Ministry of Higher Education and Scientific Research
Salhi Ahmed University Center of Naama
Institute of Economics, Managemet and Commercial Sciences

"microeconomics 1"

Courses & Exercises

for students 1st year LMD

(economics, management and commercial sciences)

2024-2025

Course presentation:

Microeconomics is a branch of economics that studies the behavior of individual economic agents, such as consumers and firms, the choices they make, and the interactions between them. It focuses on specific markets and industries, and seeks to understand how these markets allocate scarce resources.

This course entitled " Microeconomics" allows for understanding the theoretical aspects of the behavior of partial units, using mathematical analysis tools, which are capable of simplifying the analysis and simplifying the theory of the studied reality.

This course is based on eight units of applications, each unit of applications is the same as on the road Pedagogical sequences allow for assimilation of previous concepts and was supported by a series of exercises that allow for a greater understanding of the concepts.

Content:

First course

Chapter one: Introduction to microeconomics

Microeconomics corresponds to the study of the behavior of agents (consumers, firms) from a neoclassical perspective, assuming that these agents behave rationally and that this behavior can be described as the maximization of an objective function (utility for consumers, profit for firms) subject to a resource constraint**.**

Here are the key questions that microeconomics seeks to answer:

1. How are resources allocated in an economy?
2. What are the objectives of economic agents, and how can their choices be explained?
3. What market structures best serve the interests of both consumers and producers?

Why do we study microeconomics?

1-Definition of Economics: Is a social science that studies of how society choose to use scarce limited resource to satisfy unlimited human wants.

2-Economic problems are the challenges that societies face in allocating scarce **resources** to meet unlimited wants and needs. These problems arise because resources are **limited**, but human **wants** and needs are **unlimited.**

Resources are the means or materials that can be used to satisfy human needs and wants. They can be classified into two main types:

Natural resources: Natural resources are materials that occur naturally in the environment, such as land, water, air, minerals, and forests.

Human resources: Human resources are the skills, knowledge, and abilities of people.

Wants are the desires that people have for goods and services. Wants are unlimited and vary from person to person.

The difference between resources and wants is that resources are the means to satisfy wants, while wants are the desires themselves. For example, food and water are resources that can be used to satisfy the want of hunger. A car is a resource that can be used to satisfy the want of transportation.

Another way to think about the difference between resources and wants is that resources are limited, but wants are unlimited. This means that we cannot have everything that we want. We must make choices about how to use our resources to satisfy our most important wants.

The scarcity of resources and the unlimited nature of wants is the fundamental economic problem. Economists study how societies make choices about how to allocate scarce resources to satisfy unlimited wants and needs.

Here are some examples of resources and wants:

Resources: Food ,Water, Air, Minerals, Forests, Land, Labor, Capital, Technology

Wants: Hunger ,Thirst, Shelter, Clothing, Transportation, Education, Healthcare Recreation, Entertainment

3- Type of economics analysis is divided into two main branches: microeconomics and macroeconomics.

When we study **microeconomics**, it is primarily individual human beings and individual firms, agents, that we study. This is in contrast to **macroeconomics**, where one studies whole economies, and questions such as unemployment and inflation.

- Microeconomics deals with the behavior of individual economic units. These units include consumers, workers, investors, owners of land, business firms—in fact, any individual or entity that plays a role in the functioning of our economy.
- Microeconomics explains how and why these units make economic decisions. For example, it explains how consumers make purchasing decisions and how their choices are affected by changing prices and incomes. It also explains how firms decide how many workers to hire and how workers decide where to work and how much work to do.
- By studying the behavior and interaction of individual firms and consumers, microeconomics reveals how industries and markets operate and evolve, why they differ from one another, and how they are affected by government policies and global economic conditions

Microeconomics is a broad field that covers a wide range of topics, including:

Consumer behavior: Microeconomics studies how consumers make decisions about what to consume and how much to consume. This includes understanding the factors that influence consumer preferences, such as income, prices, and tastes.

Market structure: Microeconomics also studies the different types of market structures, such as perfect competition, monopoly, and monopolistic competition. Each type of market structure has different implications for how prices and quantities are determined.

Firm behavior: Microeconomics also studies how firms make decisions about what to produce, how much to produce, and how to produce it. This includes understanding the factors that influence firms' costs, such as the prices of inputs and technology.

Government policy: Microeconomics can also be used to analyze the effects of government policies on markets. For example, economists can use microeconomic models to predict the impact of price controls, tariffs, and subsidies.

Consumer theory

Consumer theory is based on the concept of utility, which itself is a key element in determining the value of a good or service. The study of consumer choices allows us to derive the demand functions for goods and services. It attempts to formalize something as unquantifiable as tastes or individual preferences.

There are two approaches to analyzing consumer behavior: the cardinal approach and the ordinal approach.

1. Historical Reminder: Cardinal Theory and Ordinal Theory

The starting point of microeconomic theory of consumption is the debate surrounding the determination of the value of goods.

Classical English economists of the 18th and 19th centuries (particularly Adam Smith) believed that the value of a good could be determined based on production costs (an object that requires twice as many hours of labor to manufacture as another is worth twice as much). David Ricardo went further by considering that goods derive their value from two sources: their scarcity and the amount of labor required to produce them. When goods are not reproducible in large quantities, their value would be determined by their scarcity. On the other hand, for reproducible goods, value would be linked to the amount of labor they embody (capital could be analyzed as crystallized labor)[1].

In contrast, other neoclassical economists such as William Jevons (1835-1882) presented an analysis of the value of goods based on psychological elements: value would not depend on production costs, but on utility and scarcity. Utility is the way in which the consumer endows a good with an abstract quality, that of providing pleasure through consumption or avoiding effort. Consumption is presented as a problem of individual choice.

What matters to the consumer is not the total number of units of a good that they could potentially acquire, but the value they attach to each unit consumed one by one, as the order of consumption is not indifferent (the value of a glass of water is not the same if the consumer is thirsty or has already drunk a whole bottle). Above a certain quantity consumed, the good would be less essential. In general, additional quantities of the good would have varying degrees of utility. The principle of diminishing marginal utility constitutes the basic assumption of microeconomic analysis of consumption.

2. General Approach of the Two Theories: Cardinal Utility or Ordinal Utility

The founding fathers of marginalist theory (Jevons, Menger, Walras) reasoned as if utility was measurable (cardinal utility assumption). In fact, it is difficult for a person to assess in a numerical way the utility they derive from consuming a particular good.

Vilfredo Pareto, followed by Slutsky and Hicks, considered that utility cannot be measured due to its subjective nature. According to them, the consumer is only able to rank the utilities provided by consuming different goods (ordinal utility assumption). For Pareto, when a person expresses their preferences through consumption choices, they show that there are combinations of goods between which they have no reason to prefer one over the other; they are indifferent to them.

Second course

Chapter 02:theory of Consumer behavior

theory of consumer behavior Description of how consumers allocate incomes

among different goods and services to maximize their wellbeing.

- Consumer behavior is best understood in three distinct steps:
  - ✓ 1. Consumer Preferences
  - ✓ 2.Budget Constraints
  - ✓ 3. Consumer Choices

▶ Rational behaviour essentially means the following:

● Individuals will take decisions to maximise their own utility or satisfaction.

● Individuals have access to all the information that they need to make a

decision at zero cost

● Decisions will be taken by individuals based on changes at the margin.

● The preferences of individuals and their attitude to risk are assumed to

be fixed

2-1:Utility:

Indeed, people obtain "utility" by getting things that give them pleasure and by avoiding things that give them pain. In the language of economics, the concept of utility refers to the numerical score representing the satisfaction that a consumer gets from a market basket. In other words, utility is a device used to simplify the ranking of market baskets.

