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Ministry of Higher Education and Scientific Research
Salhi Ahmed University of Naâma

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Department of : Electrical Engineering
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Teaching Handout

Antennas and Transmission Lines

Course

Speciality : Telecommunications
Level : 3rd

Teaching Handout prepared by :

BRAHIMI Mohamed – Lecturer

In accordance with the standards of the National Reference Framework (Order N°. 164 of 18 October 2025); relating to the structuring of lecture, tutorial (TD) and Practical work (TP)

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Syllabus

1. General Information

- Course Title: Antennas and Transmission Lines
- Teaching Unit: ■ Fundamental ☐ Methodological ☐ Transversal ☐ Discovery
- Program / Specialty: Bachelor/ Telecommunications
- Level (Target Audience): .Third
- Semester: 6
- Academic Year: 2025-2026
- Credits: [4]
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- Weekly Hour Load: [3]
- Instructor's Full Name: BRAHIMI Mohamed
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- Office:
- Teaching Room: 2. | Schedule: 8:30-11:30
- Access to the course via Moodle platform (if blended learning): ☞https://

2. Course presentation

- Context setting: *(A presentation designed to stimulate student interest).*
- General objectives

This course is intended for students in the third year of the Bachelor–Master–Doctorate (LMD) program in higher education, specialising in telecommunications within the field of science and technology. The aim of this course is to introduce students to the technologies involved in radio-frequency wave transmission, various types of antennas, and transmission lines in general. In addition, the course provides students with essential background knowledge of the fundamental principles of microwave engineering.

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3. Targeted Competencies

- Analyse telegrapher's equations to model voltage/current waves on real transmission lines.
- Calculate primary parameters (R, L, G, and C) and the characteristic impedance of lossy lines.
- Design radiation patterns and determine antenna parameters like gain, directivity, and polarisation.
- Derive electromagnetic fields from electric/magnetic dipoles using vector potentials.
- Select and optimise practical antennas (Yagi-Uda, helical, parabolic) for specific applications like high gain or circular polarisation.

4. Content

To facilitate students' understanding of this course, the document is organised into five (5) chapters, designed to address the essential and fundamental questions of this discipline. The first and second chapters cover the principles of guided wave propagation, the telegrapher's equations, and the primary parameters of real transmission lines.
Chapter 3 systematically introduces all the key parameters characterising an antenna, ranging from the radiation pattern to polarisation.
Chapter 4 presents the theory of radiation starting from the electric dipole and isolated dipole, including complete calculations of the electromagnetic fields.
Chapter 5 discusses concrete technological solutions for antennas, including folded antennas for impedance matching, compact magnetic loops for portable applications, Yagi–Uda antennas for high directivity, helical antennas for circular polarisation, and parabolic antennas for very high gain.

5. Prerequisites:

Electromagnetism fundamentals (electrostatics, magnetostatics, Maxwell's equations).

Complex numbers and phasors.

Basic circuit theory (RLC circuits, impedance).

6. Methodology

Lectures (CM, 45 hours): Theoretical explanations with derivations (e.g., telegrapher's equations and dipole fields) using a blackboard and slides.

Tutorials (TD, 22.5 hours): Problem-solving sessions on impedance matching, radiation patterns, and MATLAB simulations for array synthesis.

Active Learning: Flipped classroom for Chapter 5 (pre-read tech solutions), quizzes, and peer discussions.

7. Assessment procedure

Type of assessment
The assessment measures learning outcomes while supporting student progress through a continuous, balanced approach with these three types:

Diagnostic assessment: At semester start and/or each sequence beginning to identify prerequisites and initial student levels (e.g., 20-min electromagnetism quiz).

Formative assessment: Low-stakes/non-graded activities (quizzes, TD exercises, chapter summaries, in-class work) for practice, self-correction, and continuous improvement.

Summative assessment: Final exam or graded project to validate defined competencies.

Weighting

- Continuous assessment: 10 points
  - Collaborative work (group projects, oral presentations)
  - Active participation (flipped classroom activities, discussions, in-class engagement)
- Final exam: 10 points

- Written examination covering theoretical knowledge and expected applied skills

Assessment Criteria
The evaluation is based in particular on:

- Scientific rigor and accuracy of concepts,
- Analytical and problem-solving skills,

8. Support Resources

Documents: Course handout (PDF/Word), tutorial sheets, practical work guides, MATLAB scripts for radiation patterns and array synthesis.
Software: MATLAB, CS Studion, Ansys HFSS for antenna simulations.
Academic Support: Office hours/tutoring, cyber library access, student WhatsApp group for Q&A.

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List of Abbreviations

Abbreviation	Meaning
EM / EMW	Electromagnetic / electromagnetic wave
RF	Radio frequency
HF, VHF, UHF, SHF, EHF	High, Very-high, Ultra-high, Super-high, Extremely-high frequency bands
DC	Direct current (used for “DC–GHz” frequency ranges)
EMI	Electromagnetic interference
RFI	Radio-frequency interference
CATV	Community antenna television (cable TV)
MRI	Magnetic resonance imaging
LAN	Local area network
UTP	Unshielded twisted pair
STP	Shielded twisted pair
PCB	Printed circuit board
MMIC	Monolithic microwave integrated circuit
NVIS	Near vertical incidence skywave
DX	Long-distance HF communication
EIRP	Effective isotropic radiated power
Db	Decibel
DBi	Decibels relative to isotropic radiator
dBm	Decibels relative to 1 mW

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dBW	Decibels relative to 1 W
WiFi	Wireless Fidelity (IEEE 802.11 wireless LAN)
GPS	Global Positioning System
GNSS	Global Navigation Satellite System

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List of Symbols

Symbol	Meaning
z	Position coordinate along the transmission line
Z_0	Characteristic impedance of the transmission line
Z_L	Load impedance at the line termination
Z_g	Generator (source) internal impedance
Z_{in}	Input impedance of the line at a given point
Γ	Voltage reflection coefficient on the line
T or τ	Voltage transmission coefficient
V^+, V^-	Incident and reflected voltage amplitudes
$V_{\max}, V_{\min}$	Maximum and minimum voltage of standing wave
I^+, I^-	Incident and reflected current amplitudes
R, L, C, G	Primary line constants (series resistance, inductance, shunt capacitance, shunt conductance per unit length)
Z_c or Z_0	Characteristic impedance expressed via L and C
γ	Complex propagation constant
α	Attenuation constant (nepers per meter)
β	Phase constant (radians per meter)
v_p	Phase velocity on the line
η	Efficiency (power transfer or antenna, by context)
$P_{\text{inc}}, P_{\text{ref}}, P_L$	Incident, reflected, and load (delivered) power
$\vec{S}$ or $\vec{W}$	Poynting vector (power flux density)
$\vec{E}$	Electric field vector
$\vec{H}$	Magnetic field intensity vector
$U(\theta, \phi)$	Radiation intensity
D	Antenna directivity
G	Antenna gain (context-dependent vs. line conductance)
λ	Wavelength
f	Frequency
c	Speed of light in vacuum

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ε_0, μ_0	Permittivity and permeability of free space
ε_r	Relative permittivity of a dielectric
σ	Electrical conductivity
δ	Skin depth
a, b	Inner and outer conductor radii of coaxial cable
d	Conductor diameter or spacing (see context)
D (capital)	Center-to-center spacing in parallel-wire line Directivity of antenna
r	Radial distance from antenna (spherical coordinates)
w, h, t	Width, substrate thickness, and conductor thickness of microstrip line
HPBW	Half-power beamwidth of radiation pattern
FNBW	First-null beamwidth
SLL	Side-lobe level
SWR, VSWR, ROS	Standing-wave ratio (Voltage Standing Wave Ratio)

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General Introduction

This course handout provides a comprehensive foundation in transmission lines, wave propagation, and antenna theory, essential for telecommunications students and researchers working on RF systems, antenna design, and electromagnetic applications. It progresses logically from fundamental wave behaviours and line modelling to advanced antenna characteristics, radiation analysis, and practical antenna configurations, equipping learners with tools for simulation, optimisation, and real-world implementation.Chapter 1 delves into propagation and transmission lines, reviewing incident, reflected, and standing waves along with key metrics like reflection coefficient, transmission coefficient, and standing wave ratio. It covers modelling of parallel-plane lines, their equivalent circuits with and without losses, solutions to telegrapher's equations, power flow across generator-line-load sections, and the Smith chart for impedance matching.Chapter 2 examines types of transmission lines and their applications, highlighting examples such as coaxial, two-wire, and twisted-pair lines. The focus includes calculating primary parameters like inductance, capacitance, resistance, and conductance for two-wire lines and coaxial cables, bridging theory to practical design.Chapter 3 introduces basic antenna characteristics, splitting into radiation aspects like characteristic area, radiation pattern, power density, radiated power, radiation intensity, directivity, efficiency, gain, and EIRP. Electrical properties cover the antenna model, frequency response, matching conditions, bandwidth, and polarisation, laying groundwork for performance evaluation.Chapter 4 analyses radiation from elementary antennas, detailing far-field electromagnetic calculations for electric dipoles and space-isolated dipoles. It derives key parameters including characteristic area, radiation pattern, radiated power, equivalent height, and radiation resistance, emphasising analytical techniques for dipole behaviour.Chapter 5 surveys antenna types and applications, from folded and loop antennas in various shapes like square, triangle, or rhombus; to wire dipoles for HF; Yagi-Uda arrays with parasitic elements for high directivity and gain; omnidirectional quarter-wave verticals for VHF/EHF; small magnetic loops; helical antennas for decimetre waves with circular polarisation; and parabolic reflectors for microwaves.

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1 Chapter 1: Propagation and Transmission Lines

1.1 Introduction

This Chapter explores the essential principles of wave propagation and transmission lines, forming the groundwork for analysing signal behaviour in systems like antennas and RF circuits. It begins with a review of incident waves travelling from source to load, reflected waves arising from impedance mismatches, and the resulting standing waves that form due to their interference. Incident waves carry energy along the transmission line toward the load, while reflections occur when the load impedance differs from the line's characteristic impedance, sending waves back toward the source. These interacting waves create standing waves, whose severity is quantified by the reflection coefficient a measure of reflected amplitude and the standing wave ratio, which indicates the mismatch level, with a value of one signalling a perfect match.

The chapter models a transmission line between two parallel planes using distributed parameters that represent resistance, inductance, conductance, and capacitance per unit length. An equivalent circuit for a small line segment shows series inductance and resistance paired with shunt capacitance and conductance; losses introduce dissipation, altering wave propagation. Solving the telegrapher's equations reveals voltage and current distributions along the line, distinguishing forward and backward waves across generator, line, and load sections. Power analysis covers incident power from the source, reflected power due to mismatches, and delivered power at the load, highlighting efficiency losses in mismatched systems. The Smith chart provides a graphical tool for visualising normalised impedances on a reflection coefficient plane, simplifying impedance- matching tasks. Users plot the load position, apply line lengths or stubs through chart rotations, and identify configurations that minimise the standing wave ratio without complex calculations.

1.2 Review: Incident Wave, Reflected Wave, and Standing Wave

1.2.1 Fundamental Concepts

When an electromagnetic wave encounters an interface or a discontinuity in a transmission line, part of its energy is transmitted and another part is reflected. The superposition of incident and reflected waves creates complex interference patterns, notably the phenomenon of standing waves.

Incident Wave: The electromagnetic wave propagating from the source toward the load, traveling in the positive direction along the transmission line.

Reflected Wave: The portion of the incident wave that bounces back from the load or discontinuity, traveling in the negative direction along the transmission line.

Standing Wave: The interference pattern resulting from the superposition of incident and reflected waves propagating in opposite directions.

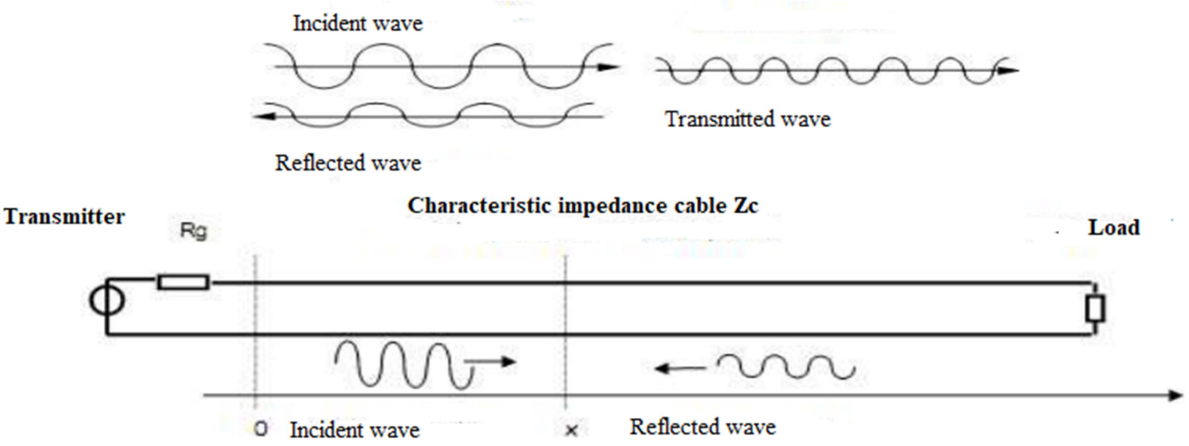


Figure 1.1: Incident and reflected waves on a transmission line between transmitter and load

1.2.2 Wave Representation

For a transmission line with characteristic impedance (Z_0), the incident voltage wave is given by:

$$V^+(z,t) = V_0^+ e^{j(\omega t - \beta z)} \quad (1.1)$$

The reflected voltage wave is represented as:

$$V^-(z,t) = V_0^- e^{j(\omega t + \beta z)} \quad (1.2)$$

Where:

V_0^+ : Incident wave amplitude

V_0^- : Reflected wave amplitude

ω: Angular frequency

β : Phase constant (propagation constant)

z : Position along the transmission line

The total voltage at any point on the line is:

$$V(z,t) = V^+(z,t) + V^-(z,t) \quad (1.3)$$

1.2.3 Reflection Coefficient

The voltage reflection coefficient at a point z on the transmission line is defined as the ratio of the reflected voltage to the incident voltage:

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{V_0^-}{V_0^+} e^{j2\beta z} \quad (1.4)$$

At the load plane (z = 0), the reflection coefficient becomes:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (1.5)$$

Where (Z_L) is the load impedance and (Z_0) is the characteristic impedance of the transmission line.

Properties of the Reflection Coefficient:

- For a matched load ($Z_L = Z_0$): ($\Gamma_L = 0$) (no reflection)
- For an open circuit ($Z_L = \infty$): ($\Gamma_L = +1$) (complete reflection, in-phase)
- For a short circuit ($Z_L = 0$): ($\Gamma_L = -1$) (complete reflection, 180° out-of-phase)

Transmission Coefficient

The voltage transmission coefficient is defined as the ratio of the transmitted voltage to the incident voltage:

$$\tau = \frac{V_t}{V_i} = 1 + \Gamma \quad (1.6)$$

Alternatively, in terms of impedances:

$$\tau = \frac{2Z_L}{Z_L + Z_0} \quad (1.7)$$

Relationship: The transmission and reflection coefficients are related by:

$$\tau = 1 + \Gamma \quad (1.8)$$

This relationship ensures energy conservation at the boundary.

1.2.4 Standing Wave Ratio (SWR)

The Standing Wave Ratio is a key parameter describing the degree of mismatch between the transmission line and the load. It is defined as:

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (1.9)$$

Where V_{max} and V_{min} are the maximum and minimum voltage magnitudes along the transmission line.

The Standing Wave Ratio (SWR) offers clear physical insight into transmission line matching. An SWR of 1 indicates perfect matching, where all incident energy transmits to the load with no reflection. Values greater than 1 reveal a mismatch, causing partial reflections

that reduce efficiency. An infinite SWR signifies a total mismatch, as in a short or open circuit, where all energy reflects back.

The SWR can also be expressed in terms of load and characteristic impedances:

$$SWR = \frac{max(Z_L, Z_0)}{min(Z_L, Z_0)} \tag{1.10}$$

When both impedances are purely resistive.

1.2.5 Voltage and Current Distribution

The voltage distribution along the transmission line is:

$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma_L\, e^{j\beta z}\right) \tag{1.11}$$

The current distribution is:

$$I(z) = \left(\frac{V_0^+}{Z_0}\right)\left(e^{-j\beta z} - \Gamma_L e^{j\beta z}\right) \tag{1.12}$$

The impedance at any point z along the line is:

$$Z(z) = \frac{Z_0\left(1 + \Gamma(z)\right)}{1 - \Gamma(z)} \tag{1.13}$$

1.3 Parallel-Plate Transmission Line Model

1.3.1 Transmission Line Structure

A transmission line can be considered as an electrical circuit composed of four circuit elements:
The quantities R, L, C and G are named 'primary constants of the line'.
L: stored magnetic energy (H/m)
G: dielectric loss in the insulator that is not perfect (S/m) (Siemens/m), it's the conductance between the 2 drivers.
C: stored electrical energy (F/m)
R: ohmic losses (conductor) (Ω/m)

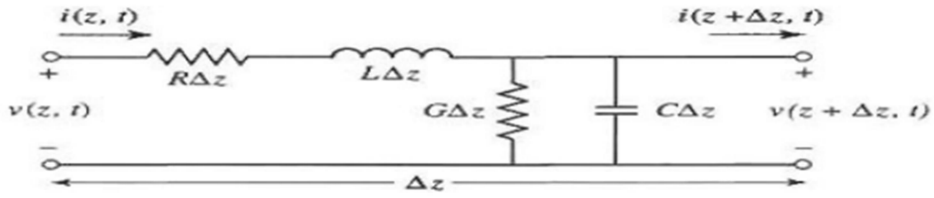


Figure 1.2: Lumped-element equivalent circuit model for a transmission line segment of length

1.3.2 Line Equations (Telegrapher's Equations)

The transmission line is characterized by its distributed series and shunt parameters:

Series Impedance per unit length

$$Z' = R' + j\omega L' \ [\Omega/m] \tag{1.14}$$

Where:

- R': series resistance per unit length
- L' : series inductance per unit length

Shunt Admittance per unit length

$$Y' = G' + j\omega C' \ [\mathrm{S}/m] \tag{1.14}$$

Where:

- G' : shunt conductance per unit length
- C' : shunt capacitance per unit length

Lossless Line (Ideal Case: R' = 0, G' = 0)

For a lossless transmission line, the Telegrapher's equations are:

$$\frac{dV}{dz} = j\omega L' I \tag{1.15}$$

$$\frac{dI}{dz} = j\omega C' V$$

Solving these coupled differential equations yields:

Voltage wave equation

$$\frac{d^2V}{dz^2} + \omega^2 L' C' V = 0 \tag{1.16}$$

Current wave equation

$$\frac{d^2I}{dz^2} + \omega^2 L' C' I = 0 \tag{1.17}$$

The solutions are:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$
$$I(z) = \left(\frac{V_0^+}{Z_0}\right) e^{-j\beta z} - \left(\frac{V_0^-}{Z_0}\right) e^{j\beta z} \quad (1.18)$$

Where the propagation constant is:

$$\beta = \omega\sqrt{L'C'} = \frac{\omega}{v_p} \qquad \left[\frac{rad}{m}\right] \quad (1.19)$$

And the phase velocity is:

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{n} \quad (1.20)$$

The characteristic impedance is:

$$Z_0 = \frac{L'}{C'} \ [\Omega] \quad (1.21)$$

Lossy Line (General Case: R' ≠ 0, G' ≠ 0)
For a lossy transmission line, the Telegrapher's equations are:

$$\frac{dV}{dz} = (R' + j\omega L')I = Z'I$$
$$\frac{dI}{dz} = (G' + j\omega C')V = Y'V \quad (1.22)$$

The solutions are:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = \left(\frac{V_0^+}{Z_0}\right) e^{-\gamma z} - \left(\frac{V_0^-}{Z_0}\right) e^{\gamma z} \quad (1.23)$$

Where the propagation constant is:

$$\gamma = \alpha + j\beta = \sqrt{Z'Y'} = \sqrt{(R' + j\omega L')(G' + j\omega C')} \quad (1.24)$$

With:

- α : attenuation constant [Np/m]
- β: phase constant [rad/m]

The characteristic impedance is:

$$Z_0 = \sqrt{\frac{Z'}{Y'}} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} [\Omega] \quad (1.25)$$

1.4 Solution of the Telegrapher's Equations and Power Calculations

1.4.1 General Solution for Lossless Lines

For a lossless transmission line, the general voltage and current solutions are:

$$V(z) = V_0^+ e^{-j\gamma z} + V_0^- e^{j\gamma z}$$
$$I(z) = \left(\frac{V_0^+}{Z_0}\right) e^{-j\gamma z} - \left(\frac{V_0^-}{Z_0}\right) e^{j\gamma z} \quad (1.26)$$

The constants (V_0^+) and (V_0^-) are determined by boundary conditions at the load.

At the load (z = 0):

$$V(0) = V_0^+ + V_0^- = V_L$$
$$I(0) = \frac{V_0^+ - V_0^-}{Z_0} = I_L = \frac{V_L}{Z_L} \quad (1.27)$$

Solving for the wave amplitudes:

$$V_0^+ = \frac{V_L}{2}(1 + \frac{Z_L}{Z_0})$$
$$V_0^- = \frac{V_L}{2}\left(\frac{Z_L}{Z_0} - 1\right) = V_0^+ \Gamma_L \quad (1.28)$$

1.4.2 Power Flow Analysis

Power in electromagnetic waves consists of active (real) power and reactive (imaginary) power. The instantaneous power density is given by the Poynting vector, but for transmission line analysis, we focus on time-averaged quantities.

Incident Power

The incident power (power flowing in the positive z-direction) is:

$$P_i = \frac{1}{2} Re(V^+ . I^{+*}) = \frac{|V_0^+|^2}{2Z_0} \quad (1.29)$$

Where the asterisk denotes complex conjugate.

