $\alpha = 45^\circ$

Questions:

1. Calculate the peak input voltage.
2. Determine the average DC output voltage for both cases.
3. Calculate the average load current for both cases.
4. For which value of α does the controlled rectifier produce the same output voltage as the diode bridge?
5. Discuss which system offers better voltage regulation.

Exercise 3: Effect of Firing Angle on Output Voltage

A single-phase fully controlled bridge rectifier is supplied by 220 V (RMS).

Questions:

1. Write the general expression of the average output voltage for:

A diode bridge

A fully controlled bridge

2. Calculate VDC for firing angles:

$$\alpha = 30^\circ$$

$$\alpha = 60^\circ$$

$$\alpha = 90^\circ$$

3. Compare the results with the diode rectifier output.

4. Explain why controlled rectifiers are suitable for DC motor speed control.

Exercise 4: RL Load – Performance Comparison

A 230 V (RMS) source supplies an RL load:

$$R=10\ \Omega$$

$$L=50\ \text{mH}$$

Case 1: Diode bridge rectifier

Case 2: Fully controlled bridge rectifier with $\alpha = 60^\circ$

Questions:

1. Explain the effect of inductance on load current.
2. Is the current continuous or discontinuous in each case?
3. What is the role of a freewheeling diode?
4. Compare the output voltage controllability in both systems.
5. Which configuration is more suitable for industrial DC drives? Justify.

Chapter III :AC/AC converters (DIMMERS)

III.1 Introduction:

Power electrical equipment known as AC/AC converters control the frequency and RMS voltage of AC inputs and transform them into controlled output values. They improve power quality and guarantee compliance with a range of loads; they are also referred to as AC voltage controllers. By enabling effective power transfer between AC systems with various needs, these converters raise the overall dependability and efficiency of the system. They are crucial in contemporary power electronics because they may provide output RMS voltage and frequency that are higher, lower, or equal to the input by varying the phase angle or duty cycle.

The dimmer (AC/AC conversion) is a static switch that converts a sinusoidal AC voltage or even a three-phase voltage system with a fixed frequency and constant RMS value into another

III.2 Adjustable AC voltage (RMS Control)

The idea behind an AC voltage controller is disarmingly simple: instead of trying to reshape the source voltage, we let the full sinusoid reach the load and then cut away a controlled portion of it, cycle after cycle. What the load actually receives is therefore a chopped version of the input sine wave, and by deciding how much of each half-cycle we keep, we decide the RMS value of the output voltage and hence the power delivered to the load while the frequency stays exactly that of the mains. This is why the dimmer is often described as a static switch rather than as a true converter: it does not generate a new waveform, it merely opens and closes the path between source and load at the right instants.

III.2.1 Operating principle (anti-parallel thyristors / triac)

The power stage that makes this possible is shown in Figure III.1. It is built from two thyristors, T_1 and T_2 , connected in inverse-parallel (anti-parallel), that is, head-to-tail so that one of them can conduct during the positive half-cycle and the other during the negative one.

In practice the pair is very often replaced by a single triac, which is nothing more than the two thyristors integrated into one three-terminal device; the behaviour, however, is identical, and we will keep reasoning in terms of T_1 and T_2 because it makes the conduction of each half-cycle easier to follow. The switch is placed in series between the AC source and the load, so that when both thyristors are off the load is simply disconnected and sees zero voltage.

Why two thyristors and not one? This is a question worth pausing on, because it is a classic trap. A single thyristor would conduct only one half-cycle and block the other, leaving the load with a half-wave rectified voltage that carries a DC component. Such a DC current is harmless on a lamp but dangerous on a transformer or a motor, where it drives the magnetic core toward saturation. The anti-parallel arrangement restores symmetry: every positive pulse delivered by T_1 is matched by a negative pulse delivered by T_2 , the average current over a period is zero, and no DC component reaches the load.

The control itself relies on phase-angle firing. Suppose the source voltage is $v_s(t) = V_m \sin(\omega t)$. During the positive half-cycle, we hold T_1 off even though it is forward-biased, and we send it a gate pulse only after a delay angle α , measured from the instant the source crosses zero. T_1 then turns on and the load voltage follows the source until the current naturally falls to zero at the end of the half-cycle, where the thyristor switches off by itself this is natural (or line) commutation, and it is the reason the dimmer needs no forced-commutation circuit. During the negative half-cycle we repeat exactly the same operation with T_2 , fired at the same angle α after the negative zero-crossing. The resulting waveforms of the source voltage, the load voltage and the gate pulses are drawn in Figure III.2: notice that for $\alpha = 0$ the load sees the complete sine wave (full voltage), whereas as α grows the conducting window shrinks and the load voltage is progressively reduced, vanishing altogether when α reaches π .

Two consequences follow directly from this picture, and we will quantify them in the next sections. First, the RMS value of the output voltage is a decreasing function of α , sweeping continuously from the full source RMS value ($\alpha = 0$) down to zero ($\alpha = \pi$); this is precisely the “adjustable RMS voltage” announced in the title. Second, because the output is no longer a pure sine wave, it contains harmonics, and the current drawn from the mains is no longer in phase with the voltage — the price we pay for such a simple circuit is a power factor that degrades as α increases. It is worth mentioning, finally, that phase-angle control is not the only way to regulate the RMS voltage: an alternative, called on–off or integral-cycle control,

connects the load to the source for a whole number of cycles and disconnects it for another whole number of cycles, which avoids chopping the sine wave inside a cycle and therefore reduces the low-order harmonics. In what follows; however, we concentrate on phase-angle control, which is by far the most common and the most instructive, and we study it first on a purely resistive load and then on an inductive one.

The basic element is formed by two thyristors mounted "head to tail" figure III.1 or a triac and placed between the source and the receiver.

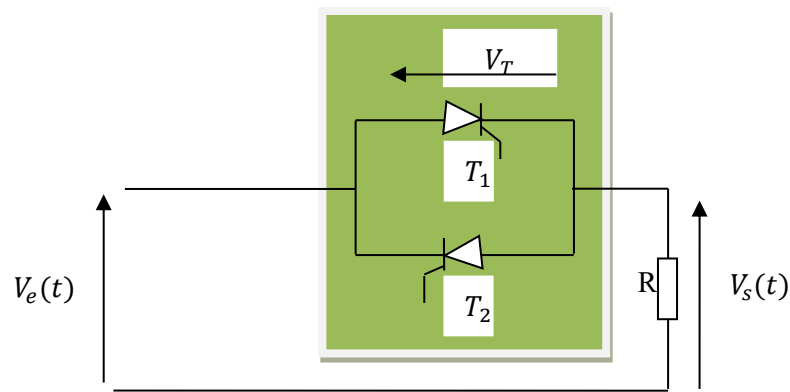


Figure III.1: Principle of a single-phase dimmer - resistive load.

III.3 Operational study

When a pulse is sent to the trigger of one of the two thyristors, it conducts if the voltage applied between its anode and cathode is positive and then blocks when the current through it is cancelled. The operation of the single-phase dimmer must be considered according to the nature of the load.

III.4 AC/AC Converter Types:

Based on their functions and uses, AC/AC converters can be divided into several varieties. Among the primary kinds are:

AC voltage controllers, or direct AC/AC converters

Without altering the frequency, these converters control the AC voltage.

They function by regulating the input AC waveform's phase angle.

For instance:

Phase-controlled AC voltage controllers that use thyristors Cycloconverters that

directly convert AC of one frequency to AC of a lower frequency

AC-DC-AC converters, or indirect AC/AC converters

With the appropriate voltage and frequency, these converters first convert AC to DC and then back to AC.

When compared to direct converters, they offer more efficiency and control.

Examples are matrix converters, which convert AC to AC directly without the need for a DC link.

Back-to-back converters, which convert using rectifiers and inverters

Converters for Matrix

a direct AC/AC conversion system that does not require a DC link and is fully controlled.

offers excellent efficiency and bidirectional power flow.

Cycloconverters

A unique kind of direct AC/AC converter that transforms an AC input with a high frequency into an AC output with a lower frequency.

frequently seen in high-power devices like industrial motor drives.