Utility: the satisfaction a consumer receives from consuming that product

2.1.1 Marginal utility theory:

- is a theory in microeconomics that attempts to explain consumer behavior. The theory is based on the idea that consumers seek to maximize utility from consumption, and that utility can be measured quantitatively.
- **Utility is measurable.** This means that consumers can assign a numerical value to the satisfaction they derive from consuming goods and services
- **Consumers are rational.** This means that they make decisions that are in their best interests, given their knowledge and constraints
- **Marginal utility is diminishing.** This means that the additional utility that a consumer derives from consuming an additional unit of a good or service decreases as the consumer consumes more of that good or service

The relationship between Total Utility and Marginal Utility:

Definition:

1-Total Utility: is the total satisfaction a consumer gets from all the units consumed.

$$UT_X = f(\varphi_{ix}) \qquad i = 1,2,3 \ldots\ldots m$$

$$U_{TX} = f(\varphi_{11}, \varphi_{12}, \varphi_{13}, \ldots\ldots \varphi_{xm})$$

In the case of a non-continuous function, total utility is the sum of marginal utilities.

$$U_{TX} = \sum_{i=1}^{n} MU_X$$

2-Marginal utility: the change in satisfaction resulting from consuming one unit more

$$MU_X = \frac{dUT_X}{d\varphi_X} = f'(\varphi)$$

$$MU_X = \frac{\Delta UT_X}{\Delta \varphi_X} = \frac{UT_2 - UT_1}{\varphi x_2 - \varphi x_1}$$

In order to ensure the condition of diminishing marginal utility, the second derivative of total utility must be negative for X

$$\frac{\delta^2 UT_X}{\delta^2 \varphi_X} < 0 \quad \text{or} \quad \frac{dUM_X}{d\varphi_X} < 0$$

To maximize a utility function, We differentiate it U`= 0 and U``< 0

Example 01:

We have a utility function for two goods in the following form and we need to find the marginal utility of each good. :

$$TU = 3X^{\frac{2}{3}}Y^{\frac{1}{3}}$$

We have

$$MU_x = \frac{\delta TU}{\delta x} \Rightarrow MU_x = \frac{\delta(3X^{\frac{2}{3}}Y^{\frac{1}{3}})}{\delta x} \Rightarrow MU_x = 2X^{\frac{-1}{3}}Y^{\frac{1}{3}}$$

And

$$MU_y = \frac{\delta TU}{\delta y} \Rightarrow MU_y = \frac{\delta(3X^{\frac{2}{3}}Y^{\frac{1}{3}})}{\delta y} \Rightarrow MU_y = 1X^{\frac{2}{3}}Y^{\frac{-2}{3}}$$

In order to ensure the condition of diminishing marginal utility, the second derivative of total utility must be negative for X and Y.

$$\frac{\delta^2 TU}{\delta x} < 0 \qquad \frac{\delta^2 TU}{\delta y} < 0$$

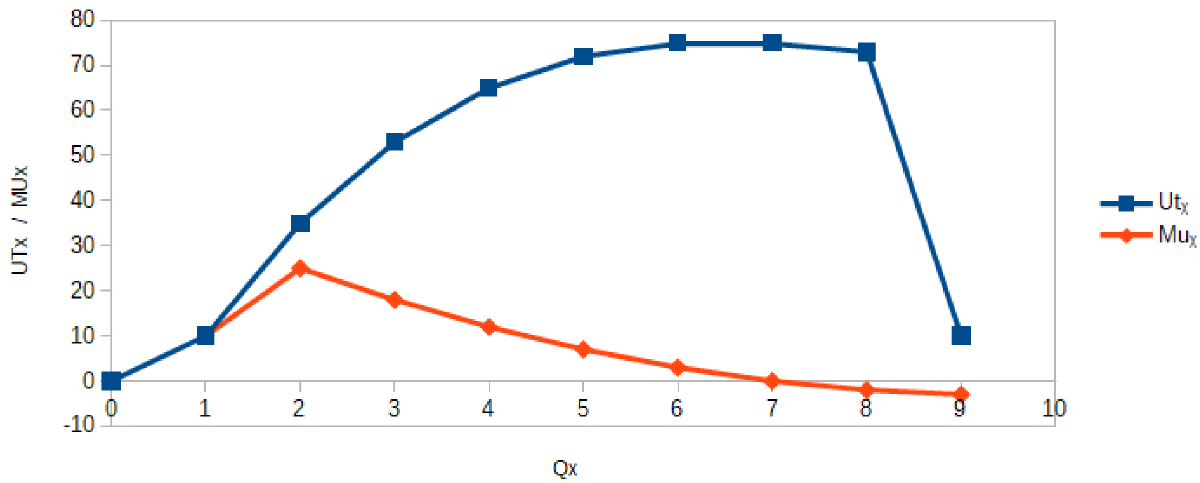
Example 2:

From the following table, we plot the marginal utility and total utility curves in different axes.

$$MU_x = \frac{\Delta TU}{\Delta x}$$

Q_x	Ut_x	Mu_x
0	0	-
1	10	10
2	35	25
3	53	18
4	65	12
5	72	7
6	75	3
7	75	0
8	73	-2
9	70	-3

Figure 01: Total and marginal utility



The consumer's total utility is the sum of the marginal utilities of each unit of goods consumed. The marginal utility is the additional utility that the consumer receives from consuming one additional unit of goods.

At the beginning, the total utility curve increases at an increasing rate until it reaches a point called the inflection point (2, 35)

2 ➡ 7 The total utility curve remains increasing but at a decreasing rate, i.e. from the inflection point until it reaches a point called the saturation point, then immediately after, 7 ⟶ 9 the total utility curve decreases.

2 ⟶ 9 In all previous parts of consumption, the law of diminishing marginal utility applies, starting from the inflection point, where the marginal utility begins to decrease from its highest value 25 , then decreases to zero, then continues to become negative.

Conclusion:

The table shows that the consumer's total utility increases with the quantity of goods consumed, up to a point of diminishing marginal utility. After the point of diminishing marginal utility, the consumer's total utility continues to increase, but at a slower rate.

The law of diminishing marginal utility states that as the quantity of a good or service consumed increases, the marginal utility of each additional unit of the good or service decreases

third course

Consumer equilibrium

Difinision: Consumer equilibrium is a state in which a consumer is maximizing their utility, given their budget constraint.

In the marginal utility theory, we always assume that the consumer is rational and can measure the utility they get from consuming goods in units called units of utility, and that the marginal utility of money is constant.

The consumer achieves equilibrium between what they will spend and what they can obtain from satisfaction.

1-Consumer equilibrium in a one-good case:

Marginal Utility sacrificed ($P_x\lambda$) = Marginal utility obtained(MU_x)

Example 01:

Determine the equilibrium position and the total utility surplus (consumer surplus) at equilibrium if the price of one unit of good x is fixed at 9 monetary units and the marginal utility of a unit of money is also fixed at 4 units of utility. The marginal utility gained from consuming the good has been estimated in the following table:

Qx	1	2	3	4	5	6	7	8	9	10
MUx	60	54	48	42	36	30	24	18	12	06

Solution:

$$MU_{\text{sacrificed}} = p_x \lambda = 9*4 = 36$$

Qx	MU_x	$MU = p_x \lambda$	TU_x	TU	$TUx - TU =$
1	60	9X4=36	60	36	24
2	54	36	114	72	42
3	48	36	162	108	54
4	42	36	204	144	60
5	36	36	240	180	60
6	30	36	270	216	54
7	24	36	294	252	42
8	18	36	312	288	24
9	12	36	324	324	0
10	06	36	330	360	-30

2:Consumer equilibrium in the case of multiple goods:

A- The method of equilibrium condition

$$\lambda = \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = \frac{MU_Z}{P_Z} \ldots\ldots\ldots$$

$$I = P_X x + P_Y y + P_Z z + \ldots\ldots$$

Example

Determine the equilibrium position of a consumer in the case of two goods x, y, their prices are 5 and 10, respectively, and he spends his entire income estimated at 95 dinars. The following table shows the marginal utilities gained from consuming the two goods:

Qxy	1	2	3	4	5	6	7	8	9	10
MUx	36	34	32	30	28	26	24	22	20	18
MUy	48	46	44	42	40	38	36	34	32	30

Solution:

Qxy	MU_x	MU_y	$\frac{MU_x}{P_x}$	$\frac{MU_y}{P_y}$
1	36	48	7,2	4,8
2	34	46	6,8	4,6
3	32	44	6,4	4,4
4	30	42	6	4,2
5	28	40	5,6	4
6	26	38	5,2	3,8
7	24	36	4,8	3,6
8	22	34	4,4	3,4
9	20	32	4	3,2
10	18	30	3,6	3

$$(x, y) = (1;7)(3;8)(5;9)(7;10)$$

$$5(9) + 10(5) = 95 \quad : (x, y) = (9;5)$$

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} = 4$$

Total utility surplus = Total utility obtained – Total Utility sacrificed

Total utility obtained ?

sum margimal utility obtained (x)= 36+34+32+30+28+26+24+22+20=252.