In terms of the incident voltage amplitude:

$$P_i = \frac{|V_0^+|^2}{2Z_0} \quad [W] \quad (1.30)$$

Reflected Power

The reflected power (power flowing in the negative z-direction) is:

$$P_r = \frac{1}{2} Re(V^- . I^{-*}) = \frac{|V_0^-|^2}{2Z_0} = \frac{(|\Gamma_L|^2 |V_0^+|^2)}{2Z_0} \quad (1.31)$$

Alternatively:

$$P_r = \frac{|V_0^-|^2}{2Z_0} = |\Gamma_L|^2 P_i \quad (1.32)$$

Transmitted (Delivered) Power

The net power delivered to the load is:

$$P_t = P_i - P_r = P_i(1 - |\Gamma_L|^2) \quad (1.33)$$

Or in terms of load parameters:

$$P_t = \frac{1}{2} Re(V_L . I_L^*) = \frac{1}{2} |V_L|^2 Re\left(\frac{1}{Z_L^*}\right) \quad (1.34)$$

For a resistive load ($Z_L = R_L$):

$$P_t = \frac{|V_L|^2}{2R_L} \quad (1.35)$$

1.4.3 Power Flow in the Three-Media System

The complete transmission system consists of three main components:

Generator

The source provides:

$$P_{source} = \frac{1}{2} Re(V_s . I_s^*) \quad (1.36)$$

The generator has internal impedance (Z_g), with source voltage (V_s).

Transmission Line

The transmission line carries the incident and reflected waves. In an ideal lossless line, the total power flow (forward + reflected) remains constant at any cross-section:

$$P_{forward} + P_{reflected} = constant \quad (1.37)$$

However, in a lossy line, power dissipates as heat:

$$P_{\{loss\}} = \int_0^l \frac{|I(z)|^2}{2} R'(z) dz \tag{1.38}$$

Where (l) is the line length.

Load

The power delivered to the load is:

$$P_{load} = \frac{|V_L|^2}{2Z_L} \tag{1.39}$$

For maximum power transfer, the load should be matched to the transmission line ($Z_L = Z_0$), and the characteristic impedance should match the source impedance ($Z_0 = Z_g$).

1.4.4 Power Transfer Efficiency

The power transfer efficiency is defined as:

$$\eta = \frac{P_t}{P_i} = 1 - |\Gamma_L|^2 = 1 - \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|^2 \tag{1.40}$$

For a matched load ($Z_L = Z_0$):

$$\eta = 1\ (100\%) \tag{1.41}$$

For a severely mismatched load:

$$\eta \approx 0 \tag{1.42}$$

1.4.5 Power Conservation (Energy Balance)

For the complete generator-line-load system:

$$P_{source} = P_{line,loss} + P_{load} \tag{1.43}$$

Where ($P_{line,loss}$) includes both resistive losses (in the conductors and dielectric) and any power absorbed by impedance mismatches.

The reflection coefficient relates these quantities:

$$P_r = |\Gamma_L|^2 P_i \tag{1.44}$$

Therefore:

$$P_{load} = P_i - P_r = P_i(1 - |\Gamma_L|^2) \quad (1.45)$$

1.5 Smith Chart and Impedance Matching

1.5.1 Introduction to the Smith Chart

The Smith Chart is a graphical tool used for solving transmission line problems, particularly for impedance matching, load analysis, and design of matching networks. It provides a visual representation of the relationship between reflection coefficient, impedance, and admittance in the complex plane. Developed in 1939 by P. Smith, it is a polar graph of Γ (the reflection coefficient).

1.5.2 Smith Chart Coordinates

The Smith Chart is constructed by mapping the normalized impedance plane into the complex reflection coefficient plane.

Normalized impedance

$$z = \frac{Z}{Z_0} = r + jx \quad (1.46)$$

Where:

- r : normalized resistance
- x : normalized reactance

Examples

$z_{L1} = 0.2 + 0.5j, z_{L2} = 0.5 - 0.2j, z_{L3} = 1.0 + 0.5j, z_{L4} = 2 + 2j, z_{L5} = 0.2 - j$

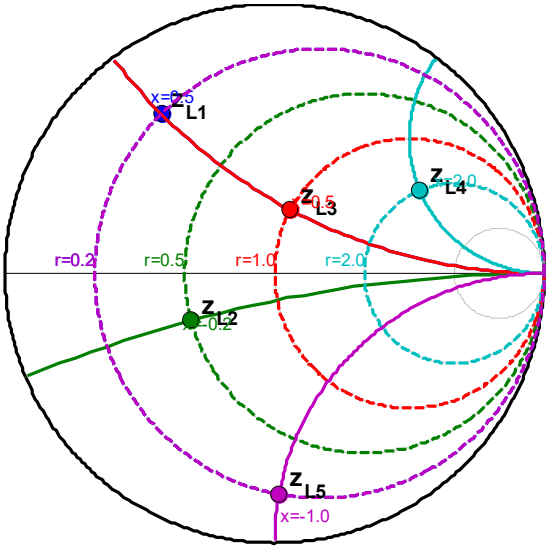


Figure 1.3: Smith Chart with plotted impedance points ZL1 through ZL5 and an approximate VSWR circle (red) passing through ZL3

Relationship to reflection coefficient

$$\Gamma = \frac{z-1}{z+1} = \frac{(r-1)+jx}{(r+1)+jx} \tag{1.47}$$

Conversely:

$$z = \frac{1+\Gamma}{1-\Gamma} \tag{1.48}$$

1.5.3 Smith Chart Features

Resistance Circles (Constant r)

Lines of constant normalized resistance are circles passing through the point (Γ = +1):

$$\left|\Gamma - \frac{r}{1+r}\right| = \frac{1}{1+r} \tag{1.49}$$

- **Center:** $\left(\frac{r}{1+r}, 0\right)$ in the complex plane
- **Radius:** $\left(\frac{1}{1+r}\right)$

Special values

- (r = 0) (short circuit): circle with center at origin, radius = 1
- (r = 1) (matched load): point at center of chart, (Γ = 0)
- $r \rightarrow \infty$(open circuit): point at (Γ = +1)

Reactance Curves (Constant x)

Lines of constant normalized reactance are circles centered on the real axis:

$$\left|\Gamma - \frac{1+jx}{1+x^2}\right| = \frac{|x|}{1+x^2} \tag{1.50}$$

- **Inductive reactance:** (x > 0) (upper half of chart)
- **Capacitive reactance:** (x < 0) (lower half of chart)

Admittance Parameters

The Smith Chart can be used for both impedance and admittance calculations. The admittance circles are rotated 180° relative to the impedance circles, or equivalently, a point on the impedance side corresponds to the diametrically opposite point on the admittance side.

Normalized admittance:

$$y = \frac{1}{z} = \frac{Y}{Y_0} = g + jb \tag{1.51}$$

1.5.4 Impedance Matching Techniques

Impedance matching is essential to maximize power transfer and minimize reflections. Several techniques utilize the Smith Chart:

Single-Stub Matching

A single shunt stub (a short or open-circuited transmission line connected in parallel) is used to cancel the reactive component of the load impedance.

Procedure:

1. Normalize the load impedance: ($z_L = r + jx$)
2. Plot the load on the Smith Chart
3. Move along the constant-conductance circle toward the source until reaching unity conductance ((g = 1))
4. At this point, add a shunt stub with normalized susceptance (b) equal and opposite to the imaginary part encountered
5. The stub length is determined by $b = -\frac{1}{\tan(\beta l_{stub})}$ for a short-circuited stub

Double-Stub Matching

Two stubs are used to provide more design flexibility, particularly when the load location is fixed or inaccessible.

Procedure

1. Space the first stub typically at a distance of $\frac{\lambda}{8}$ or $\frac{\lambda}{4}$ from the load

2. The first stub adjusts the susceptance to bring the admittance to the unity conductance circle
3. The spacing moves the admittance point along the circle
4. The second stub cancels any residual susceptance

L-Network Matching

Series and shunt reactive elements (capacitors and inductors) form L-shaped networks for broad-band matching.

Configuration: Inductor in series, capacitor in parallel (or vice versa depending on load type).

The design equations ensure that the network presents the characteristic impedance Z_0 to the source while matching the load Z_L.

Quarter-Wave Transformer

A transmission line section of length $l = \frac{\lambda}{4}$ and characteristic impedance Z_{opt} can match two different impedances:

$$Z_{opt} = \sqrt{Z_0 \cdot Z_L} \tag{1.52}$$

Properties

$$Z_{in} = \frac{Z_{opt}^2}{Z_L} \tag{1.53}$$

- Input impedance:
- Only effective at the design frequency (narrow-band)
- Simple implementation using microstrip or waveguide

Multi-Section Transformer

Multiple quarter-wave sections with gradually changing impedances provide broad-band matching (Chebyshev transformer, binomial transformer).

1.5.5 Using the Smith Chart for Line Calculations

Traveling Along the Transmission Line

As we move from the load toward the source along a transmission line, the impedance traces a path on the Smith Chart. For a lossless line, this path is a circle (constant VSWR circle) centered at $\Gamma = 0$

VSWR Circle

All points on a circle of constant radius $|\Gamma|$ represent impedances with the same VSWR.

Converting Between Impedance and Admittance

The admittance at the same point is located diametrically opposite on the Smith Chart, rotated by 180°.

Reading Parameters from the Smith Chart

Reflection coefficient magnitude: Distance from the center (origin) to the point, scaled by the radius of the chart (typically 1.0)

SWR: Read from the radial scale at the intersection of the constant |Γ| circle with the positive real axis

Wavelengths toward generator: Read from the outer radial scale (typically marked in fractions of wavelength, 0.0 to 0.5 λ/λ)

1.5.6 Analytical Solution of Matching Problems

For a purely reactive load Z_L = jX, the normalized impedance is:

$$z_L = jx = \frac{jX}{Z_0} \tag{1.54}$$

The reflection coefficient is:

$$\Gamma_L = \frac{jx - 1}{jx + 1} \tag{1.55}$$

The magnitude is: $|\Gamma| = 1$

(Pure reactance results in complete reflection)

For a resistance (R) in series with reactance (jX):

$$z = \frac{R + jX}{Z_0} = r + jx \tag{1.56}$$

The reflection coefficient is:

$$\Gamma = \frac{(r-1) + jx}{(r+1) + jx} \tag{1.57}$$

The SWR is:

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{\sqrt{(r-1)^2 + x^2} + \sqrt{(r+1)^2 + x^2}}{\sqrt{(r+1)^2 + x^2} - \sqrt{(r-1)^2 + x^2}} \tag{1.58}$$

1.6 Conclusion

The first chapter establishes the core principles of wave propagation and transmission lines, bridging theoretical foundations with practical analysis tools vital for RF and antenna

systems. It reviews essential wave interactions incident, reflected, and standing waves quantified through reflection coefficient, transmission coefficient, and standing wave ratio, highlighting mismatch impacts on signal integrity.

The transmission line model for parallel planes, complete with equivalent circuits accounting for losses, leads to telegrapher's equation solutions that reveal voltage, current, and power distributions across the generator, line, and load. Power calculations underscore efficiency losses from reflections, while the Smith chart emerges as a powerful graphical method for impedance matching, enabling efficient design without algebraic complexity.

Mastery of these concepts prepares learners for advanced topics in line types, antenna radiation, and array synthesis, forming the bedrock for simulation-driven optimisation in electromagnetic engineering.

1.7 Series of tutorial exercises

1.7.1 Exercise N°1

Over a distance of **2 km**, a telephone subscriber line operates at frequency **f = 1 kHz** with per-unit-length parameters: **R = 20 Ω/km**, **L = 7 mH/km**, **G = 8×10⁻⁶ S/km**, **C = 13×10⁻⁹ F/km**.

1. Determine if this is a lossy or lossless line.
2. Calculate the characteristic impedance **Zc** of the line.
3. Calculate the propagation constant γ.

1.7.2 Exercise N°2

Lossless Line: A dielectric-filled transmission line has characteristic impedance **Zc = 70 Ω**, phase constant **β = 3 rad/m**, at **f = 100 MHz**. Calculate the inductance L and capacitance C per unit length.
Lossy Line: A transmission line operating at **f = 500 MHz** has **Zc = 80 Ω**, attenuation constant **α = 0.04 Np/m**, and phase constant **β = 1.5 rad/m**. Determine the primary constants (**R**, **L**, **G**, **C**).

1.7.3 Exercise N°3

An ideal transmission line with characteristic impedance **Zc = 75 Ω** and length **L = 0.6λ** is terminated by a load $\mathbf{Z_L}$ **= (150+ j150) Ω**.

1. Determine the voltage standing wave ratio (VSWR) ρ and the positions of voltage maxima and minima along the line.
2. Determine the input impedance at λ/4 from the load, the line input impedance Z_{in}, and the impedances at minima/maxima ($\mathbf{Z_{min}}$, $\mathbf{Z_{max}}$).

1.7.4 Exercise N°4

A voltage generator with effective voltage **Vg = 15 V** and internal impedance **Zg = 75 Ω** feeds a load $\mathbf{Z_L}$ **= 60 Ω** via an ideal transmission line with **Zc = 75 Ω** and electrical length βl (length unspecified).

1. What is the input impedance Zin?
2. What is the power delivered to the line?
3. What is the voltage across the load at z=0?
4. What is the power dissipated by the load?

1.7.5 Exercise N°5:

A lossless transmission line of length **L = 0.3λ** is terminated by a complex load $\mathbf{Z_L}$ **= 20 - j13.5** Ω and **Zc = 50 Ω**.

1. Graphically by the Smith chart, determine :
   - Γ_L
   - ROS
   - Γin
   - Z_{in}
2. Verify results analytically.

1.8 Solutions of series of tutorial exercises

1.8.1 Solution of exercise N°1

1. The line is losy because $R \neq 0$ and $G \neq 0$

2. The characteristic impedance:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = 757 - j126 = 767.3e^{-9.43°}$$

3. The propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = 0.016 + j0.060 = 0.062e^{74.97°}$$

1.8.2 Solution of exercise N°2

Lossless Line:

$$\begin{cases} Z_0 = \sqrt{\dfrac{L}{C}} \\ \beta = \omega\sqrt{L.C} \end{cases} \Rightarrow \begin{cases} L = 3.34.10^{-7} H/m \\ C = 6.82.10^{-11} F/m \end{cases}$$

Lossy Line

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{(R + j\omega L)(R + j\omega L)}{(G + j\omega C)(R + j\omega L)}}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)^2}{(G + j\omega C)(R + j\omega L)}} = \frac{(R + j\omega L)}{\gamma}$$

$$\gamma . Z_0 = R + j\omega L \Rightarrow (\alpha + j\beta).Z_0 = R + j\omega L$$

$$R = \alpha Z_0 = 3.2\ \Omega/m$$
$$L = \frac{\beta Z_0}{\omega} = 3.81.10^{-8} H/m$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Rightarrow {Z_0}^2 = \frac{R + j\omega L}{G + j\omega C}$$

$${Z_0}^2.(G + j\omega C) = R + j\omega L$$

$$G = 0.0005\ S/m$$
$$C = 5.968.10^{-12}\ F/m$$

1.8.3 Solution of exercise N°3

Voltage standing wave ratio:

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.6202e^{29.74}$$

$$VSWR = 4.26$$

Positions from load (toward generator)

First voltage maximum at: $d_{max} = \frac{\theta_\Gamma}{4\pi}\lambda = 0.0413\lambda$

First voltage minimum at: $d_{min} = \frac{\theta_\Gamma + \pi}{4\pi}\lambda = 0.2931\lambda$

Impedance at λ/4λ/4 from load

$$Z\left(\frac{\lambda}{4}\right) = \frac{Z_c^2}{Z_L} = \frac{5625}{150 + j150} = 18.75(1 - j) = 18.75 - j18.75\ \Omega$$

Impedance at the line input (d=0.6λ)

$$\beta d = \left(\frac{2\pi}{\lambda}\right) \times 0.6\lambda = 1.2\pi\ \text{rad} = 216^\circ$$

$$\tan(216^\circ) = \tan(36^\circ) \approx 0.7265$$

$$Z_{in} = Z_c \frac{Z_L + jZ_c\tan(\beta d)}{Z_c + jZ_L\tan(\beta d)} = 98.95 - J134.12\ \Omega$$

Impedance at voltage maxima and minima

$$Z_{max} = Z_{c\rho} \approx 75 \times 4.266 = 319.95\ \Omega$$

$$Z_{min} = \frac{Z_c}{\rho} \approx \frac{75}{4.266} = 17.58\ \Omega$$

1.8.4 Solution of exercise N°4

Input impedance

$$l = \frac{\lambda}{4} \Rightarrow Z_{in} = \frac{{Z_0}^2}{Z_L} = 93.75\Omega$$

Power delivered to the line

$$V_{in} = \frac{Z_{in}V_g}{Z_{in} + Z_g} = 8.33V$$

$$I_{in} = \frac{V_g}{Z_{in} + Z_g} = 0.0889A$$

$$P = V_{in}I_{in} = 0.74W$$

Voltage across the Load

$$P = \frac{|V_L|^2}{Z_L} \Rightarrow V_L = 6{,}667V$$

Power dissipated by the Load

$$P_L = 0{,}74\ W$$

1.8.5 Solution of exercise N°5

Normalised load impedance

$$\boldsymbol{z_L = \frac{Z_L}{Z_c} = \frac{20 - j13.5}{50} = 0.4 - j0.27}$$

Load reflection coefficient

$$\Gamma_L = \frac{OA}{OP} = \frac{3.6}{7.8} = 0.461$$

$$\theta_\Gamma = -146°$$

1.8.5.1 Analytically

$$\Gamma_L = \frac{z_L - 1}{z_L + 1} = \frac{-0.6 - j0.27}{1.4 - j0.27} = -0.3774 - j0.2657$$

$$\Gamma_L = 0.4615e^{-j144.9°}$$

Voltage Standing Wave Ratio (VSWR or ROS)

$$SWR = 2$$

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.4615}{1 - 0.4615} = \frac{1.4615}{0.5385} = 2.714$$

Input reflection coefficient

Electrical length:

$$\beta L = \left(\frac{2\pi}{\lambda}\right).0.3\lambda = 0.6\pi\ rad = 108^o$$

The reflection coefficient at the input is rotated by $-2\beta L$:

$$\Gamma_{in} = \Gamma_L e^{-j2\beta L} = \Gamma_L e^{-j216°} = \Gamma_L e^{j144°}$$

Multiply magnitudes:

$$|\Gamma_{in}| = 0.4615$$

Add phases:

$$\theta_{\Gamma \mathrm{i}} = -144.9° + 144° = -0.9° \ (or\ 359.1°)$$

Thus

$$\Gamma_{in} = O.4615e^{-\mathrm{j}0.9°} = 0.4614 - j0.00725$$

Input impedance

Using:

$$Z_{in} = Z_c \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 135.65 - j2.5\ \Omega$$

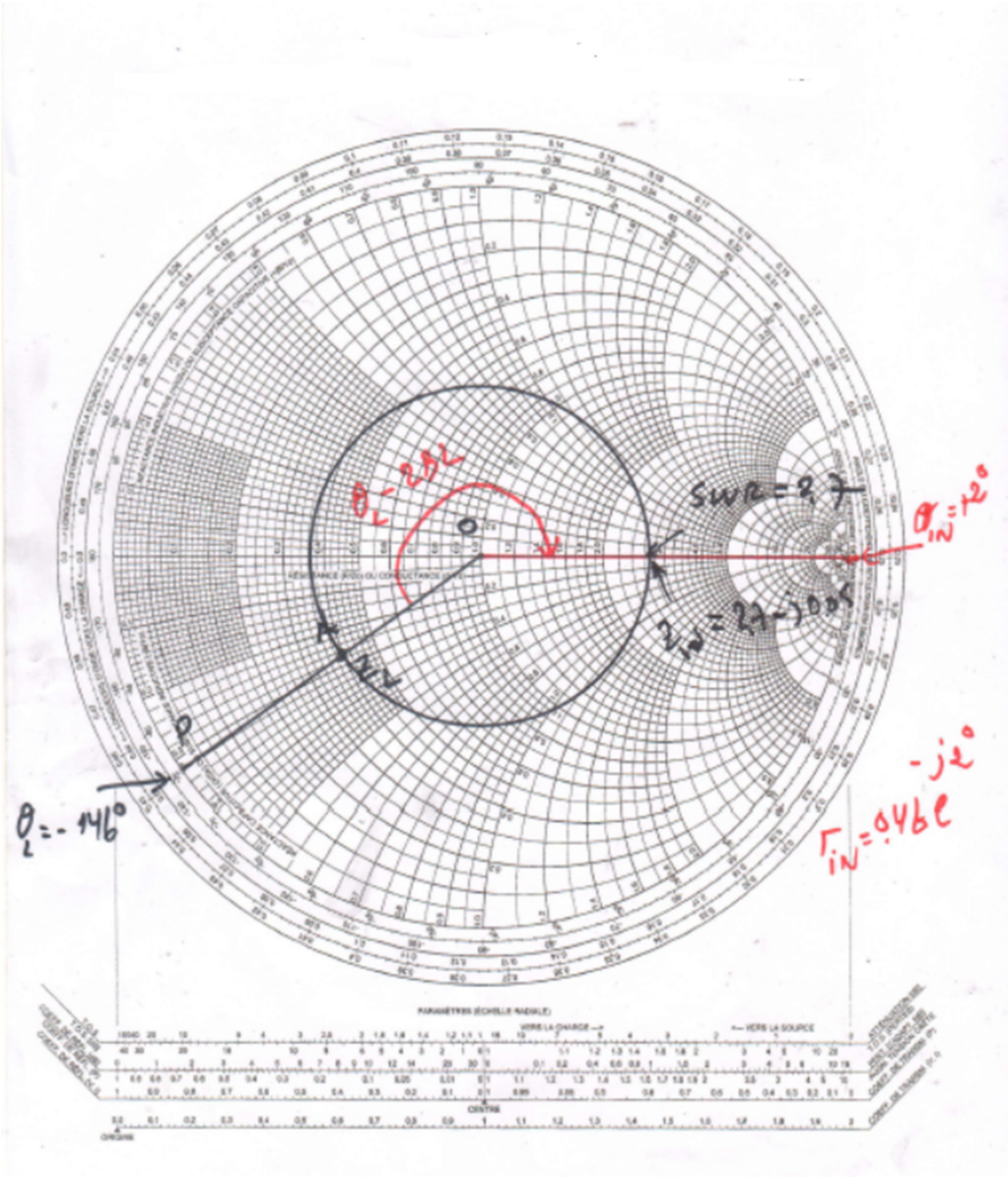


Figure 1.4: Determination of the transmission line parameters graphically using the Smith chart

1.9 Summary of chapter 1

Chapter 1 summarizes the fundamental principles of wave propagation and transmission lines essential for RF systems. Key points include incident/reflected waves, reflection coefficient $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$, VSWR $\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$, distributed parameters (R', L', G', C'), telegrapher's equations yielding $\gamma = \alpha + j\beta$ and Z_0, plus power analysis showing efficiency losses $\eta = 1 - |\Gamma|^2$. Essential concepts cover voltage/current distributions $V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$, Smith chart for impedance matching via stubs/quarter-wave transformers, and 5 solved exercises for practical mastery. Students should retain VSWR/Smith chart skills for RF analysis, as mismatches degrade antenna performance. Now that you have acquired transmission line fundamentals, you can move on to antenna parameters and radiation patterns in Chapter 2.