III.5 Single-phase dimmer on resistive load:

The basic principle of the "single-phase dimmer" is very simple. The diagram in Figure III.1 shows the associated circuit with the following characteristics:

- The input voltage, usually sinusoidal $V_e(t) = V_{max} \sin(\omega t)$
 - The load is a pure R-resistance.
 - We define θ_0 the delay angle of the thyristor firing T_n
- For $T_1 : \omega t = \theta_0 + 2k\pi$
 $T_2 : \omega t = (\theta_0 + \pi) + 2k\pi$

III.5.1 Operation:

The conduction intervals are as follows:

Interval (ωt)	Input voltage $V_e(t)$	Passing thyristors	Output voltage $V_s(t)$	Output current $i_s(t)$
$[0 \quad \theta_0]$	$V_e(t) > 0$	No	0	0

$[\theta_0 \quad \pi]$		T_1	$V_s(t) = V_e(t)$	$\frac{V_e}{R}$
$[\pi \quad \pi + \theta_0]$	$V_e(t) < 0$	No	0	0
$[\pi + \theta_0 \quad 2\pi]$		T_2	$V_s(t) = V_e(t)$	$\frac{V_e}{R}$

III.5.1.1 The value of the effective voltage

The objective of the operation will be to obtain an alternating regime in sound; we will be particularly interested in the expression of the effective values of the output quantities:

$$\begin{aligned}
V_{eff}^2 &= \frac{2}{2\pi} \int_{\theta_0}^{\pi} V_s(t)^2 d\theta \\
&= \frac{1}{\pi} \int_{\theta_0}^{\pi} V_e(t)^2 d\theta = \frac{V_{max}^2}{\pi} \int_{\theta_0}^{\pi} \sin^2 \theta d\theta \\
&= \frac{V_{max}^2}{2\pi} \int_{\theta_0}^{\pi} 1 - \cos 2\theta d\theta \\
&= \frac{V_{max}^2}{2\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_{\theta_0}^{\pi} = \frac{V_{max}^2}{2\pi} \cdot \left(\pi - \theta_0 + \frac{1}{2} \sin 2\theta_0 \right) \\
V_{eff}^2 &= \frac{V_{max}^2}{2} \left(1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0 \right) \\
V_{eff} &= \frac{V_{max}}{\sqrt{2}} \cdot \sqrt{1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0} \\
&= V \cdot \sqrt{1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0}
\end{aligned} \tag{III.1}$$

III.5.1.2 The rms value I

$$I_{eff} = \frac{V_{eff}}{R} = \frac{V}{R} \cdot \sqrt{1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0} \tag{III.2}$$

Power dissipated in the load :

$$P_R = \frac{V_{eff}^2}{R} = \frac{V^2}{R} \left(1 - \frac{\theta_0}{\pi} + \frac{1}{2\pi} \sin 2\theta_0 \right) \tag{III.3}$$

III.5.1.3 Chronograms

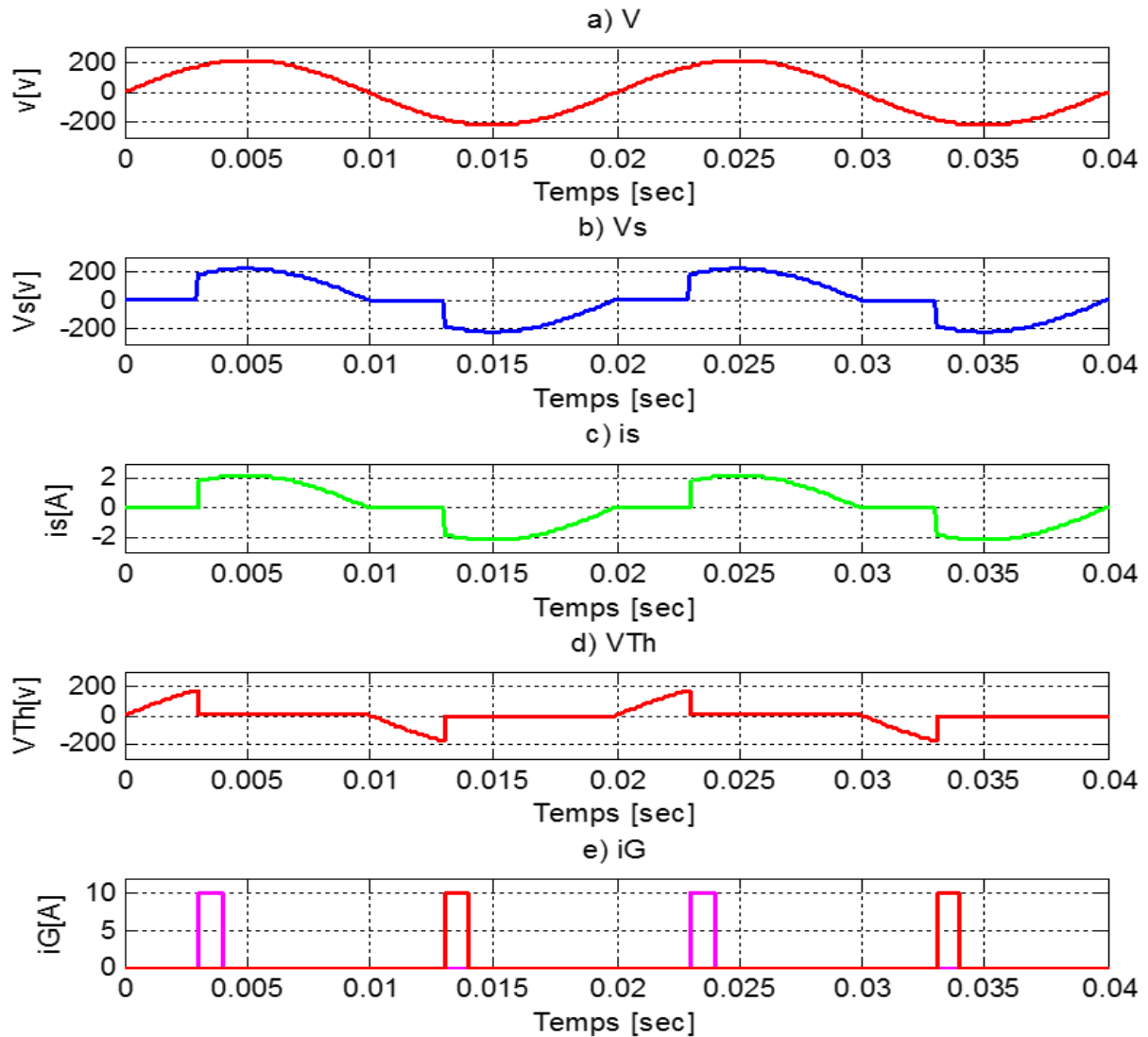


Figure III.2: Voltage and current diagrams for a load R

III.5.2 Single-phase dimmer on inductive load

The argument φ_R of the receiver reduces the variation of the angle θ_0 which ensures the passage of I from its maximum to zero.

In reality an "inductive load" imposes a delay in the extinction of the current in the load which modifies the previous waveforms and may even impose a continuous conduction thus cancelling the dead times imposed by the triac, and therefore the variation in RMS value. The diagram in figure III.2 shows the circuit associated with an inductive load

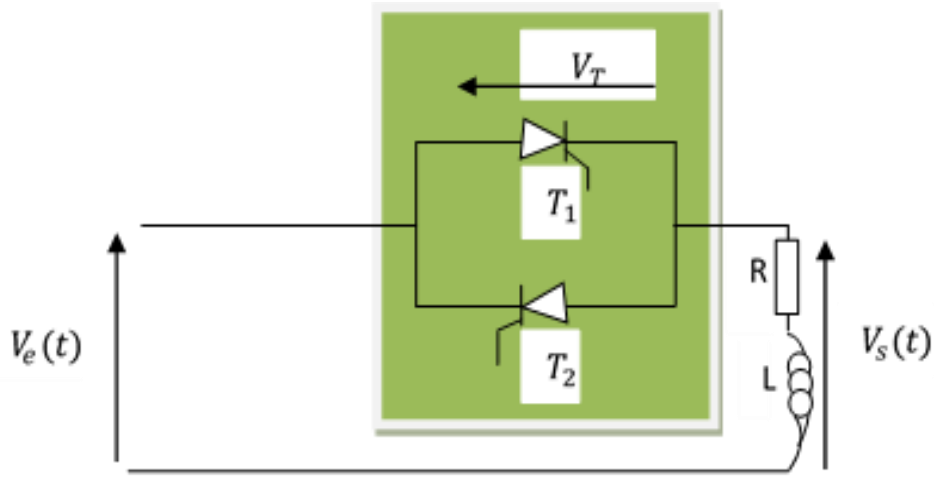


Figure III.3: Single-phase dimmer with RL load

III.5.2.1 Operation

The thyristor T_1 is unlocked for $t = t_0$.

such as $\omega \cdot t_0 = \theta_0$.

From this moment on :

$$L \frac{di_s(\theta)}{dt} + Ri_s(\theta) = V_{max} \sin \theta \quad (\text{III.4})$$

R and L are the constants of the receiver.

The current $i_s(\theta)$ has the general expression $i_s(\theta)$

$$\text{is : } i_s(\theta) = \frac{V_{max}}{Z} \sin(\theta - \varphi) + Ke^{-\frac{\theta}{\tau}}$$

With $\omega t = \theta$,

$$Z = \sqrt{R^2 + (L\omega)^2}$$

$$\varphi = \arctan\left(\frac{L\omega}{R}\right)$$

$$\tau = \frac{L\omega}{R}$$

The initial conditions: at $t = t_0$

we have $i_s = 0$

$$= \frac{V_{max}}{Z} \sin(\theta_0 - \varphi) + Ke^{-\frac{\theta_0}{\tau}}$$

So: $K = -\frac{V_{\max}}{Z} \sin(\theta_0 - \varphi) e^{-\frac{\theta_0}{\tau}}$

Therefore : $i_s(\theta) = \frac{V_{\max}}{Z} \left[\sin(\theta - \varphi) - (\sin(\theta_0 - \varphi) e^{-\left(\frac{\theta - \theta_0}{\tau}\right)}) \right] \quad \textbf{(III.5)}$

The analysis of the current $i_s(\theta)$ allows us to have three possible cases depending on the sign of $(\theta_0 - \varphi)$

Case where :

- $\theta_0 = \varphi$

$$i_s(\theta) = \frac{V_{\max}}{Z} (\sin(\theta - \varphi))$$

- $\varphi < \theta_0 < \pi$:

- $i_s(\theta) = \frac{V_{\max}}{Z} \left[\sin(\theta - \varphi) - (\sin(\theta_0 - \varphi) e^{-\left(\frac{\theta - \theta_0}{\tau}\right)}) \right]$

- dimmer operation

- $\theta_0 < \varphi$: The operation depends on the nature of the signals applied to the triggers (rectifier operation).