Sum marginal utility obtained (y)=48+46+44+42+40=220

Total utility obtained= 252+220=472

Total Utility sacrificed?

X ⟹ (9*5*4)=180

Y ⟹ (5*10*4)=200

X+y=380(Total Utility sacrificed)

Total utility surplus=472-380=92

B- Equilibrium Using Compensation Method and Lagrange Multiplier Method

The budget line equation:

The budget line equation is a mathematical equation that represents all the combinations of two goods that a consumer can afford given their income and the prices of the goods. The equation is:

"Income = Expenditure".

$$R = XPx + YPy$$

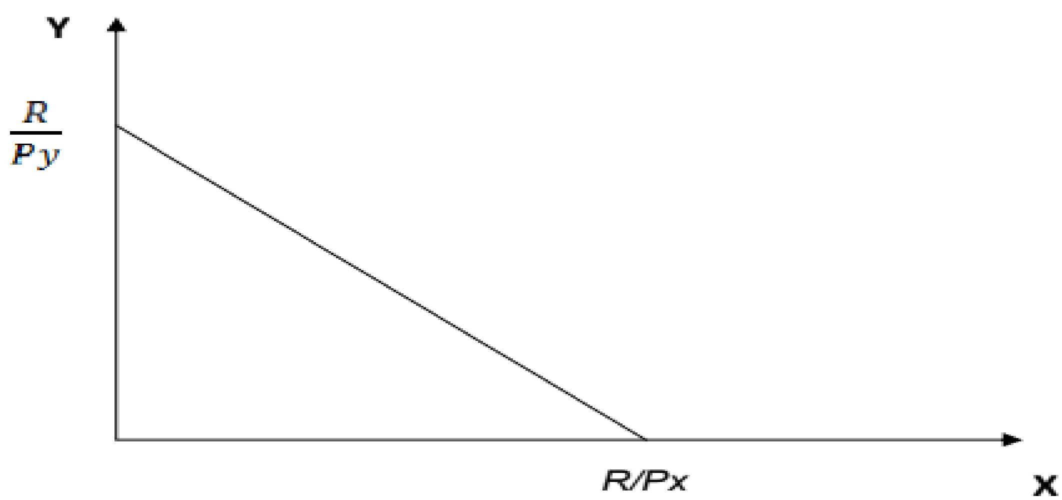
$$Y = f(X)$$

$$Y = \frac{R}{Py} - \frac{Px}{Py} X$$

It is a straight line with a negative slope :

$$-\frac{Px}{Py}$$

So, can be drawn Budget line as follows:



Maximizing Utility Function Using the Compensation Method

$$U = f(X \cdot Y)$$

$$U = f\left(X \cdot \frac{R}{Py} - \frac{Px}{Py} \quad X \quad\right)$$

A function with one variable X

To maximize a utility function, we differentiate it

$U'_X = 0$

and

U''<0

Maximizing Utility Function Using the Lagrange Multiplier Method

The goal of the consumer is to maximize utility

$$\begin{cases} \text{MAX } U = f(X, Y) \\ \text{S/C} \quad R = XPx + YPy \end{cases}$$

λ is the Lagrange multiplier

$$L = f(x, y, z) + \lambda(I - P_X x - P_Y y)$$

Necessary condition

$$L'_x = \frac{\delta L}{\delta x} = 0 \Rightarrow \frac{\delta \cdot f \cdot (x, y)}{\delta x} - P_x \lambda \Rightarrow MU_x - P_x \lambda = 0 \Rightarrow \lambda = \frac{MU_x}{P_x}$$

$$L'_y = \frac{\delta L}{\delta y} = 0 \Rightarrow \frac{\delta \cdot f \cdot (x, y)}{\delta y} - P_Y \lambda \Rightarrow MU_Y - P_Y \lambda = 0 \Rightarrow \lambda = \frac{MU_Y}{P_y}$$

$$L'_\lambda = \frac{\delta L}{\delta \lambda} = 0 \Rightarrow I - p_x x - p_y y = 0$$

We now have three equations with three variables.Therefore, it is possible to solve this system of equations by dividing the first equation by the second.

Sufficient condition: The Hessian matrix is positive H > 0

$$|H| = \begin{vmatrix} L''_{xx} & L''_{xy} & L''_{x\lambda} \\ L''_{yx} & L''_{yy} & L''_{y\lambda} \\ L''_{\lambda x} & L''_{\lambda y} & L''_{\lambda\lambda} \end{vmatrix}$$

Example:

Let's have the following utility function: **U=XY and R=100, Px=5 , Py=2**
L=XY+λ (R- XPx-YPy)

Necessary condition:

$L'_x = Y - \lambda Px = 0$ ....(1)

$L'_y = X - \lambda Py = 0$ ....(2)

$L'_\lambda = R - XPx - YPy = 0$ ......(3)

$$\frac{Y}{X} = \frac{\lambda Px}{\lambda Py} = \frac{Px}{Py}$$

$$\rightarrow Y = \frac{Px}{Py} X \text{ ... ... } (4)$$

$$R = XPx + \left(\frac{Px}{Py} X\right) Py = 2XPx$$

$$X = \frac{R}{2Px} \text{ ... ... } (5)$$

this is demand function for x

$$Y = \frac{R}{2Py}$$

This demand function for y

Y*= 25   and X*=10

$$\lambda = \frac{Y}{Px} = 5$$

Sufficient condition:

$$|H| = \begin{vmatrix} L''_{xx} & L''_{xy} & L''_{x\lambda} \\ L''_{yx} & L''_{yy} & L''_{y\lambda} \\ L''_{\lambda x} & L''_{\lambda y} & L''_{\lambda\lambda} \end{vmatrix} = \begin{vmatrix} 0 & 1 & -5 \\ 1 & 0 & -2 \\ -5 & -2 & 0 \end{vmatrix} > 0$$

Criticisms of marginal utility theory:

The difficulty of measuring utility: Utility is a subjective measure, and it is difficult to quantify. This makes it difficult to test the predictions of marginal utility theory.
The assumption of rationality: Marginal utility theory assumes that consumers are rational and make decisions that maximize their utility. However, consumers may not always be rational, and they may make decisions based on factors other than utility.
The assumption of independence: Marginal utility theory assumes that the utility of one good is independent of the utility of other goods. However, this assumption may not always be realistic. For example, the utility of a car may be higher if the consumer also owns a driver's license.

Course 04

Chapter 03: Ordinal utility theory

The indifference curve

The indifference curve is a geometrical device developed by J.R.Hicks and R.G.D. Allen for explaining how choices between two alternatives are made based on ordinal utility approach. It may be viewed as a replacement or improvement over the neo-classical cardinal utility approach or concept. In contrast to the cardinal measurement of utility, the indifference curve measures the utility ordinally. It means, unlike cardinal utility approach, based on preference orderings how the consumers are assumed to select commodities in such a way to be remained indifferent in deriving satisfaction from the consumption of any of the available combination of two goods on the same indifference curve is explained by the indifference curve analysis.

The theory of consumer behavior begins with three basic assumptions about people's preferences for one market basket versus another.

- ✓ Completeness
- ✓ Transitivity
- ✓ More is better than less

1-indifference curve

Definition

An indifference curve is a graph showing combination of two goods that give the consumer equal satisfaction and utility. Each point on an indifference curve indicates that a consumer is indifferent between the two and all points give him the same utility,An indifference curve represents all combinations of market baskets that provide a consumer with the same level of satisfaction.