Chapter 2: Types of Transmission Lines and Their Applications

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2 Chapter 2: Types of Transmission Lines and Their Applications

2.1 Introduction

Transmission lines are used to transmit electromagnetic energy from a source to a load while minimising losses and reflections. Different geometries and configurations are suitable for different frequency ranges and applications. This chapter examines the most common types of transmission lines, their physical characteristics, main parameters and practical applications.

2.2 Coaxial Cable

2.2.1 Physical Structure and Geometry

A coaxial cable consists of:

- **Inner conductor:** A solid or stranded copper wire of radius (a)
- **Dielectric:** An insulating material (typically polyethylene, PTFE, or polyfoam) with relative permittivity (ε_r)
- **Outer conductor:** A copper shield (braid or foil) with inner radius (b)
- **Outer sheath:** Protective jacket (PVC or other polymers)

Geometric Cross-Section:

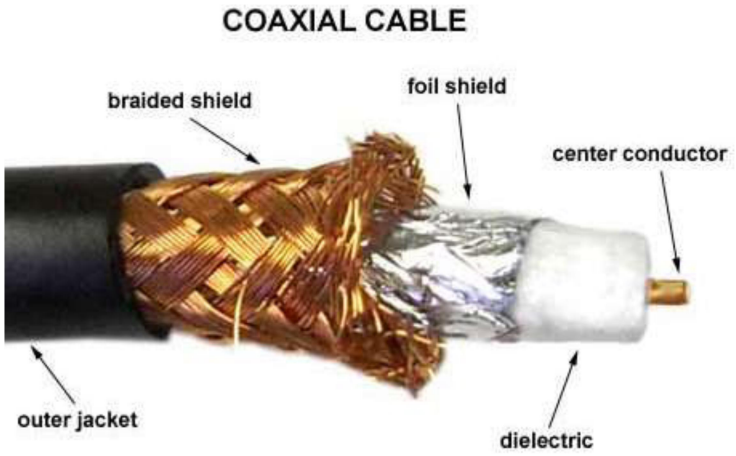


Figure 2.1: Cross section of coaxial cable

2.2.1.1 Primary Parameters of Coaxial Cable

Series Resistance per Unit Length

The resistance of the coaxial cable is distributed between the inner and outer conductors. At high frequencies (where the skin effect dominates), the current flows primarily on the conductor surfaces:

$$R' = \frac{1}{2\pi\sigma\delta}\left(\frac{1}{a}+\frac{1}{b}\right)\ [\Omega/\mathrm{m}] \tag{2.1}$$

Where:

- σ : conductivity of the conductor (copper: $5.8 \times 10^{-7}\ S/m$)
- δ : skin depth $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}\ [m]$
- a: inner conductor radius [m]
- b: outer conductor radius [m]

Simplified form at high frequencies

$$R' = \frac{1}{\pi\sigma\delta}\left(\frac{1}{a}+\frac{1}{b}\right)\ [\Omega/\mathrm{m}] \tag{2.2}$$

Series Inductance per Unit Length

The magnetic field exists only in the dielectric region between the conductors. Using Ampere's law and integrating the magnetic field energy:

$$L' = \frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right)\ [\mathrm{H/m}] \tag{2.3}$$

For a coax with $\mu_r \approx 1$ (non-magnetic):

$$L' = \frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right) = \frac{2 \times 10^{-7}}{2\pi} ln\left(\frac{b}{a}\right)\ [\mathrm{H/m}] \tag{2.4}$$

Shunt Capacitance per Unit Length

The electric field is radial, with dielectric between inner and outer conductors:

$$C' = \frac{2\pi\varepsilon_0\epsilon_r}{ln\left(\frac{b}{a}\right)}\ [\mathrm{F/m}] \tag{2.5}$$

Where:

- $\epsilon_0 = 8.854\ \times 10^{-12}\ [F/m]$ (permittivity of free space)
- ϵ_r : relative permittivity of the dielectric

Shunt Conductance per Unit Length

Represents dielectric losses:

$$G' = \omega C' tan(\delta_d)\ [\mathrm{S/m}] \tag{2.6}$$

Where: $tan(\delta_d)$ is the dielectric loss tangent of the insulation material.
For low-loss dielectrics (typical polyethylene), $G' \ll \omega C'$, so shunt conductance is often neglected.

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2.2.2 Characteristic Impedance of Coaxial Cable

The characteristic impedance is determined by the ratio of series inductance to shunt capacitance:

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right)}{\frac{2\pi\varepsilon_0\epsilon_r}{ln\left(\frac{b}{a}\right)}}} \tag{2.7}$$

Simplifying:

$$Z_0 = \frac{\mu_0 ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0\epsilon_r ln\left(\frac{b}{a}\right)} \sqrt{\frac{1}{4\pi^2}} ln\left(\frac{b}{a}\right) \tag{2.8}$$

After algebraic simplification:

$$Z_0 = \frac{\eta_0}{2\pi\epsilon_r} ln\left(\frac{b}{a}\right) \ [\Omega] \tag{2.9}$$

Where: $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \Omega$ is the intrinsic impedance of free space.

$$Z_0 \approx \frac{138}{\sqrt{\epsilon_r}} log_{10}\left(\frac{b}{a}\right) \ [\Omega] \tag{2.10}$$

Or using natural logarithm:

$$Z_0 \approx \frac{59.95}{\sqrt{\epsilon_r}} ln\left(\frac{b}{a}\right) \ [\Omega] \tag{2.11}$$

2.2.3 Phase Velocity and Velocity Factor

The phase velocity in the dielectric is:

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \tag{2.12}$$

Where:

- $c = 3 \times 10^8\ m/s$: speed of light in vacuum.
- $n = \sqrt{\mu_r\epsilon_r}$: refractive index of the dielectric.

Velocity factor

$$v_f = \frac{v_p}{c} = \frac{1}{\sqrt{\mu_r\epsilon_r}} \tag{2.13}$$

For typical coaxial cables with polyethylene dielectric $\epsilon_r \approx 2.25$:$v_f \approx 0.66\ to\ 0.68$

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This means electromagnetic waves travel at approximately 66-68% of the speed of light.

2.2.4 Typical Coaxial Cable Specifications

Table (2.1) summarises the main characteristics of common coaxial cables, including characteristic impedance, dielectric material, operating frequency range, and typical applications ranging from low-frequency test equipment to microsystem high-frequency waves.

Table 2.1: Typical specifications of common coaxial cables used in RF and microwave applications

Cable Type	Impedance	Dielectric	Frequency Range	Common Applications
RG-58	50 Ω	Polyfoam	DC–1 GHz	Audio, RF test leads, antenna cables
RG-59	75 Ω	Polyfoam	DC–1 GHz	Video, television distribution
RG-174	50 Ω	Polyethylene	DC–1 GHz	Flexible, low-cost RF applications
RG-213	50 Ω	Polyethylene	DC–2 GHz	Military and industrial RF
LMR-400	50 Ω	Foam polyethylene	DC–6 GHz	Cellular, microwave systems
9913	50 Ω	Air-dielectric foam	DC–18 GHz	Low-loss, high-frequency systems
85 OMNI	50 Ω	Polyethylene	DC–2 GHz	Marine, outdoor installations

2.2.5 Advantages and Disadvantages of Coaxial Cable

Advantages

- Excellent EMI/RFI shielding due to outer conductor
- Wide frequency range capability (DC to microwave)
- Standard impedances (50 Ω, 75 Ω) readily available
- Various sizes and flexibility options
- Well-established industry standards and connectors

Disadvantages

- Higher cost compared to open-wire transmission lines
- Diameter increases with lower frequencies (larger cross-section needed)
- Attenuation increases with frequency
- Center conductor can experience stress if bent sharply
- Dielectric losses increase with temperature and frequency

2.2.6 Applications of Coaxial Cable

- **RF and Microwave Systems:** Antenna feed lines, waveguide transitions, measurement systems

- **Video Distribution:** Television, CATV, surveillance systems
- **Medical Equipment:** Ultrasound, MRI systems
- **Test and Measurement:** Oscilloscope probes, RF signal generators, spectrum analyzers
- **Telecommunications:** Trunk lines, cell tower installations
- **Military and Aerospace:** Harsh environment RF systems
-

2.3 Parallel Wire (Two-Wire) Transmission Line

2.3.1 Physical Structure and Geometry

A parallel wire transmission line consists of:

- **Two parallel conductors:** Solid or stranded wires of radius (a)
- **Spacing:** Center-to-center distance D
- **Dielectric:** Surrounding medium (air, polyethylene, or other)

Geometric Cross-Section (Looking Along the Wire Axis)

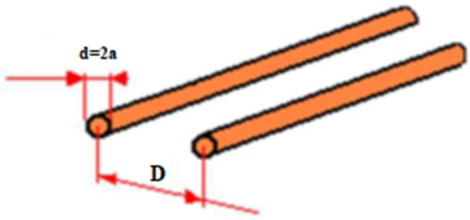


Figure 2.2: Geometric cross-section of a parallel wire (two-wire) transmission line showing conductor diameter d and centre-to-centre spacing D

The separation D must be greater than 2a to avoid conductor overlap.

2.3.2 Primary Parameters of Parallel Wire Line

2.3.2.1 Series Resistance per Unit Length

For two conductors carrying opposite currents, the resistance is:

$$R' = \frac{2}{2\pi\sigma\delta a} = \frac{1}{\pi\sigma\delta a} \ [\Omega/\text{m}] \qquad (2.14)$$

Where:

- σ : conductivity of the wire material
- δ:skin depth: ($\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$)
- a :wire radius

For copper at frequency (f) [Hz]:

$$R' = \frac{0.0159}{\sqrt{f}.a} = \ [\Omega/\mathrm{m}] \tag{2.15}$$

Where a is in meters.

2.3.2.2 Series Inductance per Unit Length

The magnetic field extends from one conductor to the other. The inductance per unit length includes both internal and external inductance:

$$L' = \frac{\mu_0}{\pi}\left[\frac{1}{8} + ln\left(\frac{d}{a}\right)\right] \ [\mathrm{H/m}] \tag{2.16}$$

Where:

- d : center-to-center spacing
- a :wire radius
- The term (1/8) accounts for internal inductance
- The $ln\left(\frac{d}{a}\right)$ term represents external inductance

Approximation for (d >> a):

$$L' \approx \frac{\mu_0}{\pi} ln\left(\frac{d}{a}\right) \ [\mathrm{H/m}] \tag{2.17}$$

2.3.2.3 Shunt Capacitance per Unit Length

The electric field exists in the dielectric medium surrounding both conductors. For two identical parallel cylinders:

$$C' = \frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d}{a}\right)} = \frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d+\sqrt{d^2-4a^2}}{2a}\right)} \ [\mathrm{F/m}] \tag{2.18}$$

Simplified form for (d >> a):

$$C' \approx \frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d}{a}\right)} \ [\mathrm{F/m}] \tag{2.19}$$

2.3.2.4 Shunt Conductance per Unit Length

For air-insulated lines, shunt conductance is negligible (G' ≈ 0). For dielectric-insulated lines:

$$G' = \omega C' tan(\delta_d) \ [\mathrm{S/m}] \tag{2.20}$$

2.3.3 Characteristic Impedance of Parallel Wire Line

$$Z_0 = \sqrt{\frac{L'}{C'}} \tag{2.21}$$

Substituting the expressions for (L') and (C'):

$$Z_0 = \sqrt{\frac{\frac{\mu_0}{\pi} ln\left(\frac{d}{a}\right)}{\frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d}{a}\right)}}} \tag{2.22}$$

Simplifying:

$$Z_0 = \frac{\mu_0}{\pi\varepsilon_0\epsilon_r} . \frac{ln^2\left(\frac{d}{a}\right)}{ln\left(\frac{d}{a}\right)} = \frac{\eta_0}{\pi\epsilon_r} ln\left(\frac{d}{a}\right) \quad [\Omega] \tag{2.23}$$

Where $\eta_0 \approx 377\ \Omega$.

Numerical form:

$$Z_0 \approx \frac{120}{\sqrt{\epsilon_r}} . ln\left(\frac{d}{a}\right) \ [\Omega] \tag{2.24}$$

Or using base-10 logarithm:

$$Z_0 \approx \frac{120}{\sqrt{\epsilon_r}} . log_{10}\left(\frac{d}{a}\right) \ [\Omega] \tag{2.25}$$

2.3.4 Comparison with Coaxial Cable

For the same conductor radius (a):

Coaxial cable characteristic impedance:

$$Z_{0,coax} = \frac{138}{\sqrt{\epsilon_r}} . log_{10}\left(\frac{b}{a}\right) \ [\Omega] \tag{2.26}$$

Parallel wire characteristic impedance:

$$Z_{0,wire} = \frac{276}{\sqrt{\epsilon_r}} . log_{10}\left(\frac{d}{a}\right) \ [\Omega] \tag{2.27}$$

For typical values:

$\frac{d}{a} \approx 3$ and $Z_0 \approx 300 - 600\ \Omega$ typically for parallel wire.

$\frac{b}{a} \approx 3$ and $Z_0 \approx 50 - 75\ \Omega$ typically for coaxial cable.
This shows that parallel wire lines naturally have much higher impedance than coaxial cables.

Velocity Factor

$$V_f = \frac{1}{\sqrt{\mu_r \epsilon_r}} \tag{2.28}$$

For air-insulated parallel wire $\epsilon_r \approx 1\ V_f = 0.95\ to\ 0.98$
For parallel wire with polyethylene covering $\epsilon_r \approx 2.25\ V_f \approx 0.66$:

2.3.5 Typical Parallel Wire Line Specifications

Common specifications for parallel wire transmission lines, including geometry, characteristic impedances, dielectrics, and applications, are assembled in Table 2.2.

Table 2.2: Typical specifications of common parallel wire (two-wire) transmission lines include geometry, characteristic impedance, dielectric material, and typical applications.

Type	Wire Diameter	Spacing	Impedance	Dielectric	Applications
Open wire (telegraphy)	0.1–0.2 cm	10–30 cm	300–600 Ω	Air	Long-distance power/signal
Bell "Ladder"	0.064 cm	2.54 cm	600 Ω	Air	Telephone trunk lines (obsolete)
Ladder line (450 Ω)	0.0566 cm	0.88 cm	450 Ω	Polyethylene spacers	Antenna feed, HF/VHF
Ladder line (300 Ω)	0.0566 cm	0.56 cm	300 Ω	Polyethylene spacers	TV antenna feed, amateur radio
Twin lead	0.0566 cm	0.8 cm	300 Ω	Polyethylene	TV reception, HF antennas

2.3.6 Advantages and Disadvantages of Parallel Wire Line

Advantages

- Lower cost than coaxial cable
- Lower attenuation at RF frequencies due to larger spacing
- Higher characteristic impedance matches many antenna types
- Flexible for outdoor installations
- Lower velocity factor variation with frequency
- Easier to manufacture and repair in field

Disadvantages

- Minimal shielding from external electromagnetic fields

- Susceptible to interference and cross-talk
- Radiation losses from the transmission line itself
- Requires careful spacing and support structure
- Higher frequency range limitation compared to coax (typically < 30 MHz effectively)
- Environmental sensitivity (weather affects performance)

2.3.7 Applications of Parallel Wire Line

- **Amateur Radio:** Antenna feed lines for dipoles and other radiators (especially at HF frequencies)
- **Broadcast Antennas:** Feed lines for AM/FM broadcast stations
- **Shortwave Communications:** Long-range HF antenna systems
- **Legacy Telephone Systems:** Historical trunk line applications (largely obsolete)
- **Power Transmission:** High-voltage overhead transmission lines
- **Specialty RF Applications:** Low-loss feeds for high-impedance loads

2.4 Twisted Pair Transmission Line

2.4.1 Physical Structure and Geometry

A twisted pair is a transmission line formed by two twisted insulated conductor wires. This configuration aims to precisely maintain the distance between the wires and reduce crosstalk. We call crosstalk the interference of a first signal with a second one (traces of the first are found in the signal of the second). There is also a significant attenuation. Less sensitive to noise. This technology is extensively utilised for local telephone and computer cabling applications.
A twisted pair consists of:

- **Two conductors:** Individual wires twisted around each other
- **Pitch:** Number of twists per unit length (typically 2–12 twists per inch)
- **Dielectric:** Surrounding insulation (plastic coating on each conductor)
- **Shielding:** Optional outer shield (shielded twisted pair, STP)

Geometric Configuration



Figure 2.3: Geometric Configuration of: (a): UTP, (b): STP Cables

2.4.2 Twisting Effect on Primary Parameters

The regular twisting of the conductors significantly affects transmission line parameters:

2.4.2.1 Series Resistance per Unit Length

Similar to parallel wire, but slightly increased due to twisting:

$$R' = \frac{2}{2\pi\sigma\delta a} = \frac{1}{\pi\sigma\delta a} \ [\Omega/\mathrm{m}] \tag{2.29}$$

The twisting negligibly affects DC resistance but increases effective path length slightly.

2.4.2.2 Series Inductance per Unit Length

The twisting reduces the external inductance by randomly orienting the loop areas:

$$L' = \frac{\mu_0}{\pi}\left[\frac{1}{8} + ln\left(\frac{d}{a}\right) - \Delta L'\right] \ [\mathrm{H/m}] \tag{2.30}$$

Where $\Delta L'$ is the reduction due to twisting.

Approximate formula for twisted pair:

$$L' \approx \frac{\mu_0}{\pi} ln\left(\frac{p}{a}\right) \ [\mathrm{H/m}] \tag{2.31}$$

Where p is the twist pitch (distance over one complete twist).

2.4.2.3 Shunt Capacitance per Unit Length

The twist geometry affects capacitance:

$$C' = \frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d}{a}\right)} \ [\mathrm{F/m}] \tag{2.32}$$

Where ϵ_r includes contributions from both the insulation and surrounding air/jacket.

2.4.3 Characteristic Impedance of Twisted Pair

For unshielded twisted pair (UTP):

$$Z_0 = \sqrt{\frac{L'}{C'}} \approx 100 - 150\ \Omega \tag{2.33}$$

For shielded twisted pair (STP):

$$Z_0 \approx 85 - 120\ \Omega \tag{2.34}$$

The characteristic impedance depends on:

- Conductor diameter
- Insulation thickness and material
- Twist pitch
- Proximity of shield (for STP)

2.4.4 Velocity Factor

Typical velocity factors for twisted pair:

UTP: $v_f \approx 0.60 - 0.68$

STP: $v_f \approx 0.65 - 0.75$

These values depend on the dielectric constant of the insulation material.

2.4.5 Common Twisted Pair Categories

Standard twisted pair cable specifications, such as frequency range, data rate, impedance, and usual usage, are shown in Table 2.3.

Table 2.3: Common Twisted Pair Cable Categories and Specifications

Category	Frequency Range	Data Rate	Impedance	Applications
Cat 3	16 MHz	10 Mbps	100 Ω	Telephone, 10BaseT Ethernet (obsolete)
Cat 5	100 MHz	100 Mbps	100 Ω	Fast Ethernet, telephone
Cat 5e	100 MHz	1 Gbps	100 Ω	Gigabit Ethernet, multimedia
Cat 6	250 MHz	10 Gbps	100 Ω	10 Gigabit Ethernet
Cat 6A	500 MHz	10 Gbps	100 Ω	High-speed, longer distances
Cat 7	600 MHz	10–40 Gbps	100 Ω	Shielded, professional installations
Cat 8	2000 MHz	40 Gbps	100 Ω	Data center, short runs

2.4.6 Advantages and Disadvantages of Twisted Pair

Advantages

- Very low cost (especially UTP)
- Ease of installation (common RJ45 connectors)
- Good for short distances
- Twisted geometry provides some interference rejection
- Standardized (IEEE 802.3)
- Wide availability

Disadvantages

- Limited frequency range (typically < 2 GHz)
- Susceptible to EMI/RFI (especially UTP)
- Higher attenuation than coaxial cable
- Cross-coupling between adjacent pairs
- Requires careful routing to avoid interference
- Less suitable for outdoor/harsh environments

2.4.7 Applications of Twisted Pair

- **Ethernet Networks:** Cat 5e, Cat 6 for local area networks (LANs)
- **Telephone Systems:** Voice-grade twisted pair for telephony
- **Data Communications:** Short-distance digital signals
- **Building Wiring:** Structured cabling in offices and homes
- **Audio Systems:** Balanced audio cables (microphone, speaker)
- **Industrial Control:** Sensor/actuator networks in manufacturing

2.5 Other Transmission Line Types

2.5.1 Microstrip Line

Structure: A conductor strip of width (w) and thickness (t) on a dielectric substrate of thickness (h) over a ground plane.

Primary Parameters:

$$Z_0 \approx \frac{60}{\sqrt{\epsilon_{eff}}} ln\left(\frac{8h}{\omega} + \frac{\omega}{4t}\right) \quad [\Omega] \tag{2.35}$$

for $\omega > t$, where ϵ_{eff} is the effective dielectric constant.

Applications: PCB circuits, monolithic microwave integrated circuits (MMICs), antenna feed networks.

2.5.2 Stripline

Structure: A conductor strip suspended between two ground planes with symmetric dielectric.

Primary Parameters:

$$Z_0 \approx \frac{30b}{\sqrt{\epsilon_{eff}}\omega} \quad [\Omega] \tag{2.35}$$

Where b is the separation between ground planes and ω is the strip width.

Applications: High-speed digital circuits, low-radiation RF circuits.

2.5.3 Waveguides

Structure: Hollow metallic rectangular or circular tubes that guide electromagnetic waves.

Characteristics:

- Supports EM waves above a cutoff frequency
- No center conductor
- Very low loss at microwave/millimeter-wave frequencies
- Larger dimensions at lower frequencies

Applications: Microwave ovens, satellite communications, radar systems, test equipment.

2.5.4 Optical Fiber

Structure: Glass or plastic core (diameter ~5–50 μm for single-mode) surrounded by cladding and jacket.