Chronograms

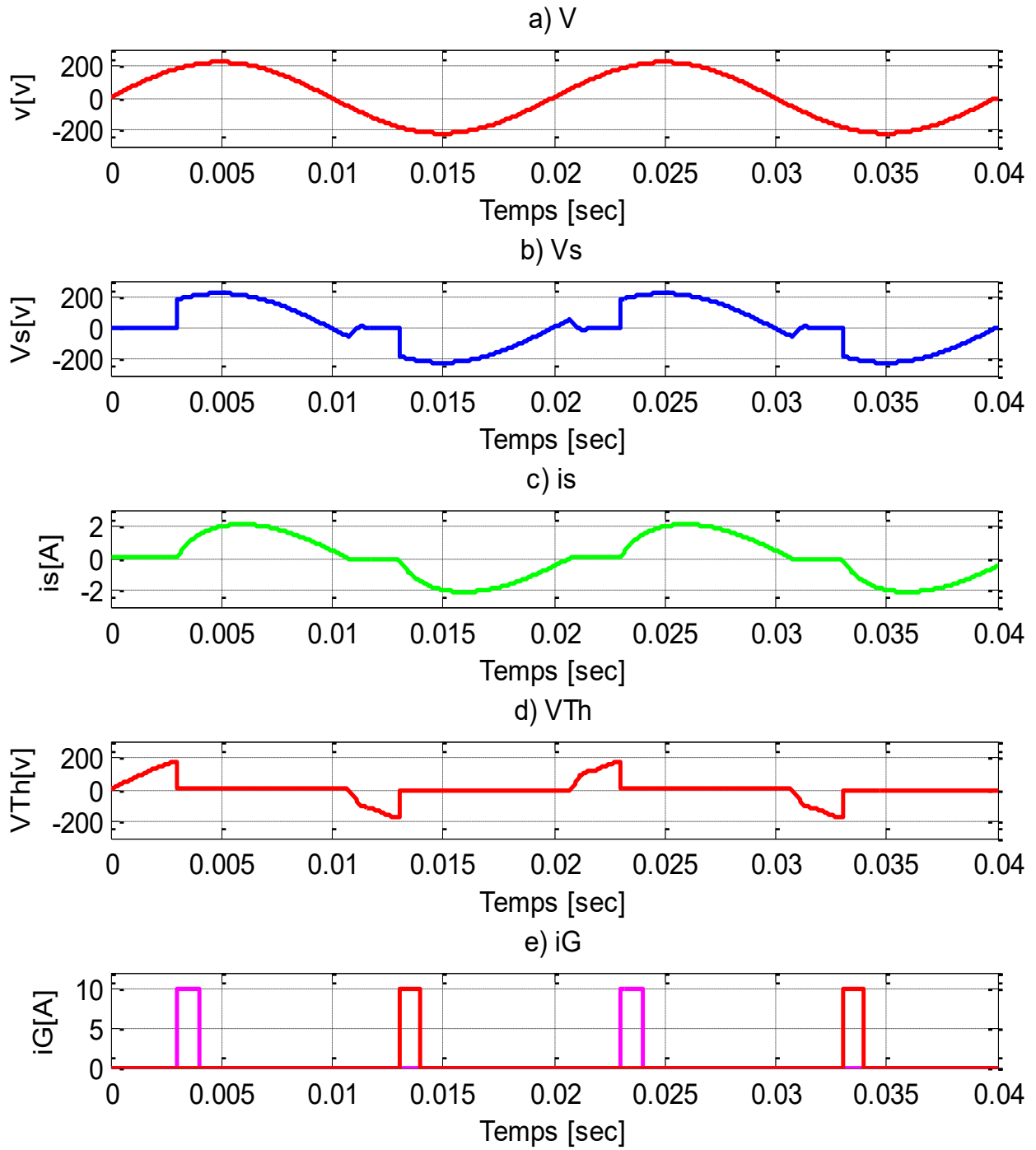


Figure III.4: Voltage and current diagrams for an RL load

III.6 Relevance and Uses of Power Electronics

In contemporary power electronics systems, AC/AC converters are essential components. In a variety of applications, these converters are quite effective at regulating the AC RMS voltage and frequency. The importance of AC/AC converters has grown significantly due to the growing demand for energy conservation and better power quality. The following explains some of the main uses and importance of AC/AC converters in power electronics:

Motor Speed Control:

It is well known that AC/AC converters can regulate the speed of AC motors by merely altering the RMS voltage and frequency that are delivered to the motor. For synchronous and induction motors, which are utilized in robotics, electric cars, and industrial automation, these provide speed control and increased energy efficiency.

Renewable Energy Systems:

AC/AC converters play a vital role in harnessing energy from renewable sources like solar, wind, and hydroelectric power. By controlling the RMS voltage and frequency of the generated power, they facilitate either direct load usage or seamless integration into the power grid, enhancing overall system efficiency.

Uninterruptible Power Supplies (UPS):

These systems incorporate AC/AC converters as essential components to deliver continuous and reliable power to critical loads during outages or disruptions. By regulating the RMS output voltage and frequency, these converters ensure the proper operation of sensitive electronic devices, such as computers, servers, and medical equipment, even in cases of power quality issues.

Exercises on AC/AC Converters (Dimmers)

Exercise 1: Single-Phase AC Voltage Controller (Resistive Load)

A single-phase AC voltage controller (dimmer) supplies a resistive load $R=30\ \Omega$.

The input voltage is 220 V (RMS), 50 Hz.

The firing angle is $\alpha = 60^\circ$.

Questions:

1. Determine the peak value of the input voltage.
2. Write the expression of the output voltage.
3. Calculate the RMS output voltage.
4. Determine the RMS load current.
5. Calculate the power delivered to the load.
6. Sketch the output voltage waveform.

Exercise 2: Effect of Firing Angle on Output Power

A dimmer is connected to a 220 V (RMS) source and supplies a purely resistive load.

Questions:

Derive the expression of the RMS output voltage as a function of firing angle α .

Calculate the RMS output voltage for:

$$\alpha = 30^\circ$$

$$\alpha = 60^\circ$$

$$\alpha = 90^\circ$$

Determine the corresponding load power in each case.

Explain how the firing angle controls the brightness of a lamp.

Exercise 3: RL Load with AC Voltage Controller

A single-phase AC voltage controller supplies an RL load:

$$R=15\ \Omega$$

$$L=40\ \text{mH}$$

Supply: 220 V (RMS), 50 Hz

Firing angle $\alpha = 45^\circ$

Questions:

1. Explain how inductance affects current waveform.
2. Is the current waveform symmetrical? Justify.
3. What happens if the firing angle increases?
4. Discuss the impact on power factor.
5. Sketch the expected voltage and current waveforms.

Exercise 4: Industrial Application

A dimmer is used to control the temperature of an electric heater rated:

3 kW

220 V (RMS)

Questions:

1. Determine the nominal resistance of the heater.
2. If the firing angle is adjusted to reduce the RMS voltage to 150 V, calculate the new power delivered.
3. What percentage of nominal power is delivered?
4. Explain why AC voltage controllers are suitable for heating applications but not ideal for induction motors.
5. State two practical applications of dimmers.

Chapter IV: DC/DC Converters (Choppers)

IV.1 Introduction:

A DC chopper, sometimes referred to as a DC-to-DC converter, is a static device that transforms a constant DC voltage source into a changing DC voltage. Applications for chopper use are numerous and include traction motor control, battery-powered vehicles, trolley cars, and the regulation of several DC motors. They also function as regulators of DC voltage.

There are two primary categories for DC choppers:

- Step-down choppers: These lower the output voltage to a level below the input voltage.
- Step-up choppers: These raise the output voltage above the input voltage.

A chopper is a static DC/DC converter, placed between the generator and the receiver, and it converts DC energy to the same type of load (Figure IV.1). The principle of this converter is based on the regular opening and closing of controllable electronic switches, such as bipolar transistors or IGBTs or GTO thyristors (Figure IV.2).

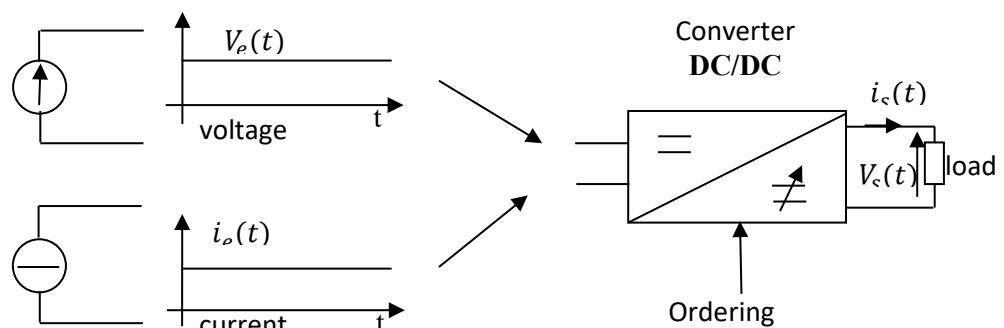
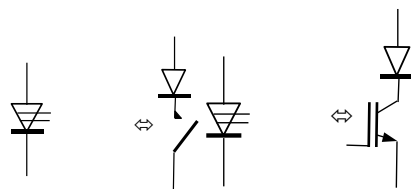


Figure IV.1 DC/DC Converter



Operation With an IGBT transistor

Figure IV.2 Static switch.