<u>Example :</u>

Table 3.1 Provides Some Examples of baskets containing various of food and clothing

TABLE 3.1 ALTERNATIVE MARKET BASKETS

MARKET BASKET	UNITS OF FOOD	UNITS OF CLOTHING
A	20	30
B	10	50
D	40	20
E	30	40
G	10	20
H	10	40

Note: We will avoid the use of the letters C and F to represent market baskets, whenever market baskets might be confused with the number of units of food and clothing.

:

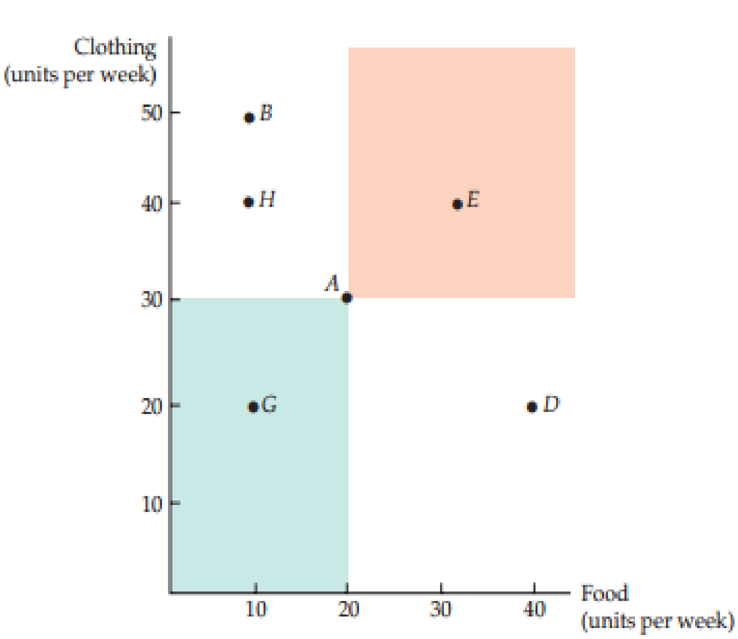
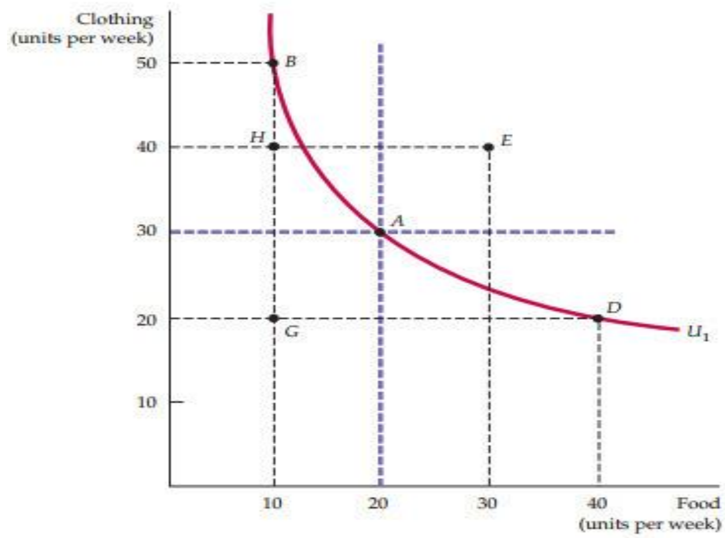


FIGURE 3.1
DESCRIBING INDIVIDUAL PREFERENCES

Because more of each good is preferred to less, we can compare market baskets in the shaded areas. Basket *A* is clearly preferred to basket *G*, while *E* is clearly preferred to *A*. However, *A* cannot be compared with *B*, *D*, or *H* without additional information.



The indifference curve U1 that passes through market basket A shows all baskets that give the consumer the same level of satisfaction as does market basket A; these include baskets B and D. Our consumer prefers basket E, which lies above U1, to A, but prefers A to H or G, which lie below U1.

Recall that indifference curves are all downward sloping. In our example of food and clothing, when the amount of food increases along an indifference curve, the amount of clothing decreases. The fact that indifference curves slope downward follows directly from our assumption that more of a good is better than less

- the properties of indifference curves:

An **indifference curve:** represents all combinations of market baskets that provide a consumer with the same level of satisfaction.

The Shape Of Indifference curve Recall that indifference curves are all downward sloping. In our Example Of food and clothing, when the amount of food increases along an indifference curve, the amount of clothing decreases. The fact that indifference curves slope downward follows directly from our assumption that more of a good is better than less

- **Indifference curves are downward sloping.** This means that as a consumer consumes more of one good, they must consume less of another good to maintain the same level of satisfaction.
- **Indifference curves are convex to the origin.** This means that the rate at which a consumer is willing to trade one good for another decreases as they consume more of the first good.
- **Indifference curves cannot intersect.** This is because if two indifference curves intersected, then they would represent the same level of satisfaction, which is not possible.

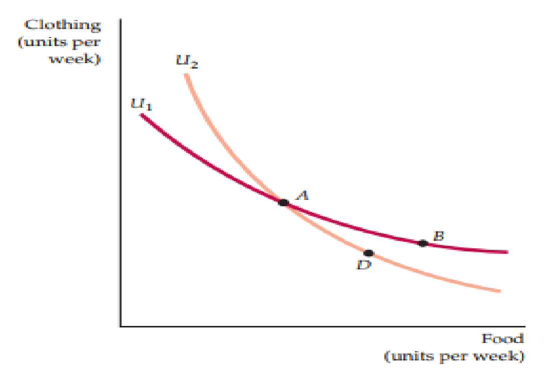


FIGURE 3.4
INDIFFERENCE CURVES CANNOT INTERSECT

Indifference map: Graph containing a set of indifference curves showing the market baskets among which a consumer is indifferent.

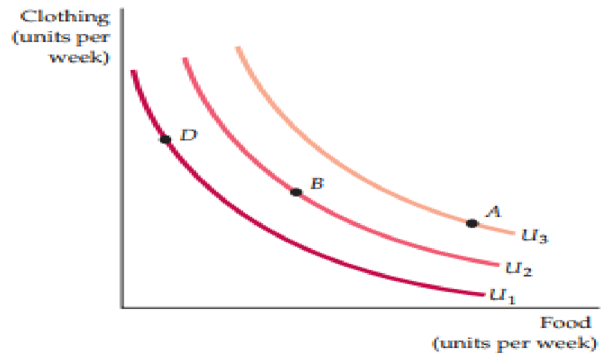


FIGURE 3.3
AN INDIFFERENCE MAP

An indifference map is a set of indifference curves that describes a person's preferences. Any market basket on indifference curve U_3, such as basket A, is preferred to any basket on curve U_2 (e.g., basket B), which in turn is preferred to any basket on U_1, such as D.

2-marginal rate of substitution (MRS):

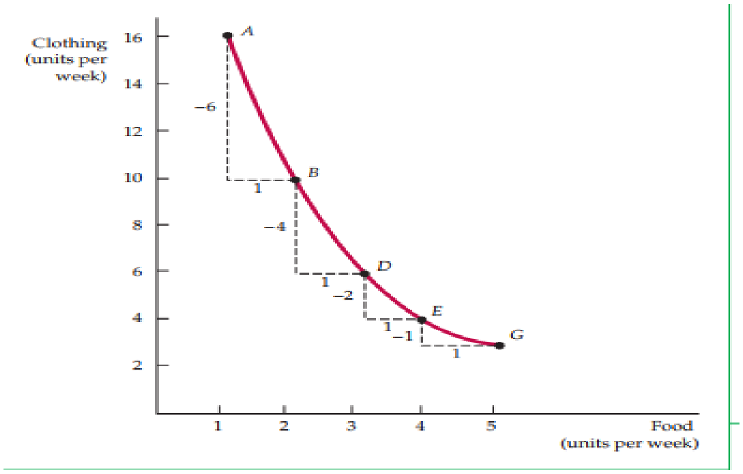
Definition

The marginal rate of substitution is the amount of a good that a consumer is willing to give up for another good, as long as the new good is equally satisfying. It's used in indifference theory to analyze consumer behaviour. The slope of the indifference curve is called the MRS which is the ratio of the marginal utilities of the two commodities.

This is expressed as $MRS_{x,y} = -\Delta Y / \Delta X = MUx/MUy$.