Key Advantages

- Extremely high bandwidth (GHz to THz)
- Immunity to electromagnetic interference
- Very low attenuation over long distances
- Compact, lightweight

Applications: Long-distance telecommunications, internet backbone, medical imaging.

2.6 Comparison of Transmission Line Types

Table 2.4 compares five types of transmission lines according to key parameters. The coaxial line provides excellent shielding and low electromagnetic interference (EMI) for radio frequencies and microwaves up to 18 GHz, but at a high cost. The twisted pair line offers very low cost and high velocity factor for antennas and HF applications but suffers from high sensitivity to IEM. The twisted pair excels in cost and ease of installation for digital and audio applications (DC-2 GHz), with acceptable shielding for the single-twisted pair (STP) but high IEMs for the double- twisted pair (UTP). The microstrip line manages the highest frequencies (DC-100 GHz) for printed circuits at an average cost, although its installation is complex due

to engraving. All these lines have a nominal impedance of 100 Ω, except at the extremes, with minimal attenuation for the twisted pair line.

Table 2.4: Comparison of Transmission Line Types

Parameter	Coaxial	Parallel Wire	Twisted Pair	Microstrip
Impedance (Ω)	50–75	300–600	100–150	25–100
Frequency Range	DC–18 GHz	DC–30 MHz	DC–2 GHz	DC–100 GHz
Velocity Factor	0.66–0.85	0.95–0.98	0.60–0.75	0.5–0.7
Attenuation (dB/100 m @ 1 MHz)	0.3–1	0.1–0.5	0.5–2	0.5–3
Shielding	Excellent	Poor	Fair–Good (STP)	Good
EMI Susceptibility	Low	Very High	High (UTP)	Medium
Cost (relative)	High	Very Low	Very Low	Medium
Ease of Installation	Good	Good	Very Good	Poor (requires etching)
Typical Applications	RF/Microwave	Antennas, HF	Digital/Audio	PCB/MMIC

2.6.1 Selection Criteria

Choose Coaxial When

- Frequency > 1 GHz
- EMI immunity required
- Standard RF impedance needed (50 Ω)
- Moderate distances with acceptable loss
- Professional RF applications

Choose Parallel Wire When

- Frequency < 30 MHz
- Low cost essential
- Long distances acceptable
- High impedance load (antennas)
- Outdoor installation feasible

Choose Twisted Pair When

- Frequency < 2 GHz
- Cost critical
- Short distances (< 100 m)
- Digital signals or audio
- Building wiring preferred

Choose Microstrip When

- Integration with PCB required

- Frequency > 100 MHz
- Miniaturization important
- Controlled manufacturing available

2.7 Conclusion

This Chapter concludes by reinforcing the comprehension of transmission line types such as coaxial, twin-wire, and twisted-core lines, as well as their practical applications in RF and communication systems. Comprehensive estimates of fundamental parameters namely inductance, capacitance, resistance, and conductance— or twin-wire lines and coaxial cables provide engineers with critical resources for precise modelling and performance forecasting. These notions connect theoretical propagation from Chapter 1 to antenna interfaces, facilitating optimised designs in practical electromagnetic applications.

2.8 Series of tutorial exercises

2.8.1 Exercise N°1

A two-wire transmission line consists of two parallel conductors with radius r = 0.5 mm, separated by a distance d = 10 mm, in air ($\varepsilon_r = 1$). Calculate the primary parameters L (inductance) and C (capacitance), using the following constants:

- Permittivity of free space: $\varepsilon_0 = 8.854187817 \times 10^{-12}$ F/m
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ H/m

2.8.2 Exercise N°2

A coaxial cable is formed by:

- Inner conductor radius: a = 0.5 mm
- Outer conductor radius: b = 5 mm
- Dielectric relative permittivity: $\varepsilon_r = 2.1$

1. Calculate the inductance per unit length (L) and capacitance per unit length (C) of the coaxial cable.
2. Using the calculated values of L and C, determine the characteristic impedance Z_0 of the coaxial cable.

2.9 Solutions of series of tutorial exercises

2.9.1 Solution of exercise N°1

Using equation 2.17: $L' \approx \frac{\mu_0}{\pi} ln\left(\frac{d}{a}\right) = 1.2 \times 10^{-6}$ [H/m].

Using equation 2.19: $C' \approx \frac{\pi\varepsilon_0\epsilon_r}{ln\left(\frac{d}{a}\right)} = 9.29 \times 10^{-9}$ [F/m].

2.9.2 Solution of exercise N°2

Using equation 2.3: $L' = \frac{\mu_0}{2\pi} ln\left(\frac{b}{a}\right) = \ 0.4605 \times 10^{-6}$[H/m]

Using equation 2.5: $C' = \frac{2\pi\varepsilon_0\epsilon_r}{ln\left(\frac{b}{a}\right)} = 50.77 \times 10^{-9}$ [F/m]

Using equation 2.7: $Z_0 = \sqrt{\frac{L'}{C'}} = 95.25\ \Omega$

Summary of chapter 2

Chapter 2 surveys transmission line types coaxial, parallel wire (twin-lead), twisted pair, microstrip, stripline, waveguides, and optical fiber—with accurate derivations for parameters like coaxial $Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right)$ and parallel wire $Z_0 = 276 \log_{10}\left(\frac{D}{a}\right)$. Essential concepts include geometry impacts on L', C', R', G'; velocity factors (coax ~0.66, twin-lead ~0.95); advantages/disadvantages (coax shielding vs. twin-lead low loss); and applications (RF coax, Ethernet Cat6 twisted pair, PCB microstrip). Students should retain selection criteria from Table 2.4 for frequency/cost/EMI trade-offs, linking Chapter 1 theory to practical RF feeds. Now that you have acquired transmission line types and applications, you can move on to antenna parameters and radiation theory in Chapter 3.

Chapter 3: Basic Antenna Characteristics

3.1 Introduction
3.2 Radiation characteristics
3.2.1 Radiation Pattern
3.2.2 Power Flux Density (Poynting Vector)
3.2.3 Radiated Power
3.2.4 Radiation Intensity
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3 Chapter 3: Basic Antenna Characteristics

3.1 Introduction

An antenna is a device used to create, transmit and receive electromagnetic waves. In general, it is designed to radiate or capture the maximum energy of an electromagnetic wave propagating in free space. Antennas are used to communicate over long distances, as communications over short distances are less expensive with the use of cables or waveguides. Today, however, we communicate wirelessly between a base station and a mobile station very close together, e.g., inside the same room, to avoid splicing.

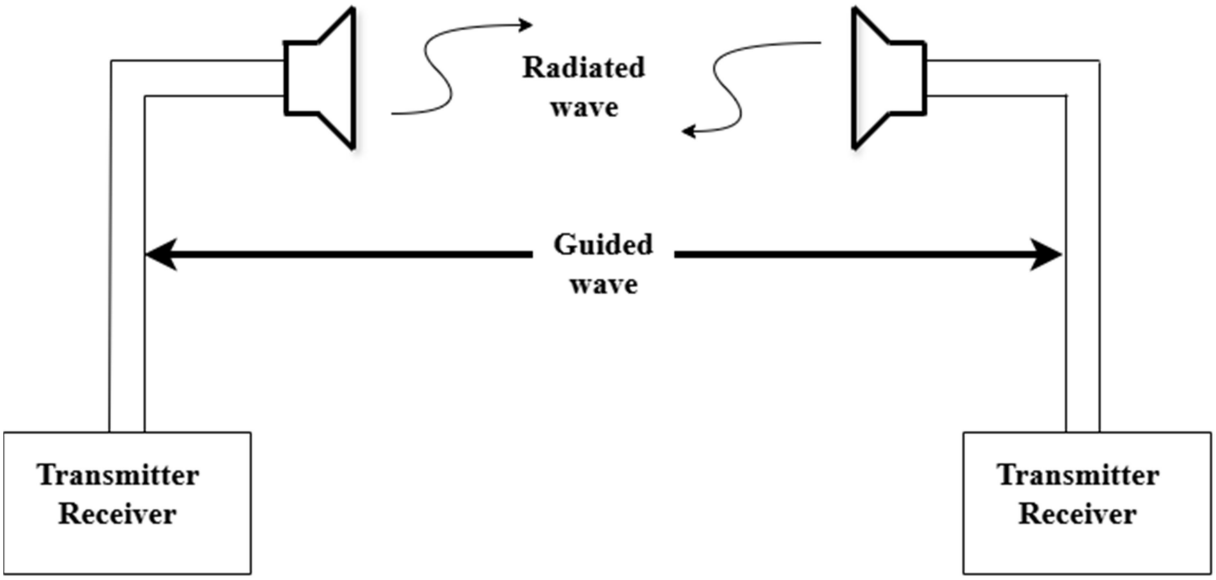


Figure 3.1: Radiated vs. Guided Waves in Communication Systems

This chapter presents the fundamental characteristics that describe how antennas radiate and receive electromagnetic energy. Understanding these parameters is essential for antenna design, analysis, and performance evaluation. The chapter is divided into two main sections: Radiation Characteristics and Electrical Characteristics.

3.2 Radiation characteristics

3.2.1 Radiation Pattern

The radiation pattern (or far-field pattern) is a graphical representation of how an antenna radiates power as a function of direction in space. It describes the directional dependence of the electromagnetic field strength from the antenna.

The radiation patterns of an antenna provide a graphical representation typically in polar or rectangular coordinates of directional far-field radiation characteristics when the distance exceeds $\frac{2D^2}{\lambda}$ (D being the largest antenna dimension and λ the wavelength). These diagrams come in several forms: field diagrams indicating the amplitude of the electric or magnetic field; power diagrams, proportional to the square of the field amplitude; normalised diagrams with respect to the maximum value; and decibel logarithmic diagrams (dB). Structurally, they

have a main maximum radiation lobe, side lobes corresponding to minor adjacent regions, back lobes in the opposite direction and zero radiation zones. Physically, this structure results from constructive interference (wave reinforcement for the main lobe) and destructive interference (wave cancellation for zero radiation zones) between electromagnetic waves emitted by different parts of the antenna.

An antenna radiation pattern is defined as a mathematical function or graphical depiction of the antenna's radiation characteristics in relation to spatial coordinates. The radiation pattern is typically ascertained in the far-field region and is expressed as a function of directional coordinates. Radiation attributes encompass power flux density, radiation intensity, field strength, directivity, phase, and polarisation. The primary concern with radiation is the two- or three-dimensional spatial distribution of emitted energy relative to the observer's position along a path or surface of constant radius.

The radiation patterns of an antenna are represented by polar diagrams, where the ray indicates the radiation intensity or gain as a function of the angle with respect to the axis of the antenna, providing intuitive 3D visualisation and common use in engineering; rectangular (Cartesian) diagrams, with the abscissa angle and the normalised gain or power (dB) on the ordinate, are particularly suitable for comparing the levels of secondary lobes and allow precise measurements; and the 3D representations offer rotating spherical views with main cuts along the planes E (containing the electric field and maximum radiation direction) and H (containing the magnetic field and maximum radiation direction).

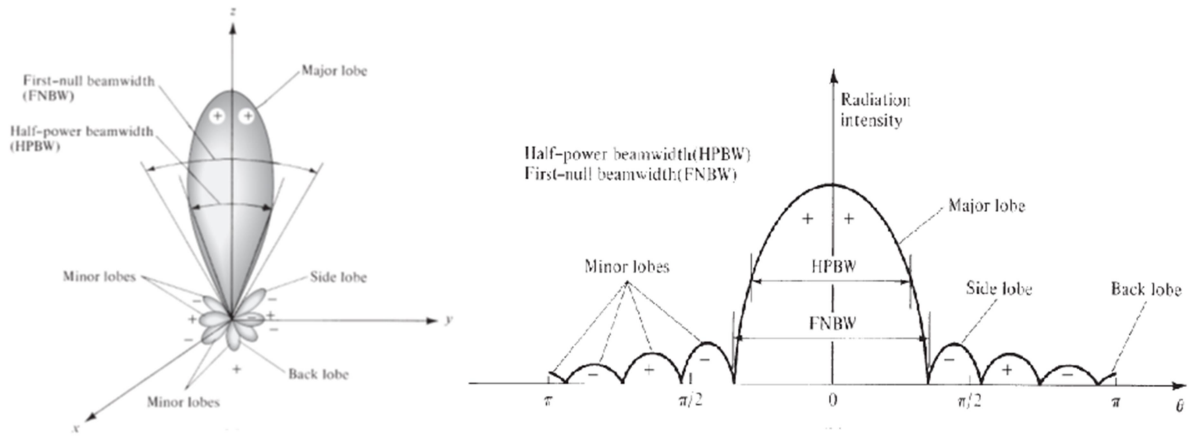


Figure 3.2: Radiation Pattern Representation Methods: (a) a 3D polar plot (b) a 2D rectangular (Cartesian) plot of normalized radiation intensity versus angle θ

Among the important parameters derived from these diagrams are the half-power beamwidth (HPBW), corresponding to the angular width at which the power falls to 50% of the maximum power (-3 dB); the first zero beamwidth (FNBW), which extends to the first zeroes on either side of the main beam; the level of the secondary lobes (SLL), which represents the power of the secondary lobes relative to the main lobe; and the front/back ratio, which quantifies the forward radiation compared to the backward radiation.

3.2.2 Power Flux Density (Poynting Vector)

The power flux density is the average power per unit area flowing in a particular direction, measured in watts per square meter (W/m²).

$$S(r,\theta,\phi) = \frac{P_t G(\theta,\phi)}{4\pi r^2} \tag{3.1}$$

Where:

- S : power flux density (W/m²)
- P_t : transmitted power
- $G(\theta,\phi)$:antenna gain in direction (θ,ϕ)
- r : distance from antenna

Radiation intensity, a key antenna parameter, decreases with the square of distance following the inverse-square law, varies directionally according to the antenna's radiation pattern reaching maximum in the main lobe direction and zero in null directions and physically represents the Poynting vector $\vec{S} = \vec{E} \times \vec{H}$ (cross product of electric and magnetic fields), quantifying far-field power flux per unit solid angle. Its practical applications encompass link budget calculations for system performance prediction, radiation safety assessments to ensure exposure limits, coverage area determination for network planning, and electromagnetic interference analysis to mitigate unwanted coupling.

3.2.3 Radiated Power

The total electromagnetic power radiated by an antenna integrated over all directions in space.

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta,\phi) sin\theta d\theta d\phi \tag{3.2}$$

Where:

- P_{rad} : total radiated power
- $U(\theta,\phi)$: radiation intensity

Relationship to Input Power:

$$P_{in} = P_{rad} + P_{loss} \tag{3.3}$$

Where:

- P_{in} : input power to antenna
- P_{loss} : power lost due to resistive losses

Energy Conservation:

- In an ideal lossless antenna: $P_{in} = P_{rad}$
- In practical antennas: $P_{rad} < P_{in}$
- The difference is converted to heat in resistive elements

3.2.4 Radiation Intensity

The power radiated per unit solid angle in a given direction, measured in watts per steradian (W/sr).

$$U(\theta, \phi) = r^2 S(\theta, \phi) \quad (3.4)$$

Where:

- $U(\theta, \phi)$: radiation intensity
- r: distance
- $S(\theta, \phi)$: power flux density

Relationship to Radiated Power:

$$U_{avg} = \frac{P_{rad}}{4\pi} \quad (3.5)$$

Where U_{avg} : average radiation intensity (isotropic reference)

3.2.5 Directivity

The ratio of the maximum radiation intensity to the average radiation intensity (isotropic radiator reference).

$$D = U_{avg} = \frac{U_{max}}{U_{avg}} = \frac{4\pi U_{max}}{P_{rad}} \quad (3.5)$$

Antenna directivity is a dimensionless quantity, typically expressed in dBi (decibels relative to isotropic radiator), that quantifies how much an antenna concentrates radiation in a particular direction compared to an isotropic source; it is always greater than 1 for any real antenna due to inherent directivity from element geometry, remains independent of radiation efficiency (which affects gain instead), and depends solely on the shape of the radiation pattern narrower beams yield higher directivity.

Directivity in antennas is commonly expressed in decibels using:

$$D(dBi) = 10log_{10}(D) \quad (3.6)$$

Example Values:

- Isotropic radiator: D = 1 (0 dBi)
- Short dipole: D = 1.5 (1.76 dBi)
- Half-wave dipole: D = 1.64 (2.15 dBi)
- Large dish antenna: D can exceed 50 dBi

Directivity represents the concentration of radiated power in the main lobe direction compared to uniform radiation in all directions. Higher directivity means narrower beam and better focused radiation.

3.2.6 Efficiency

The ratio of total radiated power to input power, representing the fraction of input power successfully converted to electromagnetic waves.

$$\eta = \frac{P_{rad}}{P_{in}} = \frac{P_{rad}}{P_{rad} + P_{loss}} \tag{3.7}$$

Efficiency Components:

a) **Radiation Efficiency (e_R):**

$$e_R = \frac{R_r}{R_r + R_l} \tag{3.8}$$

Where:

- R_r : radiation resistance
- R_l : loss resistance (conductor + dielectric losses)

b) **Mismatch Efficiency (e_M):**

$$e_M = 1 - |\Gamma|^2 \tag{3.9}$$

Where: $|\Gamma|$: reflection coefficient at antenna terminals

c) **Total Efficiency:**

$$\eta_{total} = e_R . e_M \tag{3.10}$$

Loss Sources:

1. **Conductor losses**: Resistive losses in metal conductors
2. **Dielectric losses**: Losses in substrate and surrounding medium
3. **Impedance mismatch**: Power reflected due to impedance discontinuity

Typical Values:

- High-efficiency antennas: 80-95%
- Medium efficiency: 50-80%
- Low efficiency: <50% (measurement probes, some integrated antennas)

Efficiency in antennas is commonly expressed in decibels using:

$$\eta(dB) = 10log_{10}(\eta) \tag{3.11}$$

3.2.7 Gain

The ratio of radiation intensity in a given direction to the radiation intensity that would be obtained from an isotropic antenna fed with the same input power. Gain combines directivity and efficiency.

$$G(\theta,\phi) = e.D(\theta,\phi) \quad (3.12)$$

Where:

- e: antenna efficiency
- $D(\theta,\phi)$: directivity

Gain is expressed in decibels using:

$$G(dBi) = 10log_{10}(G) \quad (3.13)$$

Antenna gain differs from directivity in that directivity describes only how strongly an antenna concentrates radiation in a given direction, ignoring any internal losses, whereas gain multiplies this directivity by the antenna's efficiency factor, thereby accounting for ohmic, dielectric, and mismatch losses. Because real antennas are never perfectly lossless, gain is always less than or, at best, equal to directivity for practical designs. This makes gain the more practically useful parameter for system design and link budgets, since it reflects the actual power delivered to or received from space rather than the idealized pattern shape alone.

Typical Gain Values

- Dipole antenna: 2.15 dBi
- Parabolic dish (large): 30-60 dBi
- Horn antenna: 10-20 dBi
- Microstrip patch: 5-10 dBi

3.2.8 EIRP (Effective Isotropic Radiated Power)

The equivalent power that an ideal isotropic antenna would radiate to produce the same power density in the direction of maximum gain as the actual antenna.

$$EIRP = P_{tx} - L_{cable} + G_{antenna} \quad (3.14)$$

All values in decibels (dBm or dBW).

$$EIRP = P_{tx} \times G \times e^{-L} \quad (3.15)$$

Where:

- P_{tx} :transmitter output power
- G :antenna gain (linear scale)
- L :cable and connector losses (linear scale)

Typical EIRP Limits (Examples):

- 802.11 WiFi: varies by country (e.g., 30 dBm in Europe)
- 5G cellular: depends on frequency band
- Satellite communications: highly restricted
- Radar systems: much higher limits allowed

3.3 Electrical characteristics

3.3.1 Electrical Model

Antenna Equivalent Circuit:

Real antennas exhibit frequency-dependent impedance characteristics that can be modeled using equivalent circuits composed of resistors (R), inductors (L), and capacitors (C).

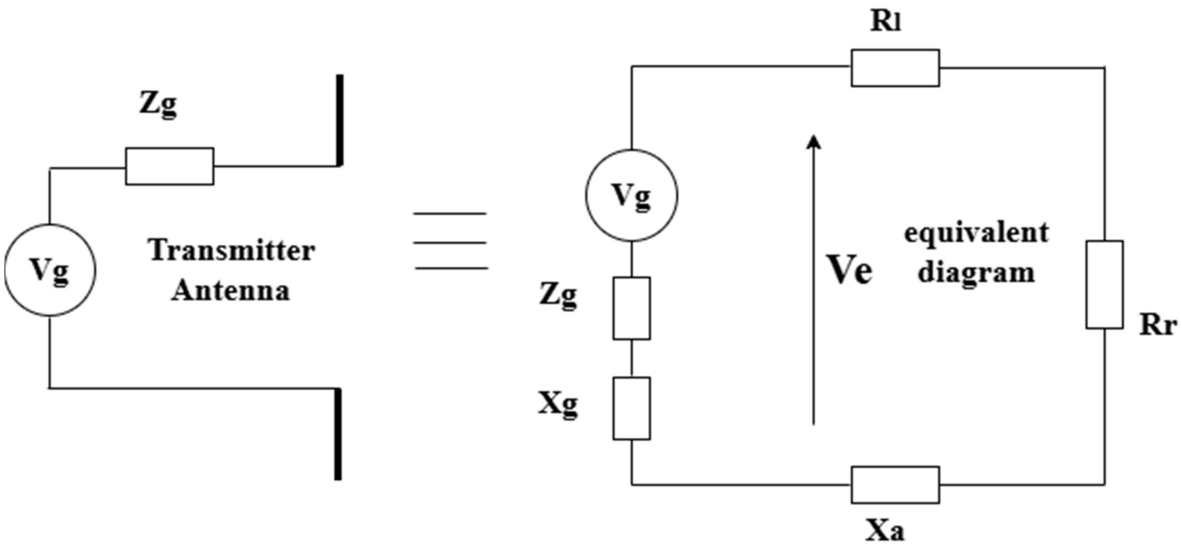


Figure 3.3: Equivalent Model of Transmitter and Its Antenna

The antenna input impedance consists of three fundamental circuit components: radiation resistance R_r, loss resistance R_l, and reactance X. The radiation resistance R_r represents the equivalent resistance that accounts for power radiated into free space by the antenna and is inherently frequency-dependent, calculated as:

$$R_r = \frac{2P_{\text{rad}}}{|I_0|^2} \tag{3.16}$$

Where P_{rad} is radiated power and I_0 is the input current.