IV.2 The cyclical relationship:

The duty cycle (α) is defined as the time t_{ON} divided by the operating period of the circuit T ,

$$\alpha = \frac{t_{ON}}{T} \text{ where } \alpha \text{ varies in the following interval } [0, 1].$$

And also, the time during which the switch is closed, defined by the equation:

$$t_{OFF} = T - t_{ON}$$

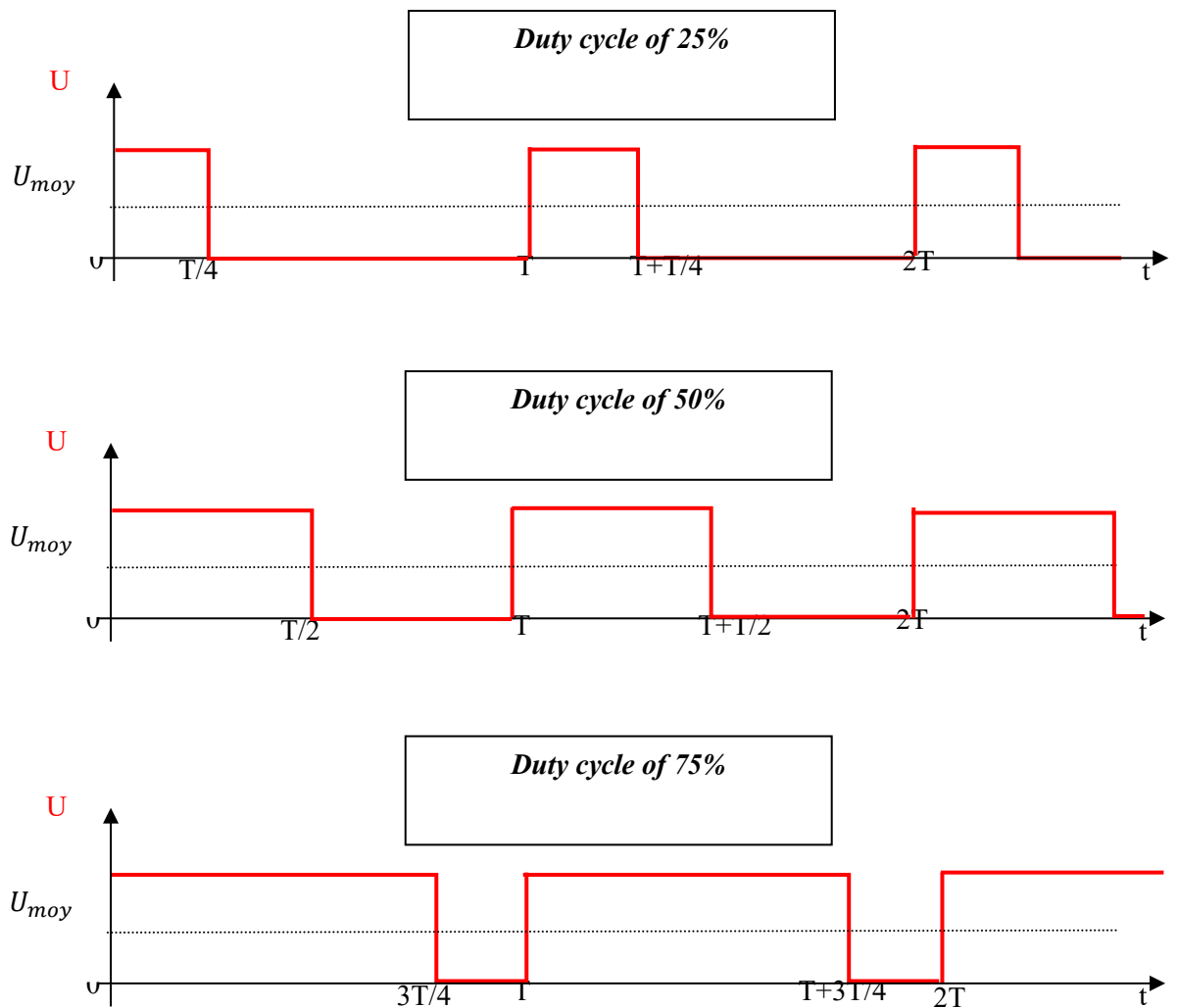


Figure IV.3: Duty Cycle

There are several types of this converter, among which the principle of the two types Buck and booster are presented.

IV.3 Series chopper (booster)

The switch's opening and closing are controlled on a regular basis. T stands for the control signals' repetition period, and α is the duty cycle, or the percentage of time the switch stays in the conducting state (H is on for a length of αT). The duty cycle α , which ranges from 0 to 1, is a dimensionless quantity.

The series chopper, also known as a **buck** converter, controls the flow:

- a voltage **generator** whose voltage is always **positive**,
- in a current **receiver** whose current cannot become **negative**.

IV.3.1 Flow on a resistive load:

The figure below shows the schematic diagram of the series chopper with a resistive load.