The Law of Diminishing Marginal Rates of Substitution states that MRS decreases as one moves down the standard convex-shaped curve, which is the indifference curve

Diminishing marginal rate of substitution



Indifference curves are usually *convex*, or bowed inward. The term *convex* means that the slope of the indifference curve *increases*(i.e., becomes less negative) as we move down along the curve. In other words, *an indifference curve is convex if the MRS diminishes along the curve.* The indifference curve in Figure is convex. As we have seen, starting with market basket *A* in Figure 3.5 and moving to basket *B*, the MRS of food *X* for clothing *Y* is - Δ*Y*/Δ*X* = -(-6)/1 = 6. However, when we start at basket *B* and move from *B* to *D*, the MRS falls to 4. If we start at basket *D* and move to *E*, the MRS is 2. Starting at *E* and moving to *G*, we get an MRS of 1. As food consumption increases, the slope of the indifference curve falls in magnitude. Thus the MRS also falls.

3- Utility Maximization: Optimal Consumer Choice (consumer's equilibrium)

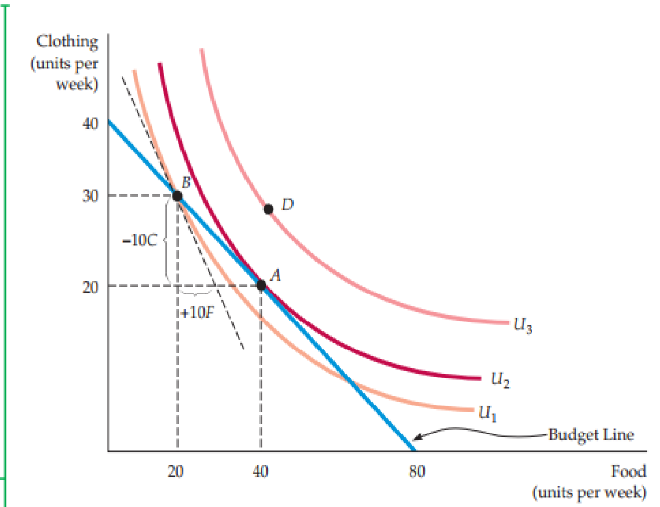
Definition

In Figure (3.3), we see the indifference curves and the budget line combined. Which of the points A – D is an optimal, utility maximizing, choice?

. At point A, an indifference curve just touches the budget line (i.e. **the budget line is a tangent to the indifference curve).**

FIGURE 3.13
MAXIMIZING CONSUMER SATISFACTION

A consumer maximizes satisfaction by choosing market basket *A*. At this point, the budget line and indifference curve U_2 are tangent, and no higher level of satisfaction (e.g., market basket *D*) can be attained. At *A*, the point of maximization, the MRS between the two goods equals the price ratio. At *B*, however, because the MRS [−(−10/10) = 1] is greater than the price ratio (1/2), satisfaction is not maximized.



the problem is solved. Here, three indifference curves describe a consumer's preferences for food and clothing. Remember that of the three curves, the outermost curve, U3, yields the greatest amount of satisfaction, curve U2 the next greatest amount, and curve U1 the least.

Note that point B on indifference curve U1 is not the most preferred choice, because a reallocation of income in which more is spent on food and less on clothing can increase the consumer's satisfaction. In particular, by moving to point A, the consumer spends the same amount of money and achieves the increased level of satisfaction associated with indifference curve U2.

In addition, note that baskets located to the right and above indifference curve U2, like the basket associated with D on indifference curve U3, achieve a higher level of satisfaction but cannot be purchased with the available income.

Therefore, **A maximizes the consumer's satisfaction**. We see from this analysis that the basket which maximizes satisfaction must lie on the highest indifference curve that **touches the budget line.** Point A is the point of tangency between indifference curve U2 and the budget line. At A, theslope of the budget line is exactly equal to the slope of the indifference curve. Because the **MRS (- ΔY/ΔX)** is the negative of the slope of the indifference curve, we can say that satisfaction is maximized (given the budget constraint) at the point where **MRS = Px/Py** This is an important result: Satisfaction is maximized when the marginal rate of substitution (of X for Y is equal to the ratio of the prices (of X to Y).

Thus the consumer can obtain maximum satisfaction by adjusting his consumption of goods x and y so that the MRS equals the price ratio.

We can relate the concept of marginal utility to the consumer's utility maximization problem in the following way. The additional consumption of food, Δx, will generate marginal utility MUx. This shift results in a total increase in utility of MUxΔx. At the same time, the reduced consumption of clothing, Δy, will lower utility per unit by MUy, resulting in a total loss of MUyΔy.
Because all points on an indifference curve generate the same level of utility,

the total gain in utility associated with the increase in x must balance the loss due to the lower consumption of y.

The relationship between MRS and Marginal utility

0 = MUx(Δx) + MUy(Δy)
Now we can rearrange this equation so that
-(Δy/Δx) = MUx/MUy
But because -(Δy/Δx) is the MRS of x for y, it follows that

MRS = MUx/MUy…………………….. (1)

Equation (1) tells us that the MRS is the ratio of the marginal utility of x to the marginal utility of y. As the consumer gives up more and more of y to obtain more of x, the marginal utility of x falls and that of y increases, so MRS decreases.

We saw earlier in this chapter that when consumers maximize their satisfaction, the MRS of x for y is equal to the ratio of the prices of the two goods:
MRS = Px/Py………………………………… (2)

Because the MRS is also equal to the ratio of the marginal utilities of consuming x and y (from equation (1)), it follows that

MUF/MUC = Px/Py
or
MUx/Px= MUy/Py…………………….. (3)
Equation (3) is an important result. It tells us that utility maximization is achieved when the budget is allocated so that the marginal utility per dollar of expenditure is the same for each good.

$$MRS = \left|\frac{\Delta Y}{\Delta X}\right| = \frac{-\Delta Y}{\Delta X}$$

$$MRS = \frac{-dy}{dx} = \frac{MU_X}{MU_Y}$$

$$U = f(x, y)$$

$$\frac{dU}{dx} = MU_X \Rightarrow dU = MU_X \cdot dx$$

$$\frac{dU}{dy} = MU_y \Rightarrow dU = MU_Y \cdot dy$$

Since the consumer is on the same indifference curve, which is constant, then the derivative of the constant is zero. Therefore, the marginal utility of all combinations is zero, meaning all combinations provide the same level of satisfaction.

$$dU = 0$$

$$MU_X \cdot dx + MU_Y \cdot dy = 0$$

$$\frac{-dy}{dx} = \frac{MU_X}{MU_Y}$$

$$MRS = \frac{-dy}{dx} = \frac{MU_X}{MU_Y} = \frac{-\Delta y}{\Delta x}$$

Determining the optimum and consumer equilibrium

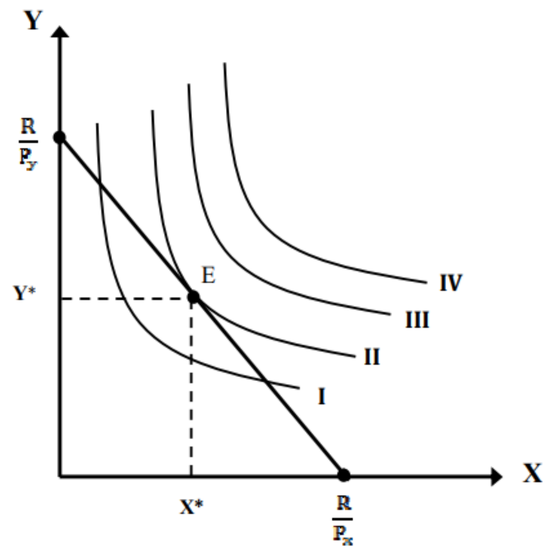
To determine the consumer's equilibrium according to this approach, two methods are used: the geometric method and the algebraic method.

Geometric Approach

The confrontation between the budget constraint line and one of the consumer's indifference curves allows us to determine the equilibrium point. On the graph below, point E represents an equilibrium point, it is the point of tangency between the budget line and the highest indifference curve.

A criterion for being exactly at the point where we maximize utility is then that

$$\mathbf{MRS} = \left|-\frac{p_1}{P_2}\right|$$



Algebraic Approach

There are two algebraic methods to determine consumer equilibrium: the Lagrange multiplier method and the substitution method.