For example, a half-wave dipole exhibits $R_r \approx 73\,\Omega$ at its resonant frequency. The loss resistance R_l captures ohmic losses within the antenna structure, including conductor losses and dielectric losses in the substrate, which directly determines the radiation efficiency through the relation:

$$\eta = \frac{R_r}{R_r + R_l} \tag{3.17}$$

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Finally, the reactance X, which can be inductive $X_L = \omega L$ or capacitive $X_C = \frac{-1}{\omega C}$ varies with frequency, becoming zero at the antenna's resonant frequency to achieve pure resistive impedance; away from resonance, non-zero reactance causes impedance mismatch, reflected power, and reduced power transfer from the transmitter.

Antenna impedance is represented by two equivalent circuit topologies: the series circuit $Z_{ant} = R_p + jX_p$ (where $R_p = R_r + R_l$ combines radiation and loss resistance) and the parallel circuit $\frac{1}{Z_{ant}} = \frac{1}{R_p} + \frac{1}{jX_p}$. Both describe the same physical antenna but suit different applications: series for narrowband analysis near resonance ($X_p \approx 0$) and matching with series L/C networks; parallel for broadband matching and high-Q resonators where $R_p \gg |X_p|$. At resonance, both reduce to pure resistance $R_r + R_l$, with radiation efficiency $\eta = R_r/(R_r + R_l)$.

3.3.2 Impedance Matching and Matching Condition

Impedance matching is the process of adjusting the antenna impedance to match the transmission line characteristic impedance (typically 50 Ω) to maximize power transfer. The matching condition is defined by the equation below:

$$Z_{ant} = Z_0^* \tag{3.18}$$

Where:

- Z_{ant} : antenna input impedance
- Z_0^* : transmission line characteristic impedance (complex conjugate).

For real impedances, most practical RF systems are designed so that the antenna input impedance equals the characteristic impedance of the transmission line, typically $Z_{ant} = Z_0 = 50\ \Omega$, which is the industry standard value chosen as a good compromise between low loss and high power-handling capability.

Impedance Parameters:

Input Impedance:

$$Z_{ant} = R_{ant} + j.X_{ant} \tag{3.19}$$

Where:

- R_{ant} : resistance component
- X_{ant} : reactance component

Reflection Coefficient

$$\Gamma = \frac{Z_{ant} - Z_0}{Z_{ant} + Z_0} \tag{3.20}$$

Return Loss (dB)

$$\text{RL} = -20\log(|\Gamma|) \tag{3.21}$$

Voltage Standing Wave Ratio (VSWR):

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{Z_{max}}{Z_{min}} \tag{3.22}$$

VSWR=1 indicates perfect match (100% power transmitted); VSWR=1.5 is good (96% transmitted, 4% reflected); VSWR=2 is acceptable (89% transmitted); VSWR>3 signals poor match with significant reflections and losses. Matching bandwidth is the frequency range where VSWR stays below a specified limit (typically 2:1 or 1.5:1).

3.3.3 Bandwidth

The frequency range over which an antenna meets specified performance criteria. There is no single universal definition; bandwidth depends on which characteristic is being specified.

It exist many antenna bandwidth types such as impedance bandwidth (most common) is the frequency range where $|S_{11}| \leq -10$ dB (VSWR ≤ 1.92:1); gain bandwidth maintains gain variation ≤ ±1 dB; pattern bandwidth preserves beamwidth/sidelobe specs; Polarisation bandwidth keeps axial ratio ≤ 3 dB for circular antennas; efficiency bandwidth ensures radiation efficiency >80%.

Fractional bandwidth is given by the equation:

$$\text{BW\%} = \frac{f_H - f_L}{f_c} \times 100\% \tag{3.23}$$

Where f_H/f_L are upper/lower limits and f_c is center frequency.

3.3.4 Antenna Polarisation

The polarisation of an antenna is the property describing the temporal variation of the direction and the modulus of the electric field vector $\vec{E}$radiated by this antenna. It is characterised by the curve described in time at the end of this vector and observed from the direction of propagation.

The polarisation can be linear, circular or elliptical. If the electric field vector $\vec{E}$ maintains a constant direction (thus having only one component), the polarisation is said to be linear or rectilinear. This condition is the case for the majority of wired antennas. In the general case, it describes an ellipse, and the polarisation is then called elliptical. The linear and circular

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polarisations are special cases of elliptic polarisation and are obtained, respectively, when the ellipse becomes a straight line or a circle.

Assuming that the instantaneous electric field $\vec{E}$ of a plane wave propagating along the z-axis, in the positive direction so that the observer placed at the origin O sees the wave moving towards him, has for expression:

$$\vec{\mathrm{E}}(\mathrm{z},\mathrm{t}) = \vec{\mathrm{E}}_x(\mathrm{z},\mathrm{t})\hat{\mathrm{x}} + \vec{\mathrm{E}}_y(\mathrm{z},\mathrm{t})\hat{\mathrm{y}} \tag{3.24}$$

Then:

$$\vec{\mathrm{E}}(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t}) = \mathrm{Re}\left[\vec{\mathrm{E}}(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t})e^{j\omega t}\right] \tag{3.25}$$

The equation (3.24) becomes:

$$\begin{aligned}\vec{\mathrm{E}}_x(\mathrm{z},\mathrm{t}) &= Re[E_{ox}e^{j(\omega t-kz+\varphi_x)}] = E_{ox}\cos(\omega t - kz + \varphi_x)\\ \vec{\mathrm{E}}_y(\mathrm{z},\mathrm{t}) &= Re\left[E_{oy}e^{j(\omega t-kz+\varphi_y)}\right] = E_{oy}\cos(\omega t - kz + \varphi_y)\end{aligned} \tag{3.26}$$

E_{ox} and E_{oy} being maximum amplitudes (positive reals), and the initial phases. The Polarisation of the wave is determined by these four parameters.

As we are interested in the temporal variation of the field, we can carry out the study of Polarisation by placing $z = 0$ in (3.26) and obtain:

a) Horizontal or vertical linear polarisation

First, if one of the field components is null (fig. 3.4), then the polarisation is linear, and we do not have to involve these four parameters at all in this simple case.

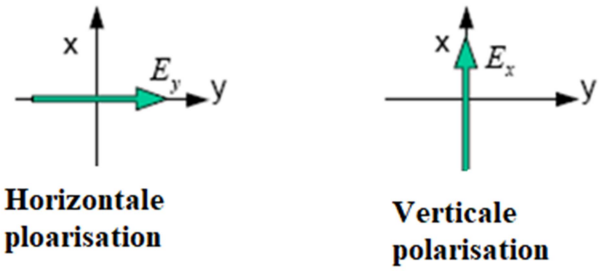


Figure 3.4: Horizontal and vertical linear polarisation

b) Circular polarisation

The polarisation is circular (fig. 3.5) when the amplitudes of the two components of the electric field are equal and their phase shift $\Delta\varphi$ is an odd multiple of $\frac{\pi}{2}$, conditions that can be formulated as follows:

$$E_{oy} = E_{ox} = E_o \tag{3.27}$$

$$\Delta\varphi = \varphi_y - \varphi_x = \begin{cases} +(2n+1)\dfrac{\pi}{2} & clockwise\ direction \\ -(2n+1)\dfrac{\pi}{2} & anticlockwise\ direction \end{cases} \quad (n = 0,1,2,3,) \tag{3.28}$$

$$E_{ox}{}^2 + E_{oy}{}^2 = E_o{}^2 \tag{3.29}$$

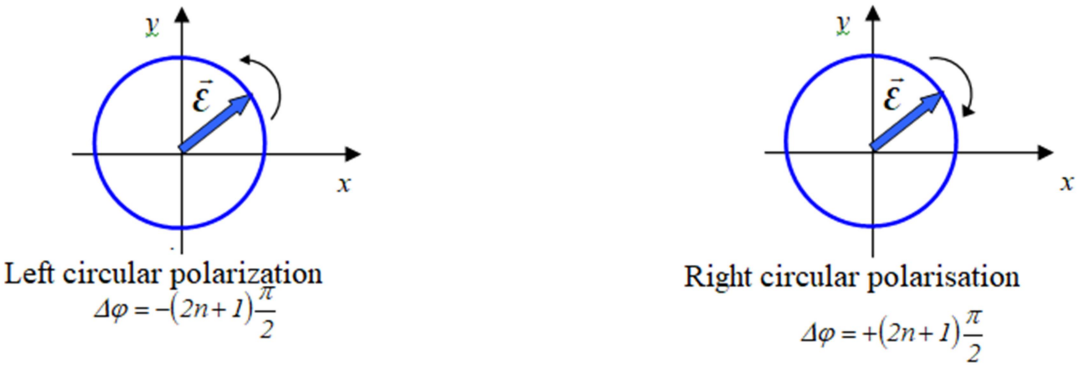


Figure 3.5: Circular polarisation

b) Elliptical polarisation

We refer to centred elliptic polarisation when the amplitudes of the two components of the electric field differ and their phase shift $\Delta\varphi$ is an odd multiple of π/2.

$$E_{oy} \neq E_{ox} \tag{3.30}$$

$$\Delta\varphi = \varphi_y - \varphi_x = \begin{cases} +(2n+1)\dfrac{\pi}{2} & clockwise\ direction \\ -(2n+1)\dfrac{\pi}{2} & anticlockwise\ direction \end{cases} \quad (n = 0,1,2,3,) \tag{3.31}$$

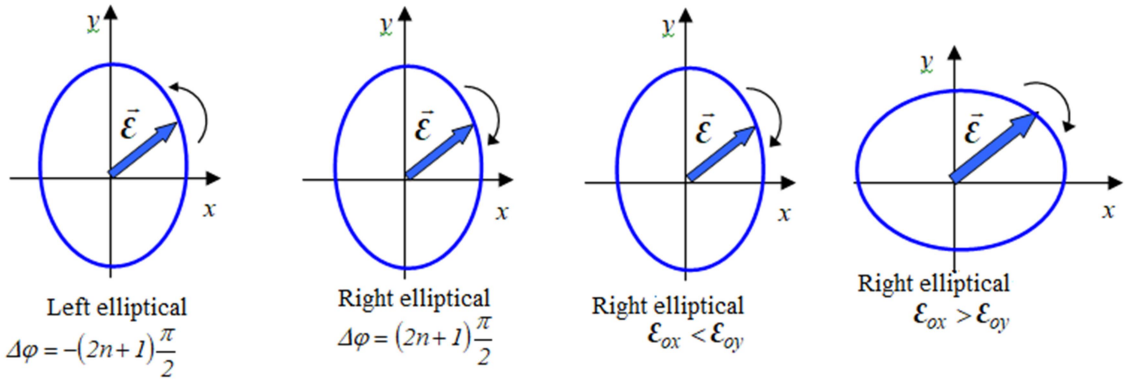


Figure 3.6: Elliptical polarisation

The choice of antenna polarisation significantly impacts the performance, cost, and reliability of a wireless communication system. Depending on the operating environment and system requirements, engineers must select between linear, circular, or elliptical polarisation. Table 3.1 provides a comparative overview of the three primary polarisation types, highlighting their typical applications, key advantages, and inherent design trade-offs.

Table 3.1: Comparative Analysis of Electromagnetic Polarisation in Antennas

Polarisation Type	Applications	Advantages	Disadvantages
Linear Polarisation	- Traditional AM/FM broadcasting - Most terrestrial radio systems - Cellular mobile communications - WiFi (most implementations)	- Simple antenna design - Standard component availability - Low manufacturing cost - Established practices and regulations	- Polarisation mismatch causes significant losses (up to -20 dB if orthogonal) - Multipath effects can rotate polarization - Faraday rotation in the ionosphere causes signal issues
Circular Polarisation	- Satellite communications (GPS, satellite TV downlinks) - Radar systems (terrain scattering rejection) - Mobile satellites (Iridium, Globalstar) - Space telemetry and tracking - Wireless sensor networks (body-worn antennas)	- Polarisation mismatch is less critical (e.g., 3 dB loss with a linear antenna) - Better multipath rejection in reflected signals - Immunity to Polarisation rotation (Faraday rotation) - Enables Polarisation diversity systems	- More complex antenna design than linear - Requires a 90° phase shifter for generation - Slightly larger antenna dimensions - More sensitive to the surrounding environment
Elliptical Polarisation	-Transition state between linear and circular designs -Flexible/conformal antennas that deform slightly -Wideband systems with Polarisation trade-offs -Multipath environments with Polarisation diversity	-Reduced Susceptibility to Orientation Mismatch -Improved Performance in Urban Environments (Multipath Mitigation) -Controlled Power Distribution for Specific Targets	-Increased Transmitter Power Requirements -Polarisation Mismatch Loss -Complexity in Design and Phasing

	needs		

3.4 Conclusion

Chapter 3 wraps up the fundamentals of antenna performance by integrating radiation characteristics like characteristic area, radiation pattern, power density, radiated power, radiation intensity, directivity, efficiency, gain, and EIRP with electrical properties such as the antenna's equivalent model, frequency response, matching requirements, bandwidth, and polarisation behaviour. These concepts provide a unified framework for evaluating how antennas convert electrical signals into radiated electromagnetic waves efficiently. This foundation sets the stage for more profound analysis of specific antenna radiation mechanisms and practical designs in subsequent chapters.

3.5 Series of tutorial exercises

3.5.1 Exercise N°1

The maximum radiation intensity of an antenna with an efficiency of 90% is 200 mW/sr (or mW/Ω). Find the directivity and the gain when:

The input power is 125.66 mW.

The radiated power is 125.66 mW.

3.5.2 Exercise N°2

The power radiated by a lossless antenna is 10 W and the radiation intensity is given by $U = B_0 \cos^3(\theta)$ where $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \varphi \leq 2\pi$. Find:

The maximum power density in W/m^2.

The directivity and the gain of this antenna.

3.6 Solutions of series of tutorial exercises

3.6.1 Solution of exercise N°1

Part a) When the input power is $P_{\text{in}} = 125.66$ mW

1. Find Radiated Power (P_{rad}):
Efficiency is the ratio of radiated power to input power:

$$\eta = \frac{P_{\text{rad}}}{P_{\text{in}}} \Longrightarrow P_{\text{rad}} = \eta \times P_{\text{in}}$$

$$P_{\text{rad}} = 0.90 \times 125.66 \text{ mW} = 113.094 \text{ mW}$$

2. Calculate Directivity (D):
Directivity is the ratio of the maximum radiation intensity to the average radiation intensity:

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi \times 200 \text{ mW/sr}}{113.094 \text{ mW}} == \frac{2513.27}{113.094} \approx 22.22, \textit{(In dB: } 10\log_{10}(22.22) \approx 13.47 \text{ dBi)}$$

3. Calculate Gain (G):
Gain is related to directivity via efficiency:

$$G = \eta \times D == 0.90 \times 22.22 = 20.0, \textit{(In dB: } 10\log_{10}(20.0) \approx 13.01 \text{ dBi)}$$

Part b) When the radiated power is $P_{\text{rad}} = 125.66$ mW

1. Calculate Directivity (D):
Using the given radiated power directly:

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} == \frac{4\pi \times 200 \text{ mW/sr}}{125.66 \text{ mW}} = \frac{2513.27}{125.66} \approx 20.0, \textit{(In dB: } 10\log_{10}(20.0) \approx 13.01 \text{ dBi)}$$

2. Calculate Gain (G):

$$G = \eta \times D = 0.90 \times 20.0 = 18.0, \textit{(In dB: } 10\log_{10}(18.0) \approx 12.55 \text{ dBi)}$$

3.6.2 Solution of exercise N°2

Step 1: Find the constant B_0

The total radiated power is the integral of the radiation intensity over the solid angle:

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} U(\theta, \varphi) \sin\theta \, d\theta \, d\varphi$$

$$10 = \int_0^{2\pi} d\varphi \int_0^{\pi/2} B_0 \cos^3(\theta) \sin\theta \, d\theta$$

$$10 = 2\pi B_0 \int_0^{\pi/2} \cos^3(\theta) \sin\theta \, d\theta$$

Let $u = \cos\theta$, then $du = -\sin\theta \, d\theta$.

When $\theta = 0, u = 1$. When $\theta = \pi/2, u = 0$.

$$\int_0^{\pi/2} \cos^3(\theta) \sin\theta \, d\theta = \int_0^1 u^3 \, du = \left[\frac{u^4}{4}\right]_0^1 = \frac{1}{4}$$

Therefore: $10 = 2\pi B_0 \left(\frac{1}{4}\right) \implies 10 = \frac{\pi B_0}{2}$, $B_0 = \frac{20}{\pi} \approx 6.366$ W/sr

Step 2: Find Maximum Radiation Intensity (U_{max})

The function $U = B_0\cos^3(\theta)$ is maximum when $\cos(\theta) = 1$ (i.e., at $\theta = 0°$).

$$U_{max} = B_0(1)^3 = B_0 = \frac{20}{\pi} \approx 6.366 \text{ W/sr}$$

Part a) Maximum Power Density (W_{max} or S_{max})

The relationship between power density W and radiation intensity U is: $W = \frac{U}{r^2}$

Since the problem does not provide a specific distance r, the generic answer in terms of r is:

$$W_{max} = \frac{U_{max}}{r^2} = \frac{20}{\pi r^2} \text{ W/m}^2$$

(Note: If your original assignment gave a specific distance, e.g., r = 1000 m, you would plug it in: $W_{max} = \frac{6.366}{1000^2} = 6.366 \times 10^{-6}$ W/m^2).

Part b) Directivity (D) and Gain (G)

$D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi\left(\frac{20}{\pi}\right)}{10} = \frac{80}{10} = 8$, *(In dB:* $10\log_{10}(8) \approx 9.03$ dBi*)*

Since the antenna is lossless ($\eta = 1$): $G = D = 8$, *(In dB:* 9.03 dBi*)*

3.7 Summary of chapter 3

Chapter 3 introduces basic antenna characteristics, covering radiation pattern (polar/3D plots, HPBW, SLL), power flux density $S = \frac{P_t G}{4\pi\ ^2}$, radiation intensity $U(\theta, \phi) = r^2 S$, directivity $D = \frac{4\pi\ \ max}{P_{rad}}$, efficiency $\eta = \frac{P_{rad}}{P_{in}}$, gain $G = \eta D$, and EIRP $P_{EIRP} = P_t G_t$. Electrical aspects include equivalent circuit $Z_A = R_r + R_L + jX$, impedance matching ($\Gamma = 0$, VSWR ≤2:1), bandwidth (impedance/gain/pattern), and polarization (linear, circular $\delta = \pm\pi/2$, elliptical). Exercises validate gain/directivity calculations accurately. Retain gain vs. directivity distinction and matching importance for RF efficiency. Now that you have acquired antenna basics, you can move on to radiation theory from dipoles in Chapter 4.

Chapter 4: Radiation from Elementary Antennas

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4 Chapter 4: Radiation from Elementary Antennas

4.1 Introduction

This chapter delves into the radiation mechanisms of elementary antennas, foundational for advanced array synthesis in electromagnetic systems like those in 5G/6G networks. It begins with the far-field electromagnetic analysis of the electric doublet a Hertzian dipole approximating infinitesimal current elements deriving essential metrics: the characteristic surface representing propagating spherical wavefronts; the $\sin^2\theta$ radiation diagram with axial nulls and equatorial maxima; radiated power via Poynting theorem integration proportional to frequency squared and length; equivalent height linking to induced voltage; and radiation resistance of approximately $80\pi^2(dl/\lambda)^2$ ohms quantifying ohmic-like dissipation in feedpoint matching. Extending to the isolated dipole antenna in free space, the chapter computes analogous far-field properties for finite-length resonators, such as half-wave dipoles with sinusoidal current distributions: a refined radiation diagram akin to $\sin^3\theta$ yielding higher directivity; total radiated power tied to ~73 Ω resistance for optimal efficiency; equivalent height near λ/π; and characteristic surfaces confirming radiation-zone dominance beyond $2D^2/\lambda$, providing benchmarks for amplitude/phase optimisations in linear arrays. These analyses equip engineers with precise tools for predicting patterns, gain, and efficiency in complex configurations. The chapter is structured to progress from theoretical foundations (Maxwell equations, retarded potentials) through specific calculations for electric dipoles and isolated dipole antennas, emphasizing radiation patterns, radiated power, effective height, and radiation resistance.

4.2 Electric dipole radiation

4.2.1 Theoretical Foundations: From Maxwell's Equations to Dipole Radiation

4.2.1.1 Maxwell's Equations and Wave Equation

Maxwell's equations form the foundational framework of classical electromagnetism, which governs the behaviour of electric and magnetic fields in space and time. In their differential vector form, these equations are expressed as follows:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ (Fraday's\ Law)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \ (Ampere - Maxwell\ Law) \tag{4.1}$$

$$\nabla . \vec{D} = \rho \ (Gauss\ Law)$$

$$\nabla . \vec{B} = 0 \ (No\ Magnetic\ Monopoles)$$

Where:

- $\vec{E}$: Electric field (V/m)
- $\vec{H}$: Magnetic field intensity (A/m)
- $\vec{B}$: Magnetic flux density [Tesla], ($\vec{B} = \mu_0 . \vec{H}$ in free space).
- $\vec{D}$:Electric displacement [C/m²], ($\vec{D} = \varepsilon_0 . \vec{E}$ in free space
- $\vec{J}$:Current density (A/m²)
- ρ : Charge density (C/m³)

Wave Equation Derivation:

Taking the curl of Faraday's law:

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) \tag{4.2}$$

Using vector identity and substituting Ampère's law:

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial \vec{J}}{\partial t} \tag{4.3}$$

In source – free regions ($\rho = 0, \vec{J} = 0$)

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{4.4}$$

This is the wave equation, which also applies to $\vec{H}$, $\vec{A}$, and scalar potential ϕ.

4.2.1.2 Vector and Scalar Potentials

Potential Definitions:

To simplify Maxwell's equations, we introduce vector potential $\vec{A}$ and scalar potential ϕ:

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \tag{4.5}$$

Gauge Condition (Lorentz Gauge):

To eliminate ambiguity in potential definitions, we impose:

$$\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} = 0 \tag{4.6}$$

Decoupled Wave Equations:

Under Lorentz gauge, Maxwell's equations become:

$$\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$
$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \tag{4.7}$$

These are driven wave equations where current $\vec{J}$ and charge ρ act as sources.