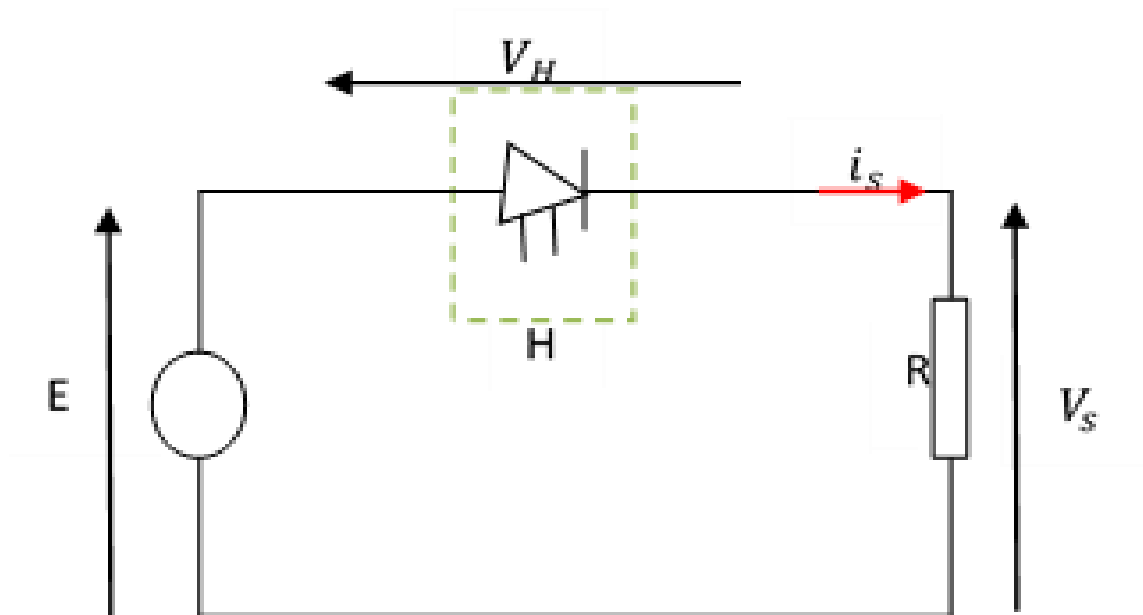


Figure IV.4: Series chopper with resistive load

Chronograms

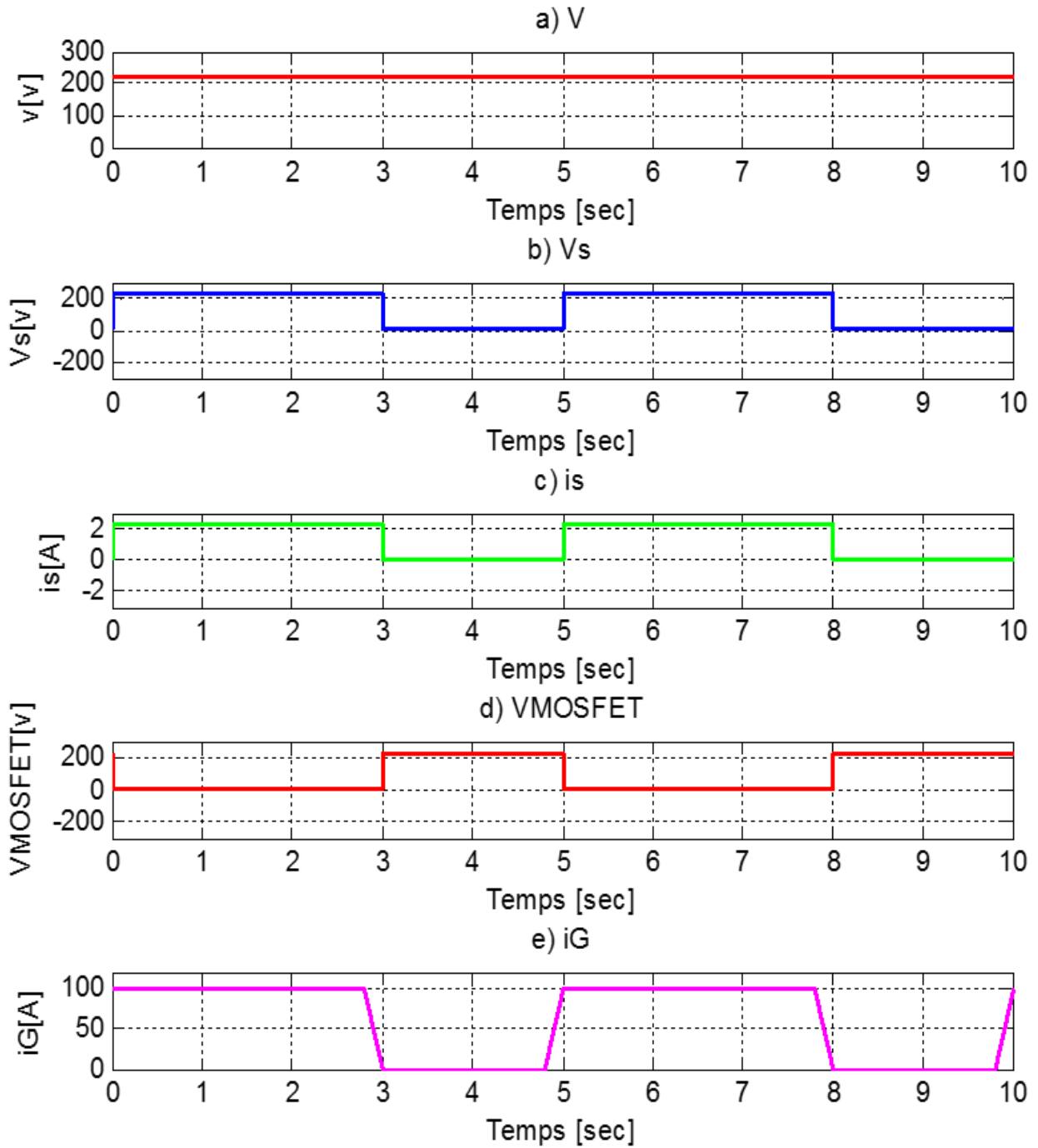


Figure VI.5: Voltage and current diagrams for a load R

The various operating phases of the assembly are then described in the following table:

Interval	Input voltage	H	V_H	V_s	i_s
$0 < t < \alpha T$	E	Closed	0	E	$\frac{E}{R}$
$\alpha T < t < T$	E	Open	E	0	0

IV.3.2. Series chopper with current smoothing (RL load)

Average value :

$$\begin{aligned}
 V_{s_moy} &= \frac{1}{T} \int_0^T V_s(t). dt = \frac{1}{T} \int_0^{\alpha T} E. dt + \frac{1}{T} \int_{\alpha T}^T 0. dt & (IV.1) \\
 &= \frac{E}{T} (\alpha T - 0) \\
 V_{s_moy} &= \alpha. E
 \end{aligned}$$

AND :

$$i_{s_moy} = \frac{V_{s_moy}}{R} = \frac{\alpha. E}{R}$$

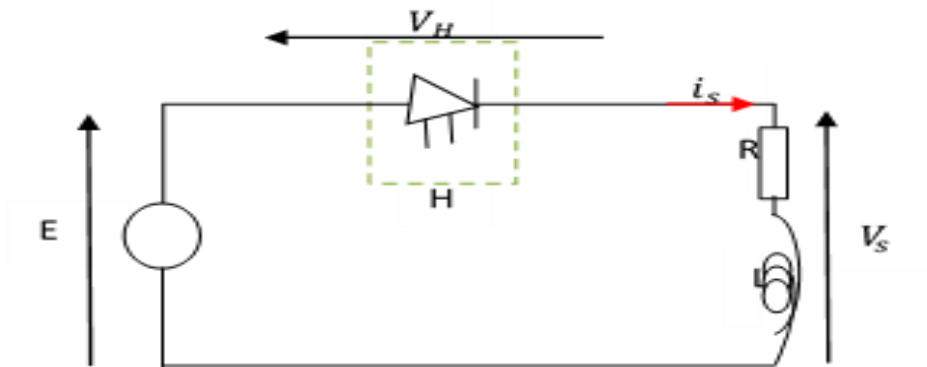


Figure IV.6: Series chopper with R-L load

We have:

$$L \frac{di_s}{dt} + Ri_s = E \quad (IV.2)$$

The solution of this equation and the sum of the particular solution

$$i_p = \frac{E}{R} \quad (\text{IV.3})$$

And the general solution of the equation without a second member :

$$L \frac{di_s}{dt} + Ri_s = 0 \quad (\text{IV.4})$$

So the current i_s

$$i_s = \frac{E}{R} + A. e^{-\frac{R}{L}t} \quad (\text{IV.5})$$

The average value of the voltage is :

$$V_{s_moy} = \frac{1}{T} \int_0^T V_s(t). dt = \frac{1}{T} \int_0^{\alpha T} E. dt = \alpha. E \quad (\text{IV.6})$$

IV.3.3. Flow rate on an active load R, L, E.

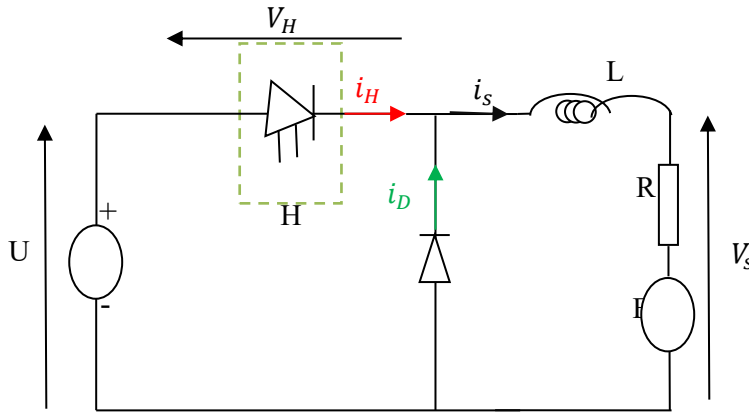


Figure IV.7: Series chopper arrangement with a load R,L,E

D: is a freewheeling (or recovery) diode, it will be assumed ideal

Analysis of the functioning:

We will adopt as a variable, for choppers, the time t (and not the angle $\theta = \omega t$ as for rectifiers).

Interval	Input voltage	H	$V_H =$	$V_s =$	$i_s =$
$0 < t < \alpha T$	U	Closed	0	U	i_H
$\alpha T < t < T$	U	Open	U	0	0

1° Interval $0 < t < \alpha T$

- The current in the load checks the following equation:

$$U = E + R\mathbf{i}_s + L \frac{d\mathbf{i}_s}{dt} \quad (\text{IV.7})$$

We will assume that R is sufficiently small that $Ri \ll E$ with this assumption. We have

$$U - E = L \frac{d\mathbf{i}_s}{dt} \Rightarrow \mathbf{i}_s = \frac{U-E}{L} t + \mathbf{i}_{\min} \quad (\text{IV.8})$$

To find the value of the current we integrate the equation in the interval $[0, \alpha T]$:

$$\int_0^{\alpha T} d\mathbf{i}_s = \int_0^{\alpha T} \frac{U-E}{L} dt$$

$$\mathbf{i}_s(t) = \frac{U-E}{L} t + \mathbf{i}_{\min} \quad (\text{IV.9})$$

2° Interval $\alpha T < t < T$

- The current in the load satisfies the following equation:

$$0 = E + R\mathbf{i}_s + L \frac{d\mathbf{i}_s}{dt} \quad (\text{IV.10})$$

$$d\mathbf{i}_s = -\frac{E}{L} dt$$

To find the value of the current, we integrate the equation in the interval $[\alpha T, T]$:

$$\int_{\alpha T}^T d\mathbf{i}_s = \int_{\alpha T}^T -\frac{E}{L} dt$$

$$\mathbf{i}_s = -\frac{E}{L} (T - \alpha T) + \mathbf{i}_{\max} \quad (\text{IV.11})$$

Let's calculate the average values:

Average current value :

$$V_{s-\text{moy}} = \alpha U$$

Average current value :

$$V_{s-\text{moy}} = E + R\mathbf{i}_{s-\text{moy}} + L \frac{d\mathbf{i}_{s-\text{moy}}}{dt} \quad (\text{IV.12})$$

$$L \frac{d\mathbf{i}_{s-\text{moy}}}{dt} = 0$$

So :

$$\mathbf{i}_{s-\text{moy}} = \frac{\alpha U - E}{R} = \frac{1}{2} (\mathbf{i}_{\max} + \mathbf{i}_{\min}) \quad (\text{IV.13})$$

The average values of the currents $\mathbf{i}_{H-\text{moy}}$ and $\mathbf{i}_{D-\text{moy}}$ a function of $\mathbf{i}_{s-\text{moy}}$

Average current in the chopper : $\mathbf{i}_{H-\text{moy}} = \alpha * \mathbf{i}_{s-\text{moy}}$

Average current in the diode : $\mathbf{i}_{D-\text{moy}} = (1 - \alpha) \mathbf{i}_{s-\text{moy}}$

Current ripple in the load :

It is given by the relation :

$2\Delta i = (\mathbf{i}_{\max} - \mathbf{i}_{\min})$; so we get

$$-2\Delta i = -\frac{E}{L} T(1 - \alpha) = -\frac{\alpha U}{L} T(1 - \alpha) \quad (\text{IV.14})$$

$$2\Delta i = \frac{UT}{L} \alpha(1 - \alpha)$$

The absolute ripple Δi depends on α : the function $\alpha(1 - \alpha)$ being maximum for $\alpha = 0.5$, we have

$$(\Delta i)_{\max} = \frac{1}{8} \frac{U \cdot T}{L} : T = \frac{1}{f}$$

To decrease Δi , the inductance L or/and the frequency f must be increased.