Course 05:

Income and Price change

1-Budget constraint

The main constraint on consumer behavior is budgetary, namely the income available to the consumer, as well as the price of the goods they are supposed to purchase.

Let two goods be X and Y with market prices Px and Py respectively, and a given income R. The expenditure on these goods are represented by X.Px and Y.Py.

The consumer's budget constraint is written as:
R = X.Px + Y.Py.

We can express Y as a function of X:
Y = R/Py - Px/Py * X

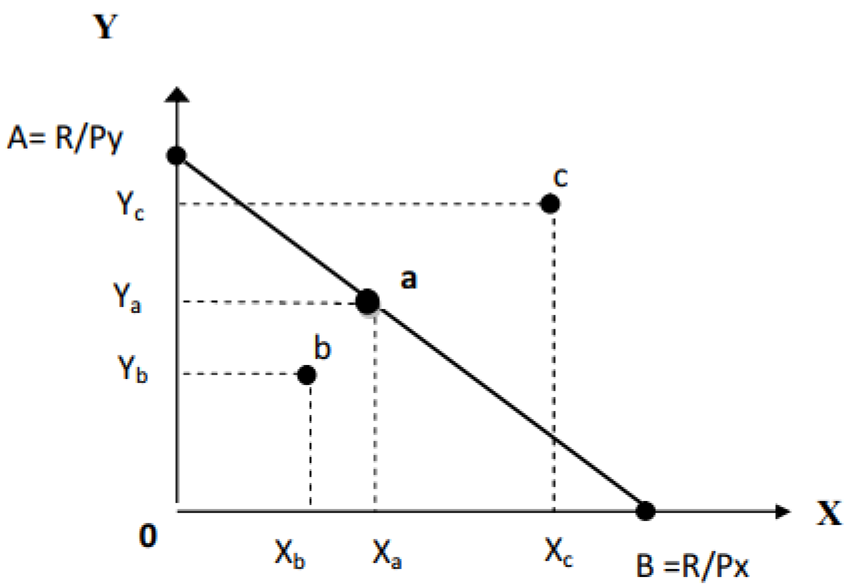
This equation reflects an important assumption: the income (R) available to the consumer is entirely spent on the consumption of X and Y.

At point A, we assume that all of R is spent on the consumption of Y.
At point B, we assume that all of R is spent on the consumption of X.

AB is called the budget line. If we take two points b and c not located on the line AB, we see that at b, the entire income R is not consumed; the combination (X_b, Y_b) is therefore not acceptable.

Similarly, at c, R is not sufficient to allow for the combination (X_c, Y_c).

The only possible combinations are therefore located on the budget line AB (for example, point (a)).



The budget set includes all the points representing baskets that are located on or below the budget line, for example, points (a) and (b) in the previous figure.

The budget line consists of points representing different baskets of goods. To acquire any of these baskets, the consumer spends their entire income: for example, point (a).

Example

A student, Pierre, has €100 of pocket money per week. He has two options for spending this money: the first is to go to the cinema, with a ticket price of €5; the second is to go to a restaurant, with a meal price of €10.

1. Graphically establish Pierre's budget constraint, knowing the different possible combinations that provide the same level of satisfaction

Cinema outings	Restaurant meals
3	**11**

5	**9**
8	**7**
12	**4**
17	**2**

2- Establish Pierre's indifference curve. Determine the optimal combination and the marginal rate of substitution of cinema/meals.

Solution:

1-If Pierre uses all his pocket money, he can go to the cinema 20 times if he doesn't want to go to a restaurant even once. Similarly, if he doesn't want to go to the cinema, he can have 10 meals.

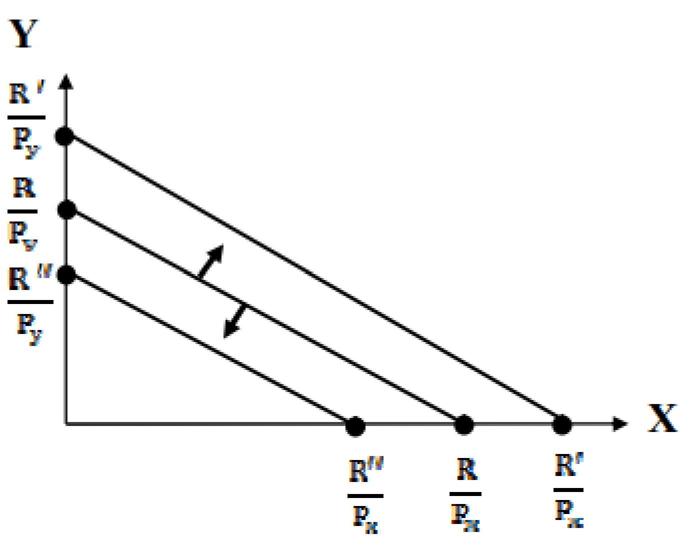
2-The optimal combination is the one where the budget line is tangent to the indifference curve, which is point E. For other combinations, the pocket money Pierre has is insufficient. At point E, the optimal combination is **4** meals [illegible]kets.

2. The effects of a change in income on the budget line

Generally, the budget line can shift parallel upwards or d[illegible] Indifference curve [illegible]nsumer's monetary income changes. It can also rotate along the X or Y axis if the prices of goods X or Y change.

In the figure below, the budget line (R/Py ; R/Px) corresponds to the consumer's initial income:

- If income increases to (R'), the budget line shifts upwards: we obtain the line (R'/Py ; R'/Px) on graph (3-2).
- If income decreases to (R"), the budget line shifts downwards: we obtain the line (R"/Py ; R"/Px) on graph (3-2).

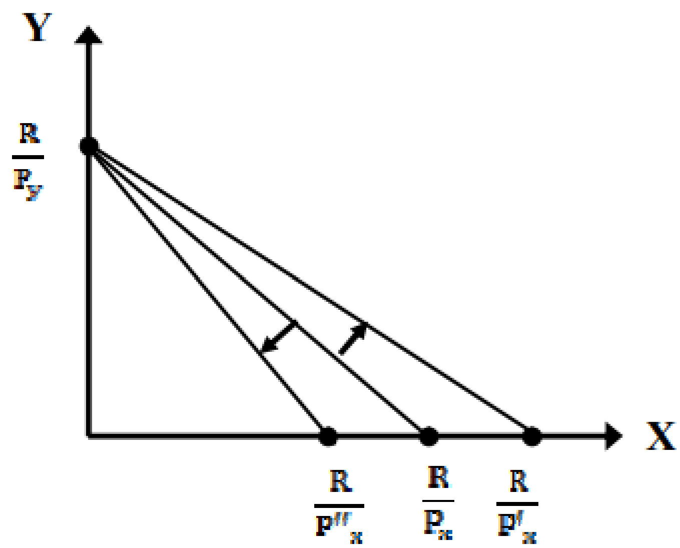


3-The effects of a change in price on the budget line

- **Case of price for change good (x):**

On graph (3-3), the budget line (R/Py ; R/Px) corresponds to the consumer's budget constraint: R = PxX + PyY.

- If the price of good X decreases to (P'x), the budget line rotates outwards to the right: we obtain the line (R/Py ; R/P'x) on graph (3-3).
- If the price of good X increases to (P''x), the budget line rotates inwards to the left: we obtain the line (R/Py ; R/P''x) on graph (3-3).