4.2.2 Electric Dipole in Radiation Fields

4.2.2.1 Dipole Moment Definition

Electric Dipole Moment

$$\vec{p}(t) = \int_V \vec{r}' \rho(\vec{r}', t) dV' \tag{4.8}$$

For a Hertzian (infinitesimal) dipole: two point charges +q and -q separated by distance $d \ll \lambda$

$$\vec{p}(t) = q.d \tag{4.9}$$

For **time-varying dipole**:

$$\vec{p}(t) = \vec{p}_0 e^{j(\omega t - \frac{\pi}{2})} \tag{4.10}$$

Or equivalently:

$$\vec{p}(t) = Re[\vec{p}(t)] = p_0 \cos(\omega t)\hat{z} \tag{4.11}$$

Dipole Moment Rate:

$$\frac{d\vec{p}}{dt} = \dot{\vec{p}}(t) = j\omega p_0 e^{j\omega t} = j\omega \vec{p}(t)$$
$$\frac{d^2\vec{p}}{dt^2} = \ddot{\vec{p}}(t) = -\omega^2 p_0 e^{j\omega t} = -\omega^2 \vec{p}(t) \tag{4.12}$$

4.2.2.2 Radiation Fields from a Hertzian Dipole

Far-Field Electric Field

At large distances $r \gg \lambda$, the electric field from an oscillating electric dipole is:

$$\vec{E}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{1}{r} sin\theta \left[\ddot{\vec{p}}\left(t - \frac{r}{c}\right) \times \hat{r} \times \hat{r}\right] \tag{4.13}$$

Or in spherical coordinates (with dipole along z-axis):

$$E_\theta(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{1}{r} sin\theta \, \ddot{p}\left(t - \frac{r}{c}\right) \tag{4.14}$$

Harmonic (Frequency Domain) Form

For time-harmonic excitation $\vec{p}(t) = \vec{p}_0 e^{j(\omega t)}$:

$$\widetilde{E_\theta}(r,\theta) = \frac{\mu_0 \omega^2}{4\pi}\frac{sin\theta}{r}\overrightarrow{p_0} e^{-jkr} \tag{4.15}$$

Where $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ is the **wave number**.

$$\widetilde{E_\theta}(r,\theta) = \frac{jk\eta_0 p_o}{4\pi}\frac{sin\theta}{r} e^{-jkr} \tag{4.16}$$

Where $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$ (free space impedance).

Far-Field Magnetic Field

The magnetic field in the radiation zone is related to the electric field by:

$$\vec{H} = \frac{\vec{E}}{\eta_0} \times \hat{r} \tag{4.17}$$

In spherical coordinates:

$$\widetilde{H_\phi}(r,\theta) = \frac{\widetilde{E_\theta}(r,\theta)}{\eta_0} = \frac{jkp_o}{4\pi\eta_0}\frac{sin\theta}{r} e^{-jkr} \tag{4.18}$$

Based on the above analysis, it can be noted that:

- Fields decrease as (1/r) (radiation field)
- Radiation pattern proportional to $sin\theta$ (donut-shaped)
- Maximum radiation perpendicular to dipole (at $\theta = \frac{\pi}{2}$)
- Zero radiation along dipole axis ($\theta = 0$ and ϕ)
- $\vec{E}$ and $\vec{H}$ are perpendicular to each other and to propagation direction ((TEM) wave)
- $\vec{E}$ and $\vec{H}$ are in phase in far field

4.2.2.3 Radiation Pattern from Electric Dipole

The radiation pattern describes the angular distribution of radiated power.

Normalized Radiation Pattern Function

$$F(\theta, \phi) = \sin\theta \tag{4.19}$$

For a z-directed dipole, the pattern is independent of ϕ (azimuthally symmetric).

Radiation Intensity (Power per Solid Angle):

$$U(\theta, \phi) = U_o sin^2\theta \tag{4.20}$$

Where U_o is the maximum radiation intensity in the equatorial plane ($\theta = \frac{\pi}{2}$).

Poynting Vector in Radiation Zone

$$\vec{S}(\vec{r}, t) = \frac{1}{2} Re[\vec{E} \times \vec{H}^*] \tag{4.21}$$

$$S_r(r, \theta) = \langle \frac{|\tilde{E}_\theta|^2}{2\eta_0} \rangle = \frac{k^2 {p_0}^2 sin^2\theta}{32\pi^2 \eta_0 r^2} \tag{4.22}$$

Radiation Pattern Diagram Characteristics

Polar Plot

- Figure-of-eight shape (two lobes)
- Lobes in equatorial plane (perpendicular to dipole axis)
- Nulls at poles (along dipole axis)
- Lobe intensity: $sin^2\theta$ function

Normalized (dB) Plot:

$$Pattern\ (dB) = 20log_{10}|sin\theta| \tag{4.23}$$

Beamwidth Measurements:

$$sin^2\theta = 0.5$$

- Half-Power Beamwidth (HPBW): where ($\theta_{HPBW} = \arcsin(0.707) \approx 44.9°$
- First Null Beamwidth (FNBW): where $sin^2\theta = 0$

$\theta_{HPBW} = 90°$ (from null to null in E-plane)

4.2.2.4 Radiated Power from Electric Dipole

Time-Averaged Radiated Power:

Integrating the Poynting vector over all directions (a sphere at radius r:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} S_r(r,\theta) r^2 sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\pi} \frac{k^2 {p_0}^2 sin^2\theta}{32\pi^2\eta_0 r^2} r^2 sin\theta d\theta d\phi \quad (4.24)$$

Integration Over Solid Angle:

$$\int_0^{2\pi} d\phi = 2\pi, \int_0^{\pi} sin^3\theta d\theta = \frac{4}{3}$$

Final Result - Larmor Formula for Dipole Radiation:

$$P_{rad} = \frac{k^2 {p_0}^2}{12\pi\eta_0} = \frac{\mu_0\omega^4 {p_0}^2}{12\pi c} \quad (4.25)$$

Comparison with Input Power:

For a harmonic dipole $\vec{p}(t) = p_0 \cos(\omega t)$:

$$P_{rad} = \frac{\mu_0\omega^4 {p_0}^2}{24\pi c}$$

(Time-averaged over one period)

4.2.2.5 Effective Height of Electric Dipole

The effective height (or effective length) is a fictitious length that a uniform antenna element carrying feed point current I_0 would need to have to produce the same far-field magnitude as the actual antenna with current I(z).

For Electric Dipole

$$h_{eff} = \frac{\vec{p}}{I_0} \quad (4.26)$$

Where $\vec{p}$ is the dipole moment and I_0 is the characteristic current.

For a dipole created by oscillating charges, the dipole moment relates to current through:

$$p(t) = q.d(t) \quad (4.27)$$

If charges oscillate with velocity $v(t) = \dot{d(t)}$:

$$\dot{p}(t) = q.v(t) = I(t).d \quad (4.28)$$

Therefore:

$$h_{eff} = d \quad (4.29)$$

For infinitesimal dipole, $h_{eff} \to 0$, but dipole moment (p) remains finite.

For Finite-Length Dipole:

$$h_{eff} = \frac{1}{I_0}\int_0^L I(z)\,dz \tag{4.30}$$

This represents the effective length weighted by current distribution.

Example: Half-Wave Dipole

For a half-wave dipole $L = \frac{\lambda}{2}$ with sinusoidal current distribution:

$$I(z) = I_0\cos(\frac{2\pi z}{\lambda}) \tag{4.31}$$

$$h_{eff} = \frac{1}{I_0}\int_0^{\lambda/2} I_0\cos(\frac{2\pi z}{\lambda})\,dz = \frac{\lambda}{\pi} \approx 0.318\lambda \tag{4.32}$$

4.2.2.6 Radiation Resistance of Electric Dipole

Radiation resistance is the equivalent resistance that, when carrying the same current as the antenna, would dissipate power equal to the radiated power.

$$P_{rad} = \frac{1}{2}{I_0}^2 R_{rad} \tag{4.33}$$

For Electric Dipole:

Starting with the radiated power:

$$P_{rad} = \frac{\mu_0\omega^4{p_0}^2}{12\pi c} \tag{4.34}$$

Relating dipole moment to current. For a time-varying dipole:

$p(t) = q.d(t)$, If $d(t) = d_0\cos(\omega t)$ and $I(t) = \frac{dq}{dt} = \dot{q}$

For oscillating charges with dipole moment $\vec{p}(t) = p_0\cos(\omega t)$ with $p_0 = \frac{I_0}{\omega}.h_{eff}$

Substituting into Power Equation

$$P_{rad} = \frac{\mu_0\omega^4}{12\pi c}(\frac{I_0 h_{eff}}{\omega})^2 = \frac{\mu_0\omega^2{I_0}^2{h_{eff}}^2}{12\pi c}$$

Setting equal to $P_{rad} = \frac{1}{2}{I_0}^2 R_{rad}$:

$$R_{rad} = \frac{\mu_0\omega^2{h_{eff}}^2}{6\pi c} \tag{4.35}$$

In Terms of Wavelength

Since $= \frac{\omega\lambda}{2\pi}$,

$$R_{rad} = \frac{\mu_0\left(\frac{2\pi}{\lambda}\right)^2{h_{eff}}^2}{6\pi} = 20\pi^2\left(\frac{h_{eff}}{\lambda}\right)^2 \quad \Omega$$

For Short Dipole (l << λ)

With $h_{eff} = l$:

$$R_{rad}^{short} = 20\pi^2 \left(\frac{l}{\lambda}\right)^2 \quad \Omega$$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \tag{4.36}$$

4.3 Dipole antenna in isolation

4.3.1 Isolated Dipole Antenna Radiation

4.3.1.1 Short Dipole (Hertzian Dipole) Antenna

An electric dipole, or Hertz dipole, is defined as a linear conductor of length l, which is significantly smaller than the wavelength (λ) in free space and possesses a negligible cross-section. Consequently, the current passing through it remains virtually constant. Examine an antenna element aligned with the Oz axis and positioned at the origin of the coordinate system (Fig. 4.1). Given these conditions, the electric current density is expressed as:

$$\overrightarrow{J_e}(x', y', z') = \begin{cases} I_0\,\hat{z} & for\ x' = y = z' = 0 \\ o & elsewhere \end{cases} \tag{4.37}$$

I_0: (in Ampere) is a constant

This is an approximation valid because the current has no time to phase-shift significantly along the short length.

Radiation Pattern - Far Field

The far-field radiation from a z-oriented short (Hertzian) dipole antenna, with uniform current I_0 and length $l \ll \lambda$, produces transverse electromagnetic fields in spherical coordinates (r, θ, ϕ) given by:

$$\tilde{E}_\theta(r, \theta) = \frac{jkI_0\eta_0 l \sin\theta\ e^{-jkr}}{4\pi r} \tag{4.38}$$

$$\tilde{H}_\phi(r, \theta) = \frac{jkI_0 l \sin\theta\ e^{-jkr}}{4\pi r} \tag{4.39}$$

Where: $\tilde{E}_\theta(r,\theta) = \eta_0 \tilde{H}_\phi(r,\theta)$, and $k = 2\pi/\lambda$ is the wavenumber and $\eta \approx 377\ \Omega$ is the free-space impedance.

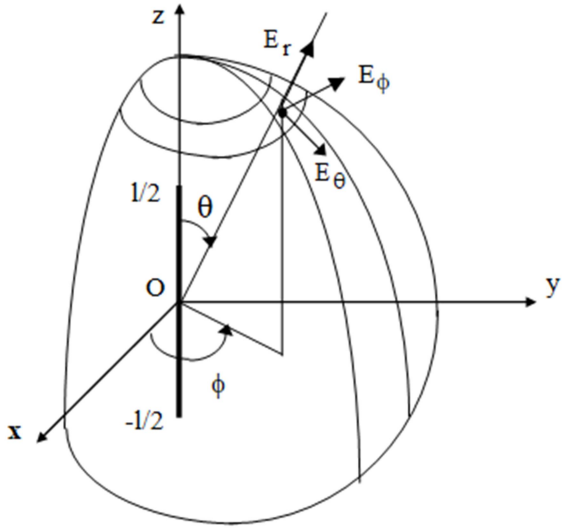


Figure 4.1: Hertzian dipole and the electric field components in spherical coordinates

Radiation Pattern Diagram

The normalized pattern is:

$$\mathrm{F}(\theta) = |\sin\theta| \tag{4.40}$$

Power density

Power density represents the radiant energy flux per unit area in the electromagnetic field, directly tied to the Poynting vector (eq 4.41), which quantifies instantaneous power flow as the cross product of electric and magnetic field vectors. In antenna contexts, average power density derives from the time-averaged Poynting vector magnitude, guiding far-field radiation analysis and received signal strength.

$$W_{rad} = \frac{1}{2} Re(E \times H^*) \tag{4.41}$$

Radiation intensity

The radiation intensity U, or steric power, is the power radiated per unit solid angle $d\Omega$.

$$\mathrm{U}(\theta,\phi) = \frac{P_{rad}}{d\Omega} \tag{4.42}$$

$$P_{rad} = \oiint \mathrm{U}(\theta,\phi)\, d\Omega = \int_0^{2\pi} \int_0^{\pi} \mathrm{U}(\theta,\phi).\sin\theta d\theta d\phi \tag{4.43}$$

$$U(\theta,\phi) = r^2 W_{rad} \tag{4.44}$$

Directivity

$$D = \frac{4\pi . U(\theta,\phi)}{P_{rad}} \tag{4.45}$$

For short dipole: $D = 1.5$

The factor 1.5 arises because although the pattern is proportional to $sin^2\theta$, not all power is concentrated in the main lobe.

Radiation Pattern Diagram Characteristics

Two equal lobes peak in the equatorial plane (θ=90°), where radiation is maximum, forming symmetric "petals" perpendicular to the z-axis. Perfect nulls occur along the z-axis (θ=0°,180°) because sinθ=0, producing no radiation endfire. The pattern is symmetric about the xy-plane, invariant under θ→180°, and rotationally symmetric in ϕ (omnidirectional azimuthally).

The H-plane radiation pattern for a short dipole (z-directed) at ϕ=90° is perfectly circular, reflecting omnidirectional radiation in the azimuthal plane. This plane cuts through the xy-equatorial plane (θ=90°). where radiation peaks, with field magnitude |Eθ|∝sinθ independent of ϕ. The pattern appears as a circle of constant radius in polar coordinates, showing uniform radiation in all azimuthal directions perpendicular to the dipole axis. No ϕ-variation exists due to cylindrical symmetry about z.

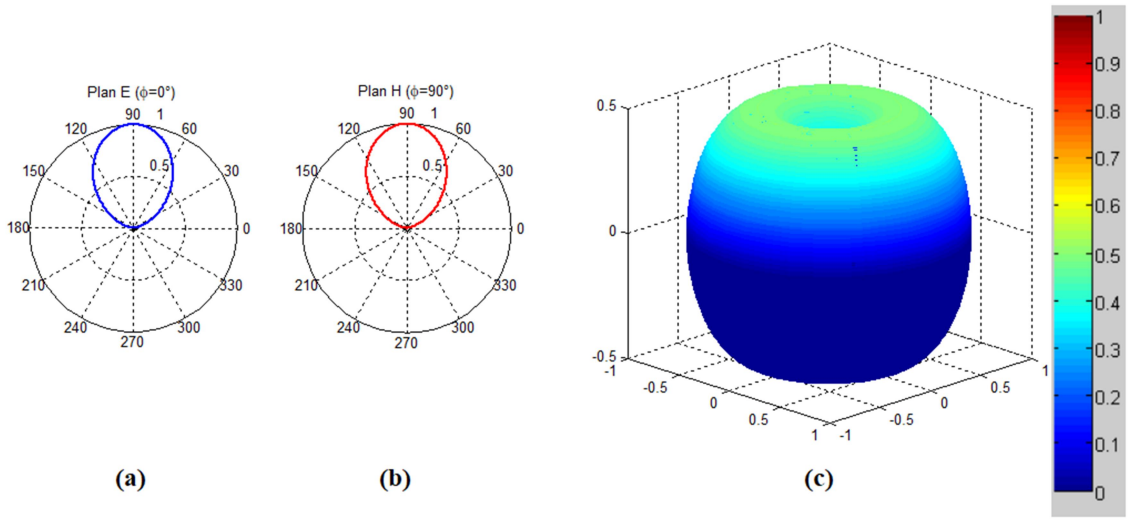


Figure 4.2: Short dipole radiation patterns in the E-plane (a), H-plane (b), and 3D radiation pattern (c).

4.3.1.2 Half-Wave Dipole Antenna

The half-wave dipole antenna, which is L=λ/2 long, is a centre-fed design that has an input impedance of about 73 Ω, making it the most practical and commonly used dipole type.
This antenna consists of two collinear arms, each of length λ/4, driven by a balanced feed at the center gap. Resonance occurs when
L≈0.48λ to 0.5λ (slightly shortened for thin wires), yielding high radiation efficiency because losses are small relative to radiation resistance. Unlike the short dipole, it supports a travelling wave from the feed that reflects at open ends, forming a standing wave pattern.

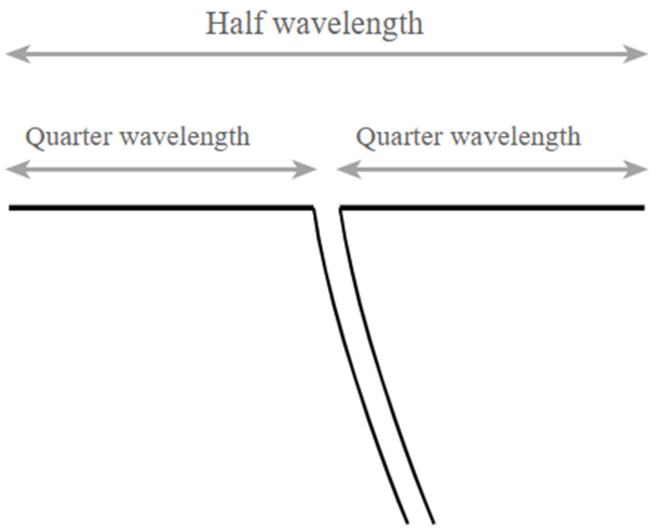


Figure 4.3: Basic half- wave antenna.

Unlike short dipole, the current varies significantly with position. For a half-wave dipole center-fed:

$$\mathrm{I}(z) = I_0 \cos\left(\frac{2\pi z}{\lambda}\right) \quad for -\frac{\lambda}{4} < z < \frac{\lambda}{4} \tag{4.46}$$

Where:

- I_0 :feed point current (maximum)
- Current = 0 at dipole ends ($z = \pm\frac{\lambda}{4}$)
- Sinusoidal distribution

At resonance ($L = \frac{\lambda}{2}$), electromagnetic wave propagates along the antenna in both directions. The reflected wave from the open ends interferes with the incident wave from the feed, creating a standing wave pattern with:

- Current anti-node at center (feed point)
- Current nodes at ends

4.3.1.3 Radiation Pattern from Half-Wave Dipole

Electric Field in Far Zone

For a sinusoidally-driven half-wave dipole:

$$\tilde{E}_\theta(r,\theta) = \frac{j60I_0}{r}\frac{\cos(\frac{\pi}{2}\cos(\theta))}{sin\theta}e^{-jkr} \tag{4.47}$$

This expression comes from integrating the contribution of each current element along the dipole using the retarded potential approach.

Pattern Function

$$\mathrm{F}(\theta) = \left|\frac{\cos(\frac{\pi}{2}\cos(\theta))}{sin\theta}\right| \tag{4.48}$$

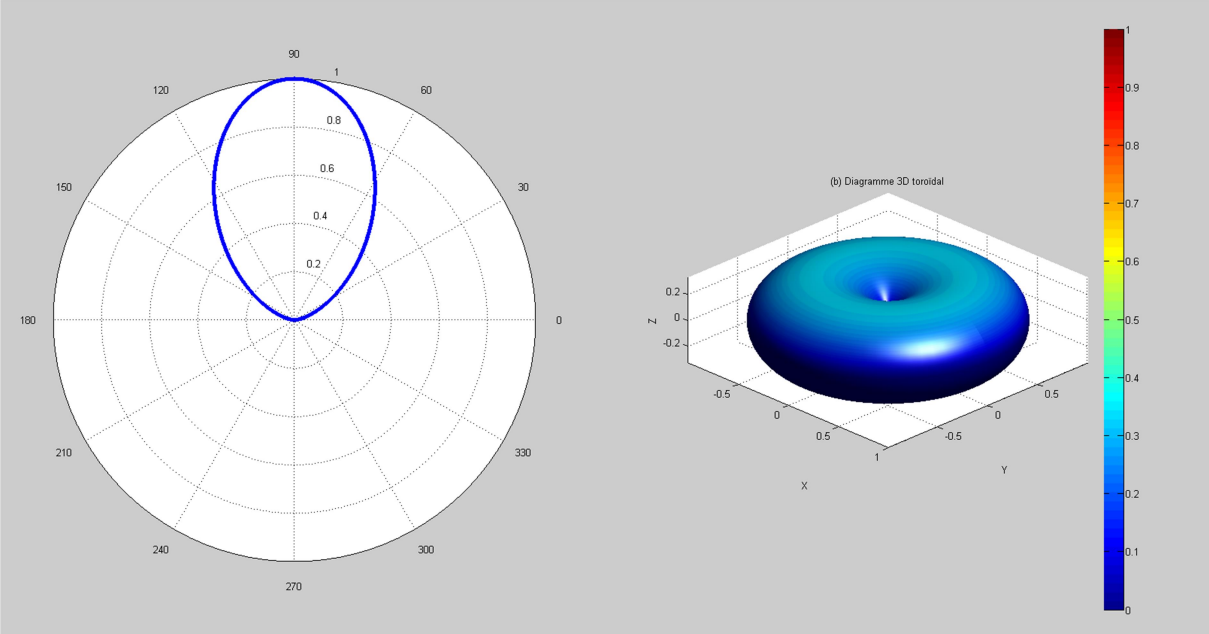


Figure 4.4: Half-Wave Dipole Radiation Patterns (E-Plane and 3D)

Radiation Intensity

$$\mathrm{U}(\theta) = \left|\frac{\cos(\frac{\pi}{2}\cos(\theta))}{sin\theta}\right|^2 \tag{4.49}$$

Key Differences from Short Dipole

The primary differences in radiation properties between half-wave and short-wave dipoles are shown in Table 4.1. Short dipoles result in a greater beamwidth, no side lobes, and a thin,

smooth main lobe with perfect nulls at end-fire angles. While both have the same directivity, half-wave dipoles have a slightly narrower beamwidth, a larger, more circular main lobe, incomplete broad nulls, and minor side lobes.