IV.4 Parallel chopper (boost) :

The parallel chopper, also known as the boost chopper, is a DC-DC converter that increases the input voltage. This circuit generates a medium output voltage U_{d0} from a lower DC voltage source (U), where $U < U_{d0}$. The principal circuit diagram is illustrated in Figure IV.8.

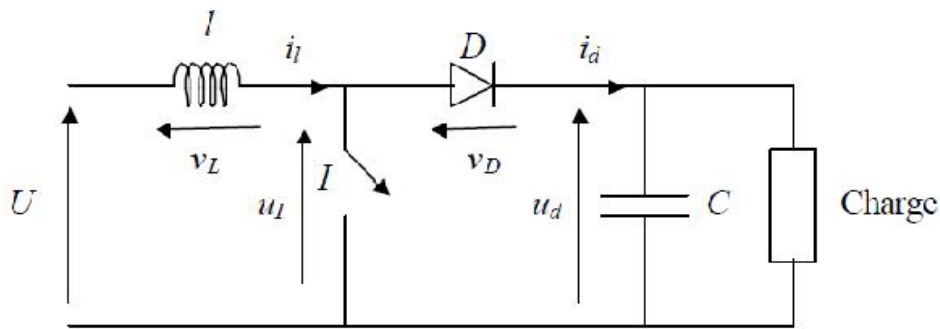


Figure IV.8 Parallel chopper.

The parallel chopper operates based on two main applications: regulated power supply and braking, where energy is recovered in DC motors operating in two phases:

First Phase (Switch Closed):

- The diode is reverse-biased ($v_D = -u_d$), and the switch I is closed.
- The load is isolated, and the power source supplies energy to the inductor L .

Second Phase (Switch Open):

- The switch I is opened.
- The output stage (capacitor + load) receives energy from both the power source and the inductor L , effectively doubling the supplied energy.

Exercises on DC/DC Converters (Choppers)

Exercise 1: Buck Converter (Step-Down Chopper)

A DC/DC buck converter is supplied by:

Input voltage $V_{in}=100\text{ V}$

Duty cycle $D=0.4$

Switching frequency $f=5\text{ kHz}$

Load resistance $R=20\ \Omega$

Assume ideal components and continuous conduction mode (CCM).

Questions:

1. Calculate the average output voltage V_{out} .
2. Determine the average output current.
3. Calculate the output power.
4. Determine the input current.
5. Explain why this converter is called a step-down converter.

Exercise 2: Boost Converter (Step-Up Chopper)

A boost converter operates with:

Input voltage $V_{in}=48\text{ V}$

Duty cycle $D=0.6$

Load resistance $R=40\ \Omega$

Assume continuous conduction mode and ideal components.

Questions:

1. Write the expression of the output voltage.
2. Calculate the average output voltage.
3. Determine the output current.
4. Calculate the output power.
5. Explain why the output voltage is higher than the input voltage.

Exercise 3: Buck Converter with RL Load

A buck converter supplies a load:

$$V_{in}=120 \text{ V}$$

$$D=0.5$$

$$R=10 \, \Omega$$

$$L=5 \text{ mH}$$

$$\text{Switching frequency } f=10 \text{ kHz}$$

Questions:

1. Calculate the average output voltage.
2. Determine the average load current.
3. Explain the role of the inductor.
4. Define continuous conduction mode (CCM).
5. What happens if the inductance value decreases significantly?

Exercise 4: Comparative Study (Buck vs Boost Converter)

A DC source of 60 V is available. The required load voltage is:

Case 1: 30 V

Case 2: 120 V

Questions:

1. Which type of converter should be used in each case?
2. Calculate the duty cycle required in both cases.
3. Compare the current stress on the switch in both converters.
4. Which converter is more suitable for battery-powered applications?
5. Give two practical applications of DC/DC converters.

Chapter V: DC/AC converters (Single phase inverter)

V.1 Introduction:

As illustrated in Figure V.1, the inverter receives a constant DC input, characterized by a stable voltage $V_e(t)$ and current $i_e(t)$, and converts it into an alternating output voltage $V_s(t)$ and current $i_s(t)$ supplied to the load. This conversion is achieved through a controlled switching operation, ensured by a dedicated command circuit that determines the frequency and the shape of the output waveform. Thanks to this control, the inverter can adapt its output to the specific requirements of the load, which explains its wide use in modern applications such as motor drives, renewable energy systems, and uninterruptible power supplies.

The figure below shows the current and voltage conversion by the inverter.

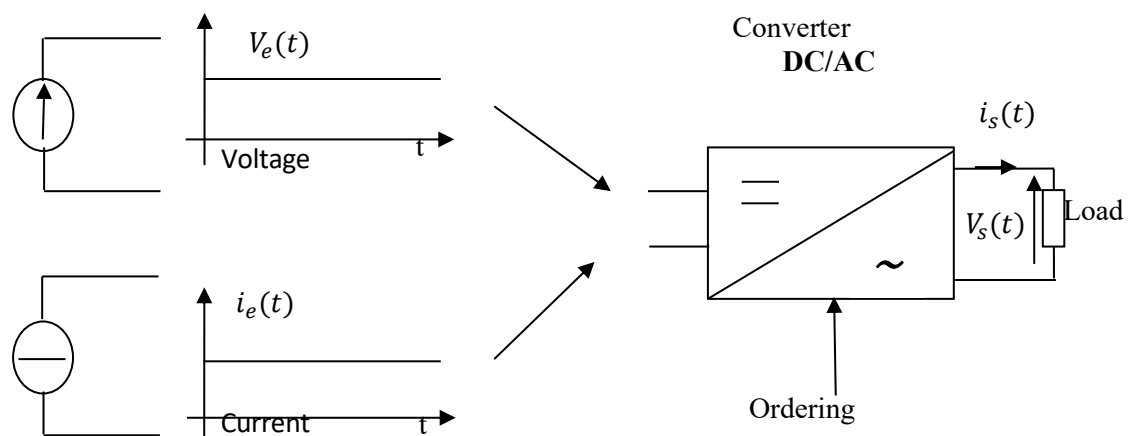


Figure V.1: DC/AC conversion

V.2 The single-phase inverter

The two basic configurations of a single-phase stand-alone inverter are:

Half-Bridge Inverter:

- Uses two electronic switches for operation.
- Requires two separate power sources to generate the AC output.

Full-Bridge (H-Bridge) Inverter:

- Consists of four electronic switches arranged in an "H" configuration.
- Operates with a single power source to provide a more efficient AC conversion.

V.2.1 Inverter with two electronic switches (half bridge)

V.2.1.1 Resistive load

The schematic diagram of the converter is presented in the figure below.

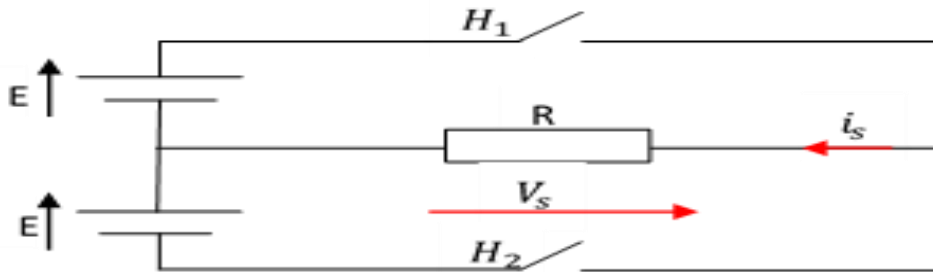


Figure V.2: Half-bridge inverters

with a resistive load The figure above is composed of the following elements:

R: resistive load. **E:** DC voltage sources.

H_1, H_2 Two unidirectional controlled electronic components (**transistors** or **thyristors**).