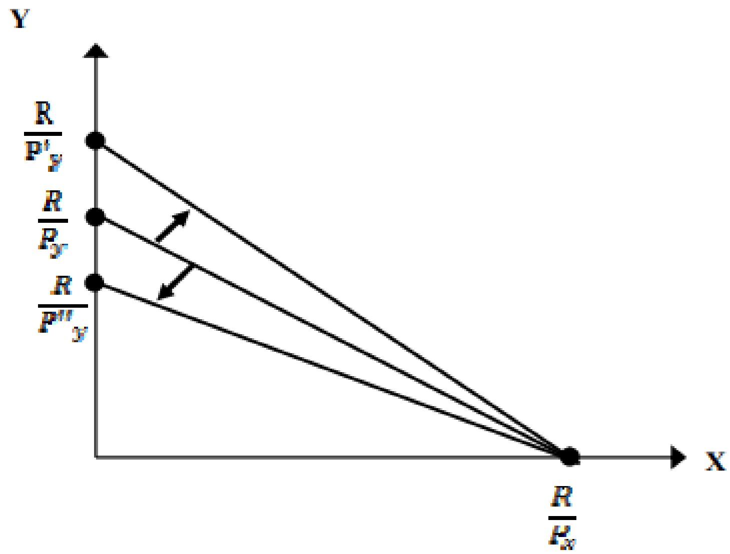


- **Case of price for change good (y):**

On graph (3-4), the budget line (R/Py ; R/Px) corresponds to the consumer's budget constraint:

R = PxX + PyY.

- If the price of good Y decreases to (P'y), the budget line rotates outwards upwards: we obtain the line (R/P'y ; R/Px) on graph (3-4).
- If the price of good Y increases to (P''y), the budget line rotates inwards downwards: we obtain the line (R/P''y ; R/Px) on graph (3-4).



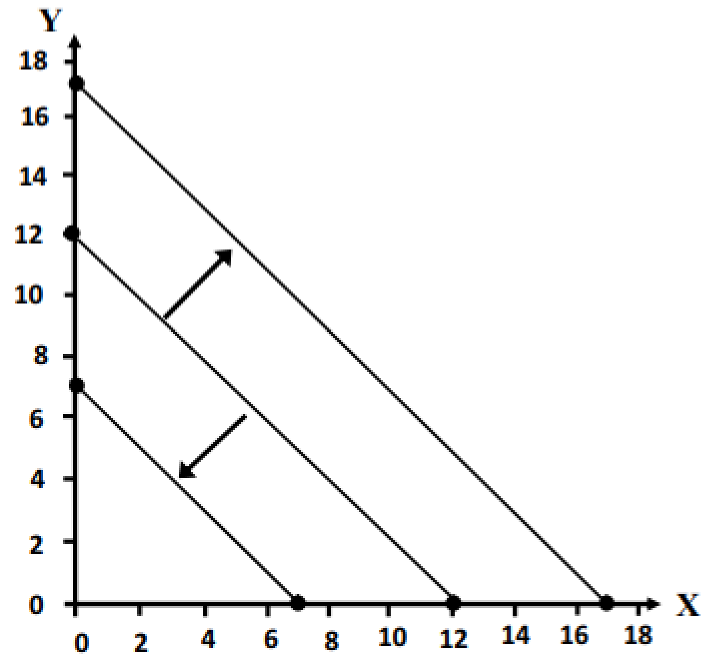
Example

Let's assume that the prices of goods X and Y are identical (Px = Py = 2DA), and the consumer's nominal income (to be spent entirely) is equal to 24DA.

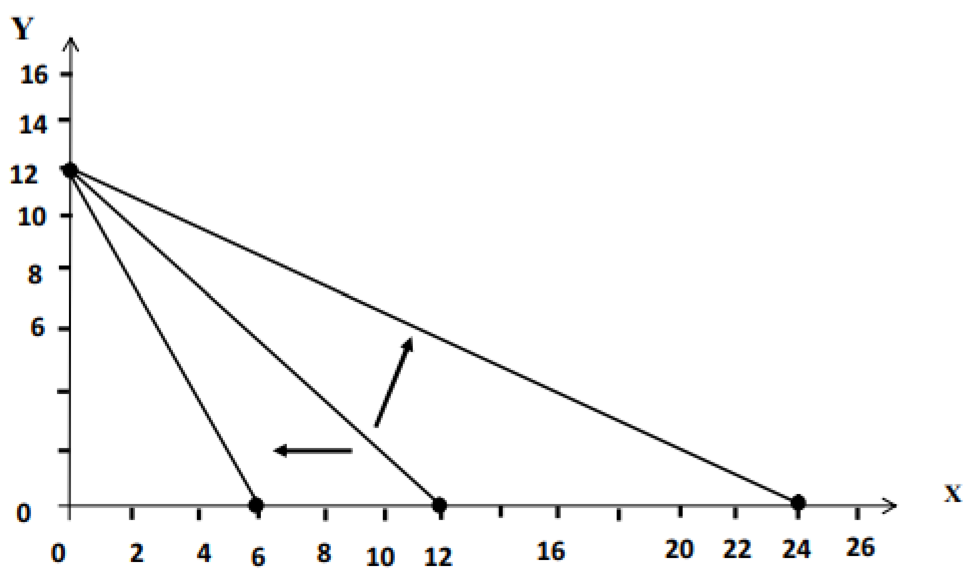
1. Draw the budget line "DCB" corresponding to this statement. Then, draw a new budget line "DCB" for the case of a 10DA income increase and another for the case of a 10DA income decrease.
2. In the cases where the price of X first changes from 2DA to 1DA, and then from 2DA to 4DA, draw the corresponding new budget lines.

Solution:

For the variation in income, the three cases (the initial position and the two other positions) are represented in the figure below:



For the variation in the price of good X, the budget lines corresponding to the three situations (the initial situation and the two other positions) are represented in the figure below:



4-Determining the optimum and consumer equilibrium

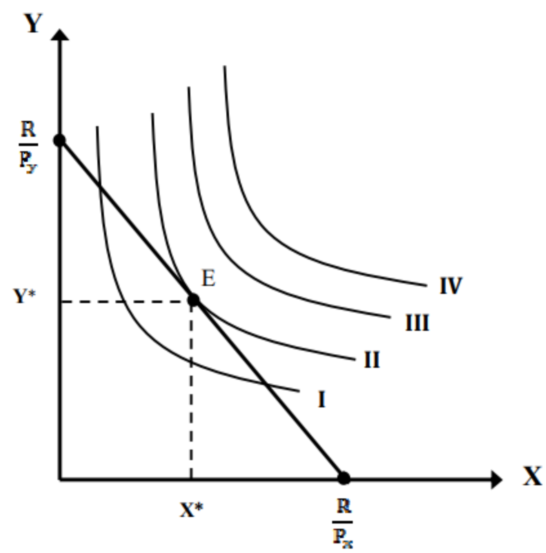
To determine the consumer's equilibrium according to this approach, two methods are used: the geometric method and the algebraic method.

Geometric Approach

The confrontation between the budget constraint line and one of the consumer's indifference curves allows us to determine the equilibrium point. On the graph below, point E represents an equilibrium point, it is the point of tangency between the budget line and the highest indifference curve.

A criterion for being exactly at the point where we maximize utility is then that

$$\textbf{MRS} = \left|-\frac{p_1}{P_2}\right|$$



Algebraic Approach

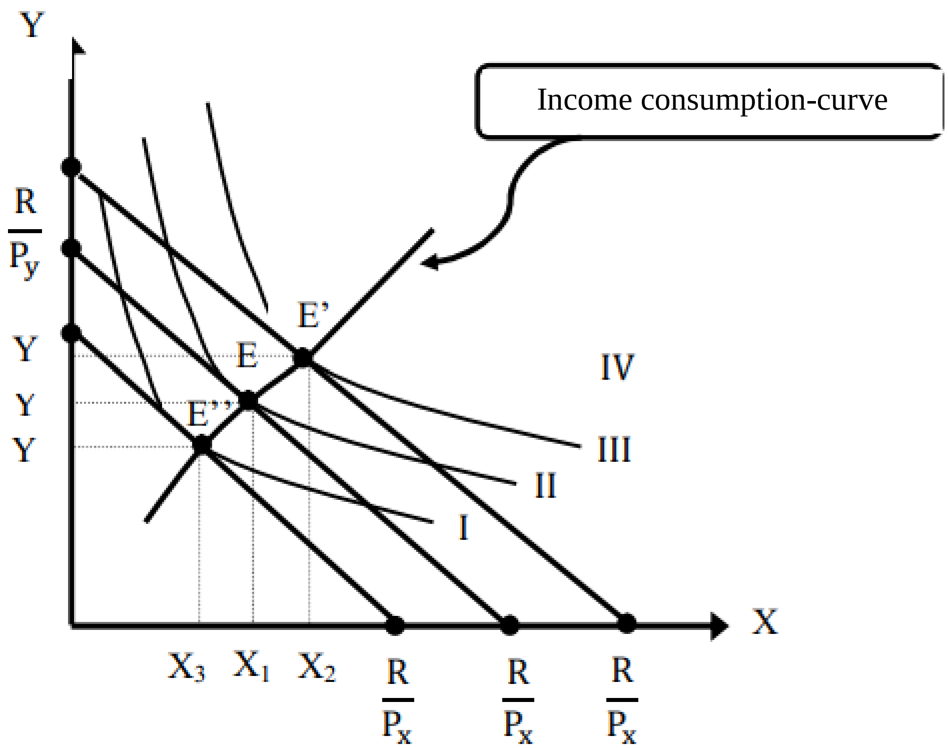
There are two algebraic methods to determine consumer equilibrium: the Lagrange multiplier method and the substitution method.