Table 4.1: Comparative Radiation Characteristics of Short vs. Half-Wave Dipoles

Characteristic	Short Dipole	Half-Wave Dipole
Pattern function	$sin\theta$	$\frac{\cos(\frac{\pi}{2}\cos(\theta))}{sin\theta}$
Main lobe shape	Narrow (smooth)	Broader, more circular
Nulls	Perfect at θ = 0, 180°	Not perfect; broad minimum
Directivity	1.64 (2.15 dBi)	1.64 (2.15 dBi)
Side lobes	None	Minor
Beamwidth (-3 dB)	~90°	~78°

4.3.1.4 Radiated Power from Half-Wave Dipole

Integration Over Solid Angle

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} S_r(r,\theta) r^2 sin\theta d\theta d\phi = \frac{{I_0}^2}{2} R_{rad}^{1/2\lambda} \tag{4.50}$$

Where the radiation resistance is: $R_{rad}^{1/2\lambda} = 73.1\ \Omega$

4.3.1.5 Effective Height of Half-Wave Dipole

Effective height describes an antenna's ability to convert an incident electromagnetic wave's electric field into an open-circuit voltage at its terminals.

$$h_{eff} = \frac{1}{I_0}\int_{-L/2}^{L/2} I(z)dz = 0.318\lambda \tag{4.51}$$

Where the current distribution is:

$$I(z) = I_0\cos(\frac{2\pi z}{\lambda}) \tag{4.52}$$

The effective height of $\frac{\lambda}{\pi}$ represents the average "seeing height" of the antenna from the perspective of radiating current. Although the physical length is $\frac{\lambda}{2}$, because current is concentrated near the center, the effective height is smaller.

Table 4.2 compares effective heights relative to physical length across antenna types: short dipoles match physical length to effective height, quarter-wave monopoles achieve half that of a half-wave dipole, and half-wave dipoles set the reference standard.

Table 4.2: Comparison of Effective Heights

Antenna Type	Physical Length	Effective Height
Short dipole	l	L
Quarter-wave monopole	$\frac{\lambda}{4}$	$\frac{\lambda}{4} \approx 0.25\lambda$
Half-wave dipole	$\frac{\lambda}{2}$	$\frac{\lambda}{\pi} \approx 0.318\lambda$

Table 4.3 contrasts short- and half-wave dipoles, showing short dipoles with low radiation resistance and directivity of 1.5 versus half-wave dipoles offering much higher resistance at 73.1 Ω and directivity of 1.64.

Table 4.3: Comparison with Short Dipole

Dipole Type	Radiation Resistance	Directivity
Short $\boldsymbol{l = 0.05\lambda}$	0.197 Ω	1.5
Half-wave $\boldsymbol{l = 0.5\lambda}$	73.1 Ω	1.64

Input Impedance (at resonance):

For a resonant half-wave dipole:

$$Z_{in} = R_{rad} + JX \approx 73 + j43.5\ \Omega \tag{4.53}$$

At exact resonance (after minor length adjustment):

$Z_{in} \approx 73$ (nearly purely resistive)

This is why the half-wave dipole is preferred: at resonance, the reactive component nearly vanishes, facilitating impedance matching.

4.4 Field Zone Regions and Boundary Determination

The electromagnetic field from a radiating dipole exhibits three distinct regions based on radial distance r. In the near-field (reactive zone), $r < \lambda/(2\pi)$, non-propagating terms dominate with $E \propto 1/r^3$ (quasi-static) and $H \propto 1/r^2$ (induction), producing stored reactive energy, imaginary Poynting vector, complex E/H phase relationships, and highly distance-dependent patterns. The intermediate zone ($\lambda/(2\pi) < r < 2D^2/\lambda$) transitions with both reactive and radiating components contributing, where power begins flowing outward but pattern shape still varies with distance. Finally, the far-field (radiation zone) at $r > \max(2D^2/\lambda, \lambda/2\pi)$ features pure $1/r$ decaying TEM waves with E perpendicular to H and propagation direction, constant phase fronts, real outward Poynting vector, and angular pattern independent of distance (except amplitude) critical for array factor analysis where $|\vec{E}|/|\vec{H}| = \eta_0 = 377\Omega$. For a $\lambda/2$ dipole ($D = \lambda/2$), far-field measurements begin at $r > 0.8\lambda$.

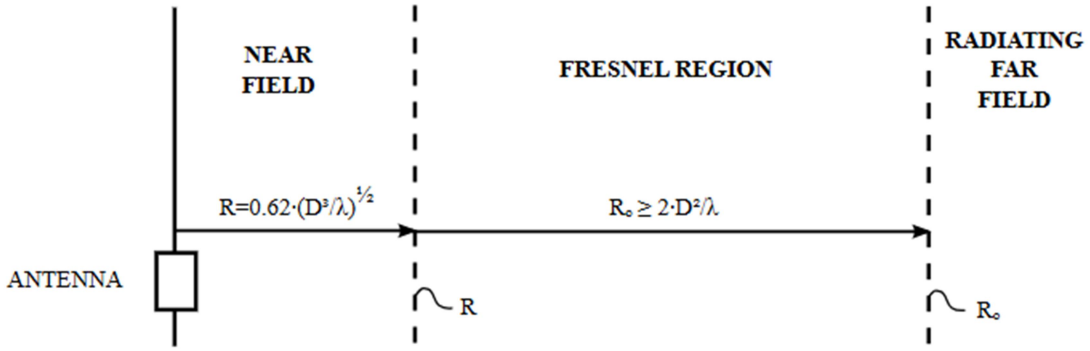


Figure 4.5: Antenna radiation zones

4.5 Dipole Array Concepts

4.5.1 Two Dipole Array - Pattern Multiplication

The radiation pattern of a linear antenna array factors into the product of the single element pattern and the array factor:

$$F_{\text{array}}(\theta) = F_{\text{element}}(\theta) \times AF(\theta) \tag{4.54}$$

Where $F_{\text{element}}(\theta)$ represents the individual dipole pattern such as $|\cos(\pi/2\cos\theta)/\sin\theta|^2$ for a half-wave dipole and $AF(\theta)$ encodes the geometric interference effects from element spacing d and progressive phase excitation $\Delta\alpha$. For a two-element array with equal amplitude excitation I_0 and spacing d, the array factor simplifies to $AF(\psi) = 2\cos(\psi/2)$ where $\psi = kd\cos\theta + \Delta\alpha$, producing beam narrowing, steering capability ($\Delta\alpha \neq 0$).

Example: Two Half-Wave Dipoles, ($d = \frac{\lambda}{2}$, In-Phase

Array factor:

$$F_{\text{array}}(\theta) = 2\cos(\frac{\pi}{2}\sin\theta) \tag{4.55}$$

Combined pattern:

$$F_{\text{total}}(\theta) = \left|\frac{\cos(\frac{\pi}{2}\cos(\theta))}{sin\theta}\right| \times 2\cos(\frac{\pi}{2}\sin\theta) \tag{4.56}$$

This demonstrates pattern multiplication principle used in antenna arrays.

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4.6 Conclusion

Finally, this chapter provides analytical methods for far-field radiation from elementary antennas, including electric dipoles and isolated space dipoles, deriving key metrics like characteristic area, radiation pattern, radiated power, equivalent height, and radiation resistance. These calculations reveal fundamental differences in pattern shape, directivity, and efficiency between short and half-wave configurations, establishing benchmarks for antenna performance. In a subsequent chapter, this foundation enables the precise modelling and optimisation of more complex radiating structures.

4.7 Series of tutorial exercises

4.7.1 Exercise N°1

A short dipole antenna operates at 1 GHz with physical length $l = 0.1\lambda = 3$ cm and feed current $I_0 = 1$ A. Determine the radiation resistance R_{rad} and total radiated power P_{rad}.

4.7.2 Exercise N°2

A half-wave dipole antenna operates at 100 MHz ($\lambda = 3$ m, $L = 1.5$ m) with peak feed current $I_0 = 5$ A. Compute all radiation characteristics.

4.8 Solutions of series of tutorial exercises

4.8.1 Solution of exercise N°1

For electrically short dipoles ($kl \ll 1$), the radiation resistance is given by $R_{\text{rad}} = 20\pi^2(l/\lambda)^2 = 20\pi^2(0.1)^2 = 1.97$, and the time-average radiated power is $P_{\text{rad}} = \frac{1}{2}I_0^2 R_{\text{rad}} = \frac{1}{2}(1)^2(1.97) = 0.493$ W.

4.8.2 Solution of exercise N°1

- Wavelength $\lambda = c/f = 3 \times 10^8/100 \times 10^6 = 3$ m
- Length $L = \lambda/2 = 1.5$ m
- Effective height $h_{\text{eff}} = \lambda/\pi = 0.955$ m
- Radiation resistance $R_{\text{rad}} = 73.1$ (standard)
- Radiated power $P_{\text{rad}} = \frac{1}{2}I_0^2 R_{\text{rad}} = \frac{1}{2} \times 5^2 \times 73.1 = 914$ W
- Directivity $D = 1.64$ (2.15 dBi)

- Max intensity $U_{\max} = P_{\text{rad}} D/(4\pi) = 75.3$ W/sr

4.9 Summary of chapter 4

Chapter 4 analyzes radiation from elementary antennas, deriving far-fields from Maxwell's equations, Lorentz potentials, and retarded times for Hertzian (short) dipoles: $E_\theta = j\frac{\eta k I_0 dl \sin\theta e^{-jkr}}{4\pi r}$, $\sin^2\theta$ pattern $(D = 1.5)$, $R_{rad} = 20\pi^2(dl/\lambda)^2 \approx 80\pi^2(l/\lambda)^2\Omega$, $P_{rad} \propto \omega^4 p^2$, effective height $h_e = l/2$. For half-wave dipoles $(\lambda/2)$, sinusoidal current yields $\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}$ pattern $(D = 1.64)$, $R_{rad} \approx 73\Omega$, $h_e = \lambda/\pi$. Defines zones (reactive $r < \lambda/2\pi$, far-field $r > 2D^2/\lambda$); introduces array pattern multiplication. Exercises confirm calcs accurately. Retain dipole benchmarks for array synthesis. Now that you have acquired elementary radiation theory, you can move on to practical antenna types in Chapter 5.

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Chapter 5: Basic Antenna Characteristics

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5.2.2 Applications of Folded Dipole

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5.12.2 Solution of exercise N°2: Small Magnetic Loop for 7 MHz

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5 Chapter 5: Antenna Types and Their Applications

5.1 Introduction

This chapter presents a detailed survey of the most important antenna types used in modern communication systems, spanning from decametric (low frequency) through microwaves and beyond. Each antenna type is characterised by specific radiation patterns, impedance properties, frequency ranges, polarisation capabilities, and practical applications. Understanding the strengths, limitations, and design principles of these elementary antenna configurations provides a foundation for developing complex antenna arrays and specialised systems.

5.2 Folded dipole antenna

A folded dipole antenna consists of a half-wavelength dipole with an additional parallel conductor connecting the two ends, forming a closed loop, where the additional element is typically identical in length and diameter to the primary dipole, though variations exist. In its basic configuration, two parallel conducting rods, each approximately λ/2 long, connect at both ends to form a closed cylindrical loop and feed at the center with opposite terminals, acting as a two-wire transmission line shorted at the ends that excites currents in both transmission line mode and antenna mode. The parallel rods form a transmission line with characteristic impedance: $Z_0 \approx 240 \log(s/r)$ ohms, where s is the spacing between rods and r is the wire radius; for equal-diameter wires, the total input impedance at resonance is $Z_A = 4*Z_d$ (around 292 ohms, given $Z_d \approx 73$ ohms for a half-wave dipole), which suits matching to 300-ohm twin-lead lines unlike a standard dipole's 73 ohms.

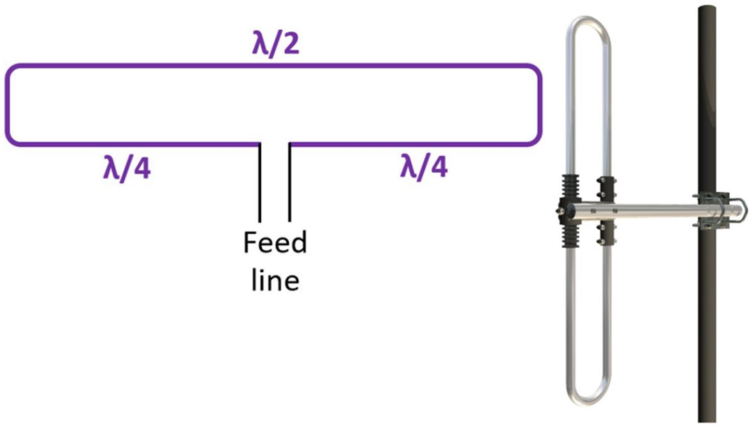


Figure 5.1: Folded dipole antenna

5.2.1 Radiation Pattern of Folded Dipole

The radiation pattern of a folded dipole is essentially identical to that of a conventional half-wave dipole:

$$F(\theta) = \left| \frac{\cos(\pi \cos(\frac{\theta}{2}))}{\sin\theta} \right| \qquad (5.1)$$

The folded dipole antenna exhibits a radiation pattern with two lobes in the equatorial plane, maximum gain perpendicular to the antenna, and approximate nulls along the dipole axis. It features linear polarisation along the axis, a 3 dB beamwidth of about 78° matching the half-wave dipole, and directivity of 1.64 (2.15 dBi). In the azimuthal end view, the pattern appears omnidirectional in the plane perpendicular to the antenna, ideally forming a perfect circular radiation with no variation by angle.

The current distribution in antenna mode is:

$$\mathrm{I(z)} = \mathrm{I_0}\cos(\frac{2\pi z}{\lambda}) \tag{5.2}$$

This is identical to a single dipole; the second element just carries the complementary return current, which has negligible radiation contribution in the far field.

Reflection Coefficient at 292 Ω Input

For transmission line with 292 Ω impedance to 50 Ω:

$$\Gamma = \frac{292 - 50}{292 + 50} = \frac{242}{342} = 0.708 \tag{5.3}$$

This represents a significant impedance mismatch.

Gain

Since pattern and radiation resistance are unchanged:

$$\mathrm{G} = \eta . D = \eta \times 1.64 \tag{5.4}$$

Where η is radiation efficiency (typically >95% for good folded dipoles).

5.2.2 Applications of Folded Dipole

Television Reception Antenna (Classic Application):

- Frequency: VHF (54-216 MHz)
- Impedance: 300 Ω (matched to twin-lead)
- Configuration: Crossed pair for all-channel coverage
- Reflector and directors for gain enhancement

RF Power Amplifier Load (Impedance Matching):

- Power transistor outputs typically 50 Ω
- Folded dipole steps up to 200-300 Ω
- Direct matching to high-impedance loads

Mobile Communication Antennas:

- GPS/GNSS receivers
- Marine VHF systems

- Wearable antennas

Broadband and Multiband Systems:

- 2-4 octave bandwidth achievable
- Multiple folded elements for different bands
- Stacked configurations for arrays

5.3 Loop antennas

Loop antennas use a continuous conducting loop to capture or radiate electromagnetic waves, operating on magnetic field induction and classified mainly as small loops (circumference < λ/10, like ferrite loops for AM radio with edge-maximum radiation and nulls broadside) or large/self-resonant loops (> λ, efficient for HF/VHF with broadside lobes, often quad or halo shapes for higher gain). They offer simple construction; low noise in direction finding; and versatile shapes (circular, square, and triangular), with small types suited for receiving/portable use despite low efficiency and large ones for transmitting over HF bands like NVIS or DX when vertical.



Figure 5.2: Loop antenna

Table 5.1 summarises the various loop antenna configurations, highlighting their unique characteristics and typical applications across different designs.

Table 5.1: Loop Configurations

Shape	Characteristics	Applications
Circular	Simplest, symmetric	Standard designs
Square	Compact, planar	Arrays, printed
Rectangular	Flexible dimensions	Phased arrays
Triangular	Space-efficient	Compact systems
Rhombic	Long-wire characteristics	HF communication
Polygonal	Optimization possible	Specialized

5.3.1 Types of loop antennas

The antenna can be small (much smaller than wavelength) or large (resonant at operating frequency). Table 5.2 compares loop antenna types by definition, radiation pattern, radiation resistance, and key applications.

Table 5.2: Loop Antenna Types Comparison

Type	Definition (Circumference)	Radiation Pattern	Radiation Resistance Example	Key Applications/Notes
Small Loop	$C \ll \lambda/10$	Max perpendicular to plane; nulls in plane (magnetic dipole)	Very low (e.g., 1-ft at 1 MHz: 0.002 Ω); multi-turn boosts by N^2	Receiving (AM radio, direction finding); inefficient TX
Large/Resonant Loop	$C \approx \lambda$	Max in plane of loop (broad lobes); nulls perpendicular	Higher than small (efficient at resonance)	HF transmitting (NVIS/DX); polygonal variants (square/triangular/rhombic)
Polygonal Variants	$C \approx \lambda$ (square: 4s, etc.)	Similar to circular; lobes per side; adjustable directivity	Varies with geometry (rhombic: angle-dependent)	RFID, arrays; rhombic for CP via feed/gap

5.3.2 Impedance Characteristics of Loop Antennas

Input Impedance (General)

$$Z_{in} = R_{rad} + jX_{in} \tag{5.5}$$

Radiation Resistance

- Small loop: ultra-low 10^{-6} to 10^{-3} Ω
- Resonant loop: moderate (~10-100 Ω)

Reactance at Resonance

- Designed for zero reactance at operating frequency
- Variable capacitor tunes out initial inductive reactance

Loop Inductance

$$L_{loop} \approx \mu_0 C \left[\ln\left(\frac{4A}{r^2}\right) - 2\right] \tag{5.6}$$

Where:

- C: circumference
- A: enclosed area
- r:wire radius

Tuning Capacitor Design:

For resonance at frequency (f):

$$C = \frac{1}{(2\pi f)^2 L_{loop}} \tag{5.7}$$

5.3.3 Applications of Loop Antennas

Receiving Antennas

- **Direction Finding**: Sharp nulls useful for bearing determination
- **MF/HF Reception**: Small loops for compact reception
- **UHF Band**: Arrays of loops for VHF/UHF systems
- **RFID Systems**: Loop elements in RFID readers

Transmitting Antennas:

- **Amateur Radio**: Transmitting magnetic loops on HF bands
- **EMC Testing**: Loop antennas for immunity testing
- **Radar Systems**: Coupling and element design

Specialized Applications:

- **Near-field Work**: Very small loops for near-field measurements
- **Biomedical**: Implanted loop antennas (MRI compatibility)
- **Geophysical**: Loop arrays for ground-penetrating radar

5.4 Wire doublet antenna for decametric waves

A doublet antenna is a centre-fed horizontal wire antenna of any length not restricted to λ/2 typically fed with a parallel-conductor transmission line like ladder line or twin-lead. Unlike resonant half-wave dipoles, it operates across multiple frequency bands without single-frequency resonance (unless the length equals nλ/2) and requires an antenna tuner for impedance matching due to its highly variable impedance. Typical configurations feature 40-80 metre lengths for HF amateur radio, mounted 10-30 metres above ground, using 300-450 Ω feed lines to handle broadband performance.

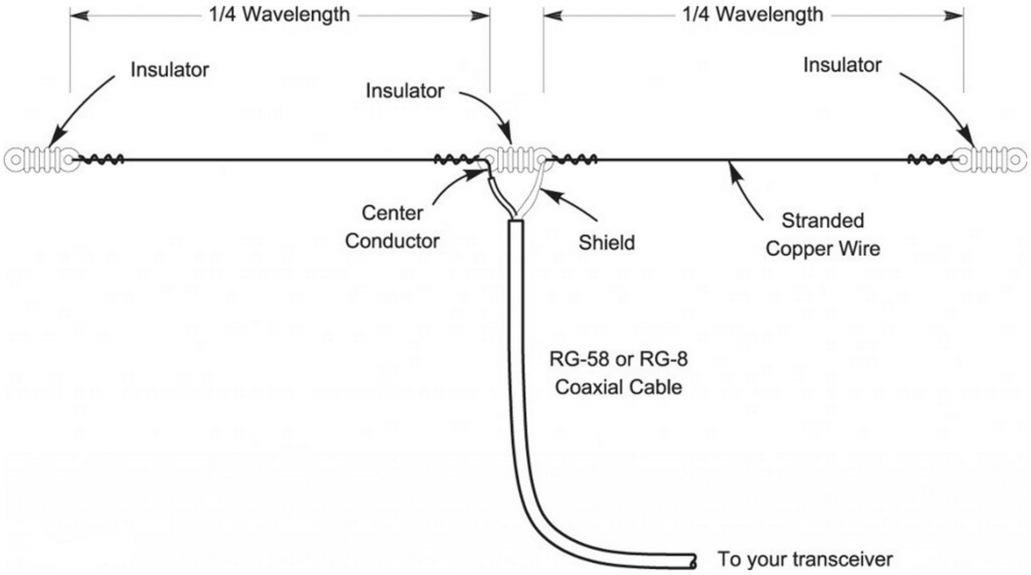


Figure 5.3: Wire doublet antenna for decametric waves

5.4.1 Impedance Variation with Frequency

For a 40-meter doublet (fundamental at 80 m = 3.6 MHz):

Table 5.3: Impedance Variation with Frequency

Band	Frequency	Length	Impedance	SWR (50 Ω)
80 m	3.6 MHz	0.5λ	51 - j0	1.0
60 m	5.35 MHz	0.67λ	400 + j1000	75
40 m	7.1 MHz	1.0λ	3600 - j2500	105
30 m	10.1 MHz	1.33λ	100 - j400	36
20 m	14.2 MHz	1.77λ	3200 + j0	65
17 m	18.1 MHz	2.25λ	100 - j200	11
15 m	21.3 MHz	2.67λ	2700 + j600	57
12 m	24.9 MHz	3.1λ	115 - j430	35
10 m	28.5 MHz	3.6λ	2100 + j650	45

5.4.2 Applications of Doublet Antennas

Amateur Radio (Primary Application)

- Multiband operation (80-10 m)
- Horizontal installation (quiet, compact)
- Economical (inexpensive components)
- Established community knowledge

Advantages

- **Multiband**: One antenna covers many frequencies
- **Flexible**: Height and orientation adjustable
- **Economical**: Cheap to build
- **Robust**: Simple, proven design
- **Efficient**: Parallel-line feed minimizes losses

Disadvantages

- **Tuner required**: Added cost and complexity
- **Space needed**: Long wires (40+ meters)
- **Variable performance**: Different pattern on each band
- **Ground sensitivity**: Height and ground affect pattern

5.5 Yagi-uda antenna

The Yagi-Uda antenna (or Yagi) is a simple, highly directive linear array of parallel dipole elements in which one element is driven (fed with a transmission line) and the others are parasitic, consisting of a slightly longer reflector behind the driven element and one or more slightly shorter directors in front, all mounted on a common boom. The parasitic elements re-radiate with appropriate phase delays so fields add constructively in the forward direction and cancel toward the rear, giving a strong unidirectional main lobe toward the directors, typical forward gains on the order of 10 dB or more, and front-to-back ratios up to about 20 dB, which makes the Yagi widely used for fixed-frequency VHF, UHF, and microwave applications such as television and amateur radio. In a basic three-element configuration, the driven element is about λ/2 long and centre-fed; the reflector is roughly 5% longer than the driven element to push radiation forward; and the directors are progressively shorter to sharpen and focus the main beam, with each added director further narrowing the beamwidth and increasing gain by roughly 1 dB until practical size limits are reached. Historically, this antenna was introduced in 1926 by Yagi and Uda in Japan and has since become a standard high-gain, linearly polarised array whose inherently narrow bandwidth can be extended by techniques such as trapped driven elements or combining with log-periodic structures.