A) Analysis of the operation:

The switches operate in a complementary manner, meaning that each switch is active for half of the period while the other remains off. As a result:

- The switch **H_1** is closed so **H_2** is open then $V_s = E$
- The switch **H_2** is closed so **H_1** is open then $V_s = -E$

b) Chronograms

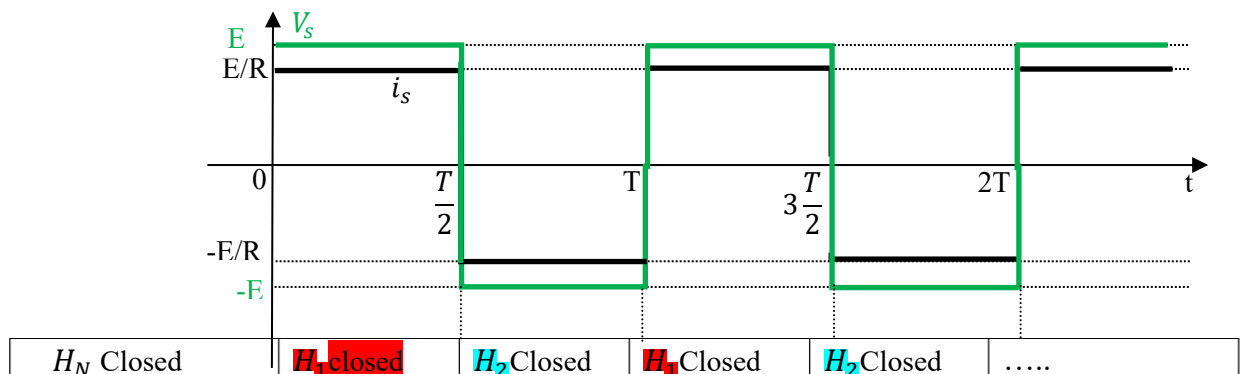


Figure V.3: Voltage and Current Waveforms for a Resistive Load (R).

From two fixed DC voltage sources, an AC current with an adjustable frequency is now generated

- The average voltage value is $V_{s-moy} = 0V$
- The average current value is $i_{s-moy} = 0A$
- The rms value of V_s is $V_{s-eff} = E$
- The rms value of i_s is $i_{s-eff} = \frac{E}{R}$

V.2.1.2 Resistive and inductive load:

To prevent current interruptions in a half-bridge inverter with a resistive-inductive (R-L) load, a diode is connected in antiparallel with each switch (H_1, H_2).

The figure below illustrates this configuration.

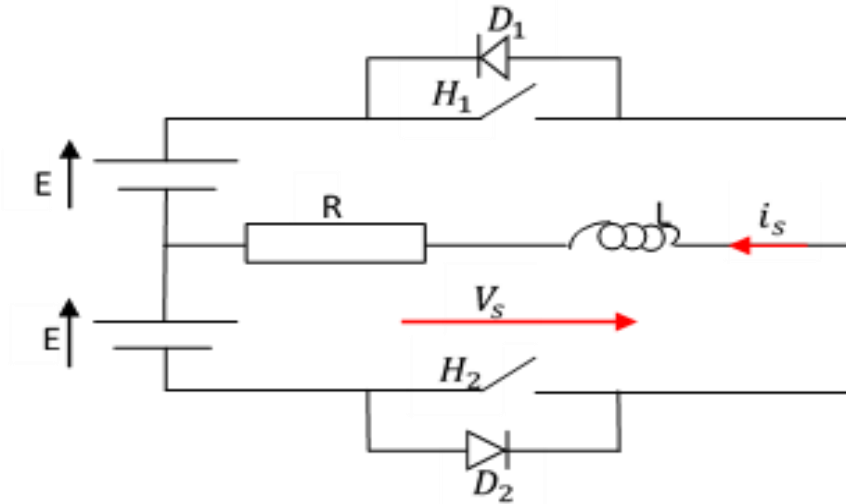


Figure V.4: Inverter assembly in R-L half-bridge load

a) Analysis of the operation:

for $0 < t < \frac{T}{2}$ The switch H_1 is closed so H_2 is open then $V_s = E$

For $\frac{T}{2} < t < T$ The switch H_2 is closed so H_1 is open then $V_s = -E$

For the current, it is governed by the following differential equations:

$$\begin{cases} \frac{L}{R} \cdot \frac{di_s}{dt} + i_s = \frac{E}{R} \text{ pour } 0 < t < \frac{T}{2} \\ \frac{L}{R} \cdot \frac{di_s}{dt} + i_s = -\frac{E}{R} \text{ pour } \frac{T}{2} < t < T \end{cases} \quad (V.1)$$

All data are contained in the following table:

t	V_s	i_s	Controlled Switches	Passing elements
$0 < t < t_1$	$V_s = E > 0$	$i_s < 0$	H_1 is closed	D_1
$t_1 < t < \frac{T}{2}$		$i_s > 0$	H_1 is closed	H_1
$\frac{T}{2} < t < t_2$	$V_s = -E < 0$	$i_s < 0$	H_2 is closed	D_2
$t_2 < t < T$		$i_s > 0$	H_2 is closed	H_2

b) Chronograms

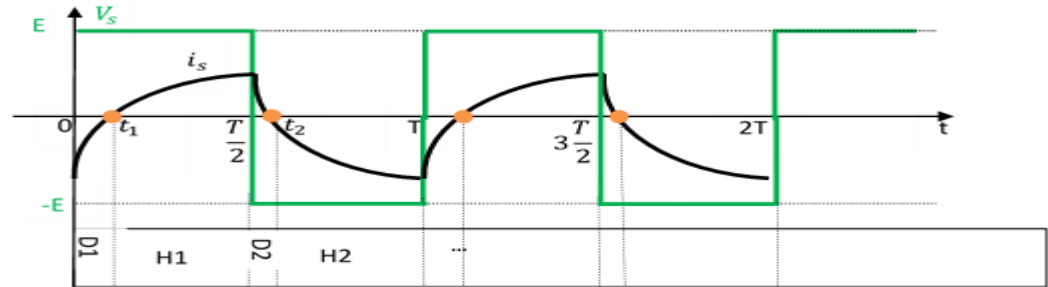


Figure V.5: Voltage and current diagrams for an R-L load

V.2.2 Inverter with four electronic switches (Bridge Inverter):

The single-phase bridge inverter, commonly known as the H-bridge inverter, is one of the most widely used topologies in power electronics for converting a DC input voltage into a single-phase AC output voltage. Its structure is illustrated in Figure V.6, where four controlled electronic switches (K1, K2, K3, and K4) are arranged in the form of the letter “H”, with the load connected between the midpoints of the two switching legs.

Each electronic switch is associated with an antiparallel diode (D1, D2, D3, D4), also called a freewheeling diode. These diodes play an essential role when the load is inductive, since they provide a path for the load current to flow during the switching transitions and prevent dangerous over voltages that could damage the semiconductor devices.

The inverter is supplied by a continuous DC voltage source E , while the load, connected between the two midpoints of the bridge, receives an alternating voltage V_s with a controllable

amplitude and frequency.

The operating principle of the H-bridge inverter is based on the complementary switching of two pairs of switches:

When switches K1 and K3 are closed simultaneously (while K2 and K4 are open), the load is connected directly to the DC source, and the output voltage becomes:

$$V_s = +E$$

When switches K2 and K4 are closed simultaneously (while K1 and K3 are open), the polarity of the voltage applied to the load is reversed, and the output voltage becomes:

$$V_s = -E$$

By alternating between these two switching states at a controlled frequency, the inverter produces an alternating square-wave voltage across the load. The frequency of the output voltage is directly determined by the switching frequency of the controlled devices, while its amplitude depends on the DC input voltage E .

To improve the quality of the output waveform and reduce the harmonic content, more advanced control strategies such as Pulse Width Modulation (PWM) are commonly used. PWM allows the output voltage to approximate a sinusoidal waveform, which is particularly important in applications such as:

- AC motor drives,
- Uninterruptible Power Supplies (UPS),
- Solar photovoltaic systems,
- Induction heating,
- Standalone AC power sources.

Thanks to its simple structure, high efficiency, and ability to control both the amplitude and the frequency of the output voltage, the H-bridge inverter is considered the fundamental building block of most modern single-phase DC/AC conversion systems.

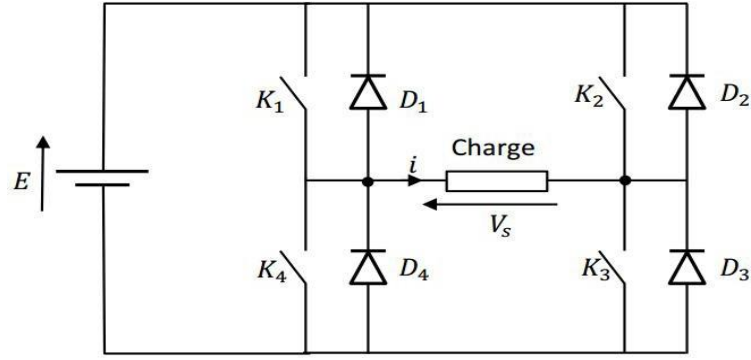


Figure V.6: Single-phase inverter with

four switches The characteristics of this circuit are as follows:

- The input voltage E
- For $0 < t < \frac{T}{2}$ H_1, H_3 are closed while H_2, H_4 are open
- For $\frac{T}{2} < t < T$ H_2, H_4 are closed while H_1, H_3 are open
- Diodes are useless for a resistive load, but they are indispensable if the load is inductive. They ensure that the current in the inductance is not interrupted.

The output current and voltage shapes, in the case of a four-switch inverter, are the same as those of a two-switch inverter.