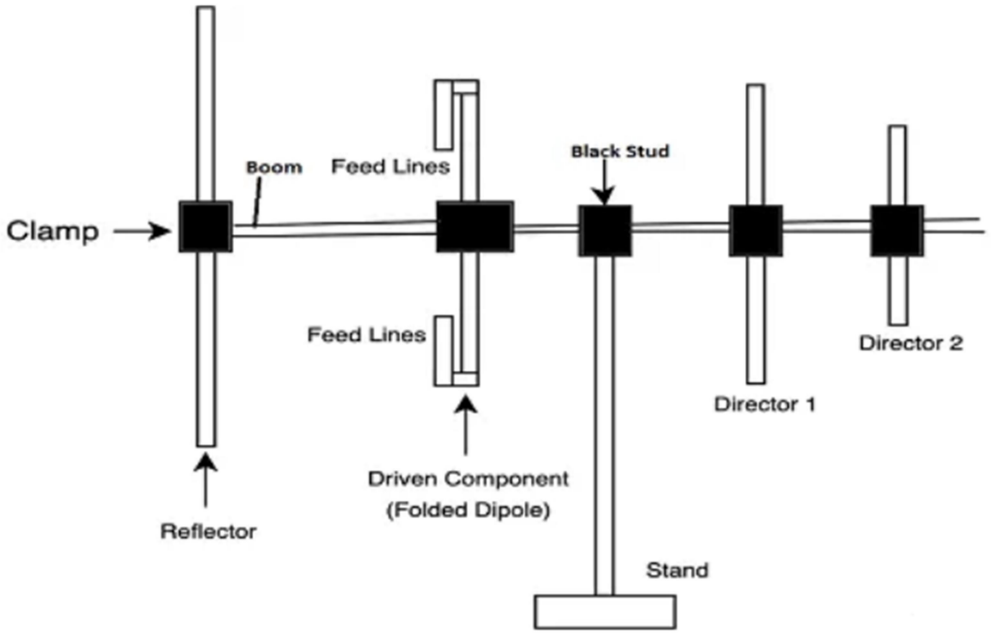


Figure 5.4: Yagi-Uda antenna structure

The Yagi-Uda antenna's radiation pattern features a narrow main beam with typical beamwidths of 30-50° and uniform gain across the beam, achieving 5-15 dBi total gain depending on the number of elements. Side lobes appear small near the main beam and sit 20-30 dB down from the main lobe, while the back lobe remains highly suppressed by the reflector, yielding front-to-back ratios of 15-25 dB. Table 5.4 illustrates how adding directors to a Yagi-Uda antenna increases gain and narrows beamwidth.

Table 5.4: Gain vs. Number of Directors

Directors	Total Elements	Gain (dBi)	Beamwidth
0	1	2.15	78°
1	3	5-6	50°
3	5	7-8	40°
5	7	8-9	30°
10	12	10-11	25°

5.5.1 Applications of Yagi Antenna

VHF Television Reception (Original Application)

- Frequency: 54-216 MHz (6 channels each)
- Gain: 8-12 dBi per Yagi
- Stacked arrays for additional gain
- Rotor for beam steering

Microwave Communications

- Point-to-point wireless links
- Frequency: 2-24 GHz

- Gain: 12-20 dBi
- Small physical size

Radar Systems

- Electronically scanned arrays of Yagi elements
- Military and civilian applications
- Phased beam steering

Amateur Radio

- VHF/UHF repeater antennas
- 50-1300 MHz bands
- Stacked and phased arrays

Satellite Communications

- Low-Earth-orbit satellite tracking
- Frequency: L-band to Ku-band
- Manual or motorized beam steering

5.6 Quarter-wave vertical monopole (omnidirectional vhf)

A quarter-wavelength monopole antenna is a vertical conductor of length L=λ/4 mounted perpendicular to an electrically large ground plane, operating on the principle that the ground plane acts as a perfect mirror to create the image of another λ/4 below it, forming an equivalent λ/2 dipole radiator. This results in a radiation pattern confined to the upper hemisphere omnidirectional in the horizontal plane with maximum gain toward the radio horizon and low-angle takeoff identical to that of a dipole above ground, providing 0 dBi azimuthal gain, linear vertical polarisation, ~36 Ω feed impedance (matchable via base loading or 42° radials), and halved radiation resistance from dipole values due to ground-plane effects, making it ideal for VHF omnidirectional applications like mobile/base stations, FM broadcast, and amateur repeaters.

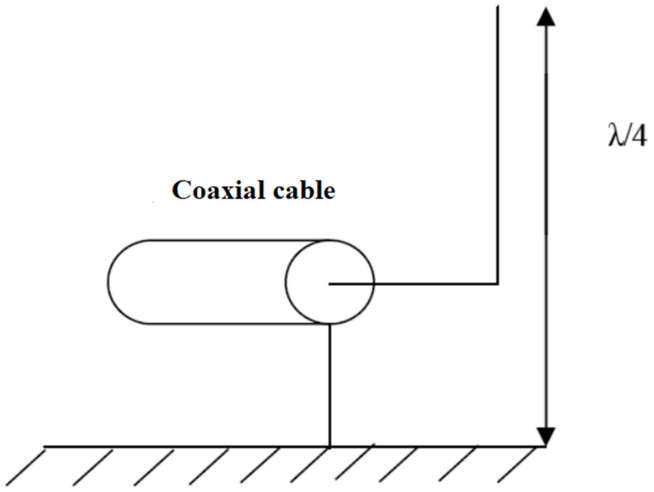


Figure 5.5: Quarter-wave vertical monopole

5.6.1 VHF Omnidirectional Applications

Table 5.5 details quarter-wave monopole antennas in VHF systems, specifying frequency, height, gain, and primary applications.

Table 5.5: VHF Communications Systems (Primary Use)

Application	Frequency	Monopole Height	Gain	Typical Use
Marine VHF	156-162 MHz	1-2 m	2-3 dBi	Boats
Aviation	118-136 MHz	1-3 m	2-4 dBi	Aircraft
Land mobile	136-180 MHz	2-5 m	3-5 dBi	Base stations
Amateur 2m	144-146 MHz	2-3 m	1.2-2.2 dBi	Fixed/mobile
Amateur 70cm	430-450 MHz	0.5-1 m	2-3 dBi	Hand-held

Advantages for VHF

- **Omnidirectional**: No need to aim antenna
- **Simple**: Single element, minimal cost
- **Compact**: Small wavelength allows manageable height
- **Robust**: Few moving parts, weather-resistant
- **Efficient**: >90% radiation efficiency typical

Disadvantages

- **Limited gain**: 2-3 dBi maximum (limited by frequency)
- **Requires ground plane**: Size and conductivity critical
- **Narrow bandwidth**: ~5-10% typically
- **Height sensitive**: Affects pattern at low frequencies

5.7 Compact magnetic loop antenna

A compact magnetic loop antenna, also called a tuned loop, resonant loop, or small loop, is a closed loop of wire with a circumference much smaller than the wavelength ($C \ll \lambda$), tuned to resonance using a high-Q variable capacitor and exhibiting magnetic dipole behaviour that distinguishes it from larger electric loops. These antennas primarily respond to magnetic fields for superior rejection of local electrostatic noise and RFI, featuring a single-turn conductor like copper pipe (0.5-1 m diameter); narrowband tuning (Q=200-400, bandwidth 10-50 kHz); very low radiation resistance (<<1 Ω, efficiency 1-10% improvable via multi-turns); and suitability for 80 kHz-50 MHz operation covering HF/MF amateur bands from 160 m to 10 m, with compact size for balconies, figure-8 radiation perpendicular to the plane (nulls broadside), excellent signal-to-noise despite low gain (-10 to -20 dBi), and remote tuning/baluns needed for TX up to 100W or amplified RX in noisy urban settings.



Figure 5.6: Compact magnetic loop antenna

5.7.1 Applications and Advantages

The key applications of small magnetic loop antennas are presented in Table 5.6, including their frequency range, loop size, tuning method, and typical gain.

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Table 5.6: Primary Applications

Application	Frequency	Loop Size	Tuning	Gain
HF Reception	3.5-30 MHz	1-3 m	Manual or motorized	-10 to -20 dB
Portable HF	3.5-14 MHz	0.5-1 m	Manual	-20 to -30 dB
Emergency/Field	7-10 MHz	0.5-1 m	Manual	-20 dB
RFI Investigation	3-50 MHz	Varies	Manual	-15 dB

Advantages Over Dipole

- **Space**: Compact (1-2 m diameter vs. 40-60 m wires)
- **Stealth**: Low visual impact, apartment-friendly
- **Portability**: Easily transported, deployed in field
- **Noise Rejection**: Sharp null capability
- **Ground Independence**: Works well low/on ground

Disadvantages

- **Inefficient Transmitting**: Radiation losses significant
- **Complexity**: Tuning capacitor and motor needed
- **Narrow Bandwidth**: Retune for each QSO
- **Size Trade-off**: More compact = much lower gain

5.8 Helical antenna for decimetric waves

A helical antenna consists of a conducting wire wound in a uniform helix (spiral shape), with each turn typically spaced by a pitch distance less than one wavelength, defined by key geometry parameters including helix diameter D, pitch angle α (angle between helix and perpendicular plane), number of turns N, circumference $C = \pi D$ of each turn, and total axial length $H = NS$ where spacing $S = C\tan\alpha$. For decimetric waves (0.1-1 GHz, λ=30-3 cm), it excels in axial mode with $C \approx \lambda$, $\alpha = 12°$-$14°$, 6-12 turns over a ground plane (>0.75λ diameter), and $S \approx \lambda/4$ between turns, radiating a circularly polarized end-fire beam along the axis with high gain (10-15 dBi via $\approx 15(C/\lambda)^2(NS/\lambda)$), narrow beamwidth (40-60°, HPBW $\approx 52°/\sqrt{NS/\lambda^3/C}$), and 100-200 Ω impedance, ideal for UHF satellite links, 5.8 GHz WiFi, and radar due to CP tolerance and directivity.

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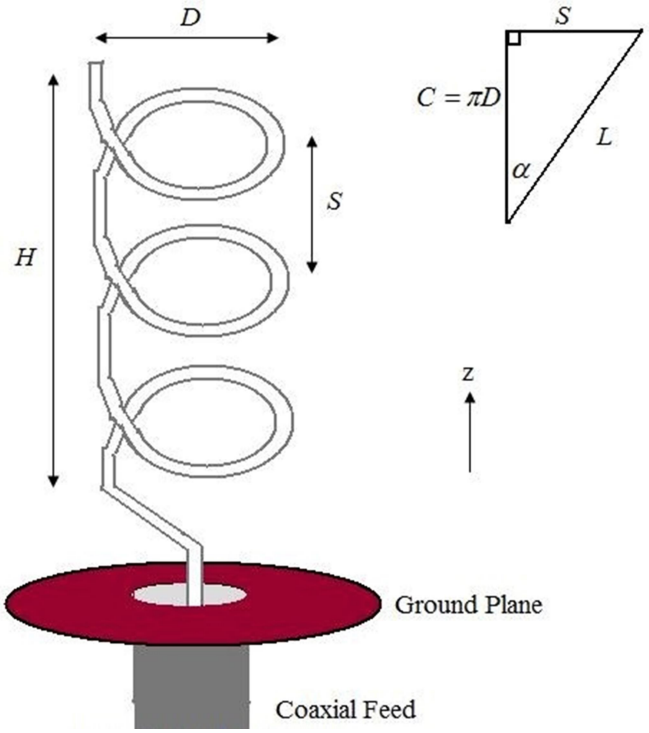


Figure 5.7: Helical antenna for decimetric waves

5.8.1 Helical Antenna Applications

Satellite Communication (Primary Application)

- Frequency: L-band (1-2 GHz) and above
- Polarization: Circular (RHCP for low-earth orbit)
- Antenna gain: 10-20 dBi (typical)
- Beamwidth: 10-30° (typical for 5-20 turns)

GPS/GNSS Receivers

- Frequency: 1.2-1.6 GHz (L-band)
- Antenna: 1-4 turns
- Compact size: ~25 mm diameter, 25-40 mm height
- Gain: 3-5 dBi
- Portable, easy integration

Deep Space Communication

- Frequency: X-band (8-12 GHz) or higher
- Antenna gain: 30-50 dBi (large number of turns)
- Beamwidth: <1° (highly directive)
- Critical for spacecraft telemetry

Satellite Downlinks

- Typical frequencies: 10-12 GHz (Ku-band)
- 10-20 turn helix antenna

- Portable receive ground station
- Motorized tracking mount

Radio Astronomy

- Frequency: 1-10 GHz typically
- Multiple helix elements in array
- Circular polarization for cosmic radiation studies
- Fixed or steerable mount

5.9 Parabolic antenna for centimetric waves

A parabolic (or paraboloid) antenna is a reflector antenna using a curved paraboloidal surface to focus electromagnetic waves, operating on the principle that its shape directs rays parallel to the axis through a single focal point where the feed horn (or dipole/Cassegrain subreflector) collects energy, producing a highly directional narrow beam with extreme gains of 30-60 dBi, beamwidths <1° (≈70λ/D), and 50-70% efficiency via optimal illumination (focal length $f=D^2/16c$ for collimated wavefronts), ideally suited for centimetric waves (1-100 GHz, practical 2-60 GHz microwave) in point-to-point links, radar, satellite comms, 5G backhaul, radio astronomy (e.g., ALMA at 100+ GHz), and deep-space probes like Voyager's 3.7m dish—though requiring precise pointing and narrow bandwidth. Key geometry includes aperture diameter D (>>λ for narrow beams), focal length f, f/D ratio (0.25 low for tight focus, 0.4 standard, 0.75 shallow), and dish depth h, with reflectors often aluminum mesh or solid for compactness.

5.9.1 Applications of Parabolic Antenna

Satellite Communications:

- Frequency: Ku-band (11-14 GHz), Ka-band (28-31 GHz)
- Dish size: 0.6-2 m (home satellite TV to teleport)
- Gain: 30-55 dBi
- Fixed or motorized mount

Microwave Links (Point-to-Point)

- Frequency: 6-38 GHz
- Dish size: 0.3-2 m
- Gain: 25-45 dBi
- Motorized mounts for mobility

Radio Astronomy

- Frequency: 400 MHz - 100 GHz
- Dish size: 10 m - 100 m diameter
- Gain: 50-70 dBi (large dishes)
- Fixed mounted on mount structure

Radar Systems

- Frequency: 3-35 GHz typically
- Dish size: 1-10 m

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- Gain: 35-55 dBi
- Rotating for scanning

Weather Radar

- Frequency: X-band (10 GHz), S-band (3 GHz)
- Dish size: 3-6 m
- Gain: 40-45 dBi
- Rapid scanning capability

Deep Space Communication

- Frequency: X-band (8 GHz), Ka-band (32 GHz)
- Dish size: 10-70 m (NASA tracking stations)
- Gain: 60-70 dBi (largest dishes)
- Precise tracking mounts

A comparison of the main characteristics of various antennas, such as frequency range, gain, polarisation, bandwidth, impedance, and typical applications, is listed in Table 5.6.

Table 5.7: Antenna Characteristics Comparison

Antenna	Frequency	Gain	Polarization	Bandwidth	Impedance	Applications
Folded Dipole	VHF	2.15 dBi	Linear	10-15%	292 Ω	TV, VHF Rx
Loop (small)	LF/MF/HF	-10 to -30 dB	Linear	<1%	0.001-0.1 Ω	HF Rx, DF
Loop (large)	HF	5-10 dBi	Linear	5-10%	10-100 Ω	Arrays, HF
Doublet	HF (multi)	1.64 dBi	Linear	Multi-octave	10-3600 Ω	Multiband HF
Yagi-Uda	VHF-UHF	5-20 dBi	Linear	5-20%	15-75 Ω	TV, Sat Rx
Monopole λ/4	VHF-UHF	1.5-2.2 dBi	Linear	5-10%	36.5 Ω	Mobile, VHF
Mag. Loop	HF	-20 dB	Linear	0.1-1%	0.1-1 Ω	Portable Rx
Helix (Axial)	UHF-SHF	10-30 dBi	Circular	20-50%	140-150 Ω	GPS, Satellite
Parabolic	Microwave	25-70 dBi	Linear	Wideband	50-150 Ω	Sat, Microwave

5.10 Conclusion

The last chapter synthesises diverse antenna types, from folded and polygonal loop antennas (square, triangle, and rhombic) in vertical or horizontal orientations to HF wire dipoles, high-gain directional Yagi-Uda arrays, omnidirectional VHF/EHF quarter-wave monopoles, compact magnetic loops, circularly polarised helical antennas for decimetre waves, and high-directivity parabolic reflectors for microwaves—each tailored to specific frequency bands and applications. This survey demonstrates how geometry, polarisation, and configuration optimise radiation patterns, gains, and bandwidth for practical RF systems. Collectively, these designs extend the theoretical foundations of prior chapters into versatile, real-world implementations for communication, broadcasting, and sensing.

5.11 Series of tutorial exercises

5.11.1 Exercise N°1 : Folded Dipole for VHF

Design a 2-element folded dipole for 146 MHz (amateur 2 m band).

5.11.2 Exercise N°2: Small Magnetic Loop for 7 MHz

Design a portable magnetic loop for 7 MHz (amateur 40 m band).

5.11.3 Exercise N°3: Yagi-Uda for 1.2 GHz

Design a 5-element Yagi for 1.2 GHz (satellite band).

5.12 Solutions of series of tutorial exercises

5.12.1 Solution of exercise N°1 : Folded Dipole for VHF

- Wavelength: $\lambda = 300/146 = 2.05$ m
- Element length: $L = \lambda/2 = 1.025$ m ≈ 102 cm
- Element spacing: 5-10 cm (optimize for 50 Ω with matching)
- Impedance: ~292 Ω (four times simple dipole)
- Matching: λ/4 coaxial transformer (50 to 86.6 Ω, then to 292 Ω)

5.12.2 Solution of exercise N°2: Small Magnetic Loop for 7 MHz

- Wavelength: $\lambda = 300/7 = 42.9$ m
- Loop diameter: D = 1 m (C = 3.14 m = 0.073λ)
- Loop area: A = 0.785 m²
- Inductance: L = 1.5 µH
- Tuning capacitor: $C = 1/(2\pi \times 7 \times 10^6)^2 \times 1.5 \times 10^{-6}) = 34$ pF
- Radiation resistance: Rrad $\approx 2 \times 10^{-4}$ Ω
- Note: Excellent receiving antenna, poor transmitter

5.12.3 Solution of exercise N°3: Yagi-Uda for 1.2 GHz

- Wavelength: $\lambda = 300/1200 = 0.25$ m
- Reflector: length $\approx 0.55\lambda = 137.5$ mm, spacing 37.5 mm
- Driven element: length $\approx 0.48\lambda = 120$ mm
- Directors (3): lengths 110, 105, 100 mm, spacings ~25 mm
- Boom length: ~300 mm
- Expected gain: ~8-10 dBi
- Beamwidth: ~45°

Summary of chapter 5

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Chapter 5 surveys practical antenna types: folded dipole (292Ω impedance, same pattern as half-wave, TV/VHF apps); small/large loops (magnetic dipole, HF Rx/DF, low R_rad); wire doublet (multi-band HF, tuner needed); Yagi-Uda (parasitic directors/reflector, 5-20 dBi VHF/UHF gain, TV/sat); quarter-wave monopole (omnidirectional VHF, 2 dBi, mobile/base); magnetic loop (compact HF Rx, Q=200-400, -20 dBi); helical (axial mode CP, 10-30 dBi UHF sat/GPS); parabolic (30-70 dBi microwave, f/D=0.25-0.75, sat/radar). Table 5.6 compares freq/gain/pol/bandwidth/impedance/apps accurately. Retain selection for gain vs. size/cost. Now that you have acquired antenna types/applications, you can move on to array synthesis/optimization.

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General Conclusion

This course handout brings together the fundamental concepts needed to understand, model, and design both transmission lines and antennas within a single coherent framework. It begins with wave propagation on lines, reflection phenomena, standing waves, and Smith chartbased matching and then extends these tools to real transmission media such as coaxial, parallel-wire, twisted pair, microstrip, and other guided structures, including the calculation of their primary parameters and practical selection criteria. Building on this foundation, the notes develop the essential radiation and electrical characteristics of antennas, analyse elementary radiators such as dipoles, and finally survey a wide range of practical antenna types folded, loop, wire, Yagi-Uda, monopole, magnetic loop, helical, and parabolic highlighting how geometry, frequency band, and polarisation are chosen to meet given system requirements.

Overall, the course provides a complete path from guided waves on lines to radiated waves in free space, giving the student not only the theoretical bases (field equations, equivalent circuits, propagation constants, power balances) but also design-oriented tools (matching techniques, typical dimensions, gains, bandwidths and application tables) needed for modern RF, microwave, and wireless communication engineering.

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