All data are contained in the following table:

t	V_s	i_s	Controlled Switches	Passing elements
$0 < t < t_1$	$V_s = E > 0$	$i_s < 0$	H_1 and H_3 : closed	D_1 et D_3
$t_1 < t < \frac{T}{2}$		$i_s > 0$	H_1 and H_3 : closed	H_1 et H_3
$\frac{T}{2} < t < t_2$	$V_s = -E < 0$	$i_s < 0$	H_2 and H_4 : closed	D_2, D_4
$t_2 < t < T$		$i_s > 0$	H_2 and H_4 : closed	$H_2; H_4$

From the table above, it can be observed that the operation of the H-bridge inverter is based on

two complementary switching sequences that alternate periodically to produce an AC output voltage from a DC source. During each half-period, either the controlled switches transfer energy from the source to the load, or the freewheeling diodes ensure the continuity of the current when the load is inductive. This alternation guarantees a stable operation of the inverter and protects the semiconductor devices from over voltages caused by sudden current interruptions.

Exercises on DC/AC Converters (Single-Phase Inverters)

Exercise 1: Single-Phase Half-Bridge Inverter

A single-phase half-bridge inverter is supplied by a DC source:

$$V_{dc}=200 \text{ V}$$

$$\text{Load resistance } R=20 \text{ } \Omega$$

Assume ideal switches and square-wave operation.

Questions:

1. Sketch the output voltage waveform.
2. Determine the peak value of the output voltage.
3. Calculate the RMS output voltage.
4. Determine the RMS load current.
5. Calculate the power delivered to the load.

Exercise 2: Single-Phase Full-Bridge Inverter

A full-bridge inverter is supplied by:

$$V_{dc}=300 \text{ V}$$

$$\text{Resistive load } R=30 \text{ } \Omega$$

The inverter operates in square-wave mode.

Questions:

1. Sketch the output voltage waveform.
2. Determine the peak output voltage.
3. Calculate the RMS output voltage.
4. Determine the RMS load current.
5. Compare the output voltage amplitude with the half-bridge inverter.

Exercise 3: PWM Inverter

A single-phase full-bridge inverter uses sinusoidal PWM.

$$V_{dc}=400 \text{ V}$$

$$\text{Modulation index } m=0.8$$

$$\text{Resistive load } R=40 \text{ } \Omega$$

Questions:

1. Write the expression of the fundamental output voltage.
2. Calculate the RMS value of the fundamental component.
3. Determine the RMS load current (fundamental).
4. Explain the advantage of PWM compared to square-wave operation.

5. Why does PWM reduce harmonic distortion?

Exercise 4: RL Load – Practical Application

A full-bridge inverter supplies an RL load:

$$V_{dc}=250 \text{ V}$$

$$R=15 \Omega$$

$$L=30 \text{ mH}$$

Questions:

1. Explain why the current waveform differs from the voltage waveform.
2. Define freewheeling operation in inverters.
3. Is the current continuous or discontinuous? Justify.
4. Explain why inverters are essential in solar energy systems.
5. Give two practical applications of single-phase inverters.

Conclusion

Conclusion

As part of my teaching responsibilities for third-year Electrotechnics students, I delivered the Power Electronics course, which represents a fundamental component of their academic and professional training in electrical engineering. The course was designed to provide students with both theoretical knowledge and practical understanding of modern power conversion systems, which play a central role in industrial processes and renewable energy technologies.

The program began with Power Semiconductor Switches, where students were introduced to the main devices used in power electronics, including power diodes, thyristors (SCRs), MOSFETs, and IGBTs. Particular emphasis was placed on their operating principles, static and dynamic characteristics, switching behavior, and selection criteria. Understanding these devices is essential, as they form the core elements of all power converters. Students also learned about switching losses, thermal considerations, and the importance of proper device protection in practical applications.

The second part of the course focused on AC/DC converters (Rectifiers), covering both uncontrolled (diode-based) and controlled (thyristor-based) configurations. Single-phase and three-phase rectifiers were analyzed in detail, including waveform analysis, average and RMS value calculations, ripple content, and the effect of firing angle control. Through this chapter, students developed the ability to evaluate and compare different rectification systems and understand their applications in DC power supplies, battery charging systems, and DC motor drives.

The third chapter addressed AC/AC converters, particularly voltage controllers (dimmers). Students studied phase control techniques used to regulate AC voltage supplied to resistive and inductive loads. The influence of the firing angle on RMS voltage, load current, and output power was examined. Practical applications such as lighting control and heating systems were discussed to connect theoretical concepts with real-world systems.

The fourth chapter was dedicated to DC/DC converters (Choppers). Step-down (Buck) and step-up (Boost) converters were studied under continuous conduction mode, with analysis of duty cycle control, output voltage regulation, and power flow. Students learned how these converters are widely used in regulated power supplies, electric vehicles, and renewable energy systems. The relationship between switching frequency, efficiency, and component sizing was also highlighted to strengthen their analytical skills.

Finally, the course covered Single-Phase DC/AC Converters (Inverters). Students examined half-bridge and full-bridge inverter topologies operating in square-wave and PWM modes. The analysis included RMS values, fundamental components, and waveform characteristics. Special attention was given to the importance of inverters in solar energy systems, backup power supplies (UPS), and standalone energy applications.

Throughout the course, emphasis was placed on problem-solving, waveform analysis, and the interpretation of converter performance under different operating conditions. The objective was not only to present theoretical formulas, but also to develop the students' ability to analyze, compare, and select appropriate power conversion solutions depending on technical requirements.

By the end of the program, students acquire a solid foundation in power electronics, enabling them to pursue advanced studies or integrate into professional fields such as electric machine control, electric vehicle systems, wind and solar energy installations, and industrial automation. Overall, this course contributes significantly to their preparation as future electrical engineers capable of understanding and designing efficient and reliable power conversion systems.

Annex

Corrected Exercises

This section presents a selection of corrected exercises covering the main topics discussed throughout this course. For each chapter, one representative exercise has been solved in detail in order to help students better understand the application of theoretical concepts and develop their problem-solving skills in power electronics.

Chapter I: Power Semiconductor Switches

Corrected Exercise (Exercise 1 – Power Diode)

Statement:

A power diode conducts a forward current of 25 A with a forward voltage drop of 1.2 V.

Solution :

1. Conduction power loss: $P = 1.2 \times 25 = 30 \text{ W}$
2. Energy dissipated in 10 hours: $E = 30 \times 10 = 300 \text{ Wh}$
3. Ideal diode: no losses. Real diode: forward drop + reverse recovery.
4. Reverse recovery time: time needed for the diode to stop conducting in reverse.
5. Important because it causes switching losses and limits frequency.

Chapter II: AC/DC Converters (Rectifiers)

Corrected Exercise (Exercise 2 – Full-Wave Bridge Rectifier)

Statement:

A single-phase full-wave bridge rectifier is connected to 230 V (RMS), 50 Hz, with $R = 25 \Omega$.

Solution:

1. $V_m = 230 \times \sqrt{2} \approx 325 \text{ V}$
2. $V_{DC} = (2 \times V_m) / \pi \approx 207 \text{ V}$
3. $I_{DC} = 207 / 25 \approx 8.28 \text{ A}$
4. Ripple frequency = 100 Hz
5. Full-wave gives higher DC voltage, lower ripple, better efficiency than half-wave.

Chapter III: AC/AC Converters (Dimmers)

Corrected Exercise (Exercise 1 – Dimmer with Resistive Load)

Statement:

A dimmer feeds a resistive load $R = 30 \Omega$ from 220 V (RMS), 50 Hz, with $\alpha = 60^\circ$.

Solution:

1. $V_m = 220 \times \sqrt{2} \approx 311 \text{ V}$
2. $V_{\text{RMS}} \approx 220 \times 0.877 \approx 193 \text{ V}$
3. $I_{\text{RMS}} = 193 / 30 \approx 6.43 \text{ A}$
4. $P = V_{\text{RMS}}^2 / R \approx 1241 \text{ W}$
5. Increasing α reduces the output voltage and power delivered to the load.

Chapter IV: DC/DC Converters (Choppers)

Corrected Exercise (Exercise 1 – Buck Converter)

Statement:

A Buck converter: $V_{\text{in}} = 100 \text{ V}$, $D = 0.4$, $R = 20 \Omega$.

Solution :

1. $V_{\text{out}} = D \times V_{\text{in}} = 40 \text{ V}$
2. $I_{\text{out}} = V_{\text{out}} / R = 2 \text{ A}$
3. $P_{\text{out}} = 80 \text{ W}$
4. $I_{\text{in}} = D \times I_{\text{out}} = 0.8 \text{ A}$
5. Since $V_{\text{out}} < V_{\text{in}}$, the converter operates as a step-down (Buck) converter.

Chapter V: DC/AC Converters (Single-Phase Inverters)

Corrected Exercise (Exercise 2 – Full-Bridge Inverter)

Statement:

A single-phase full-bridge inverter: $V_{\text{dc}} = 300 \text{ V}$, $R = 30 \Omega$, square-wave operation.

Solution:

1. $V_{\text{peak}} = 300 \text{ V}$
2. $V_{\text{RMS}} = 300 \text{ V}$
3. $I_{\text{RMS}} = V_{\text{RMS}} / R = 10 \text{ A}$
4. $P = V_{\text{RMS}} \times I_{\text{RMS}} = 3000 \text{ W} = 3 \text{ kW}$
5. Full-bridge produces twice the amplitude of the half-bridge for the same DC source.

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