5-Shifts in Equilibrium

The equilibrium point can shift due to changes in the consumer's income or the prices of the goods being considered.

- **5.1 :change of Income**

Any change in nominal income leads to a shift in the equilibrium. For most goods (normal goods), when income increases, the budget line shifts to the right and the equilibrium point also shifts to a higher indifference curve.



Income- Consumption Curve:

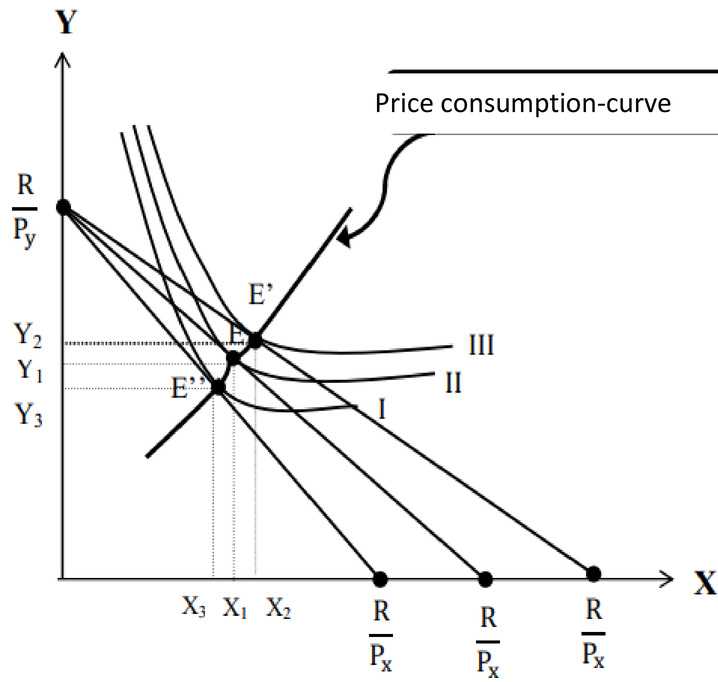
The Income-Consumption Curve represents the locus of all equilibrium points obtained from different changes in the consumer's income (assuming that the prices of goods remain constant).

5-2:Change of the price of one of the goods:

If the price of good X (or Y) increases or decreases, the ratio R/Px varies depending on the direction of the price change (increase or decrease in purchasing power) and on the economic nature of the good in question.

Price-Consumption Curve:

The Price Consumption- Curve is the geometric locus of the different equilibrium points of the consumer generated by the variation of the price of the good in question (the consumer's nominal income and the price of good Y remain unchanged).



<u>**Course 06**</u>

Income, substitution and price effects

1-The Individual Demand Curve as a function of income

1.1 The income-consumption curve: is the set of all optimal consumption points when only income varies (with the prices of goods remaining constant).

The Engel Curve

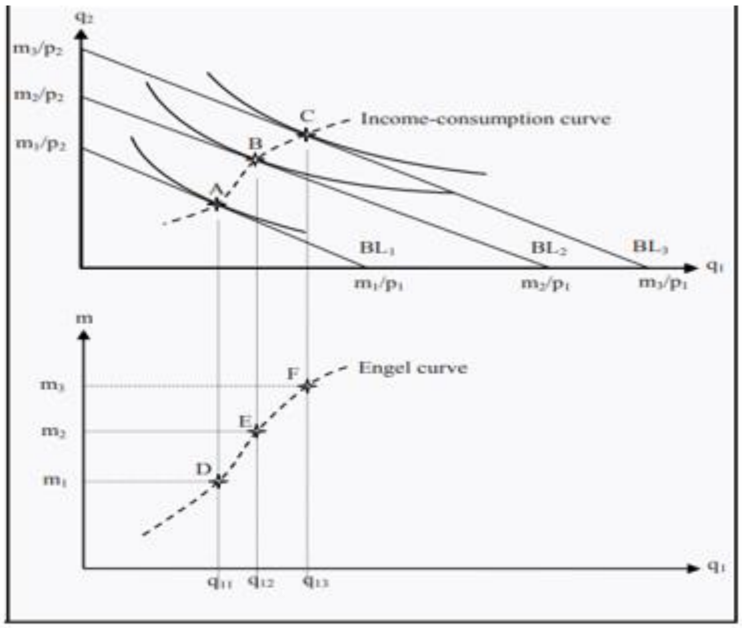
we will instead show how to derive the relation between income and the quantity demanded. The resulting curve is called the **Engel curve**.

Look at Figure., we start with the individual's maximization problem where she must choose quantities of good **1** and good **2**. However, instead of varying the price, we now vary the income **m**. This means that the budget line will shift outwards for higher incomes and inwards for lower incomes.

We assume that preferences and prices are **unchanged**. For the increasingly higher incomes **m1**, **m2**, and **m3**, the budget lines become **BL1**, **BL2**, and **BL3**.

we find **the utility maximization points** for each budget line: points **A**, **B**, and **C**. If we would do that for all possible incomes, we would get the so-called **income-consumption curve**. That curve shows the optimal consumption of good **1** and good **2** at different incomes, given preferences and prices.

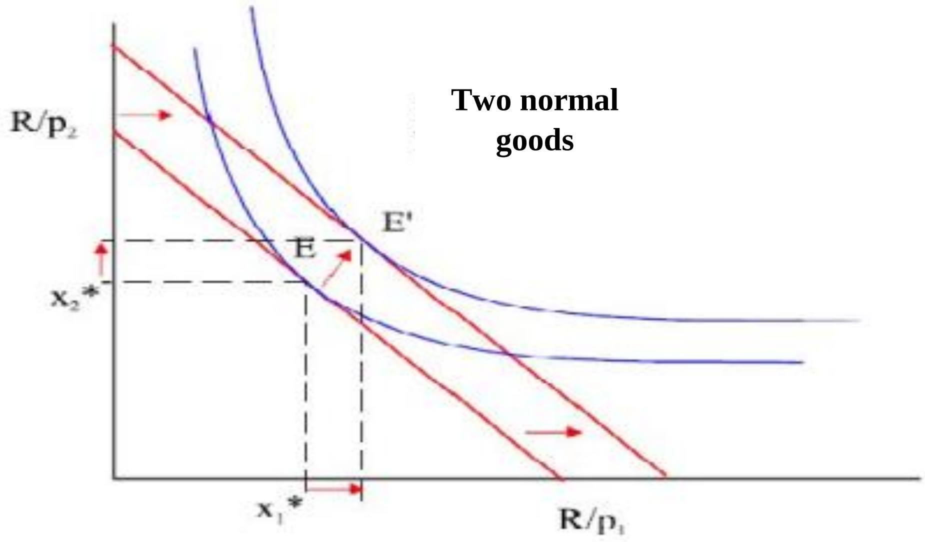
we indicate the quantities that correspond to points A, B, and C, i.e. q11, q12, and q13 in the diagram below. Then we indicate the incomes m1, m2, and m3 on the Y-axis, and the points where the incomes intersect the corresponding quantities: points D, E, and F. Thereafter, we draw a line through the points of intersection, as it would probably have looked if we had performed the same procedure for the points in between. The resulting curve is the so-called **Engel curve**, and it shows how the optimal consumption of good 1 varies with the income, given preferences and prices.



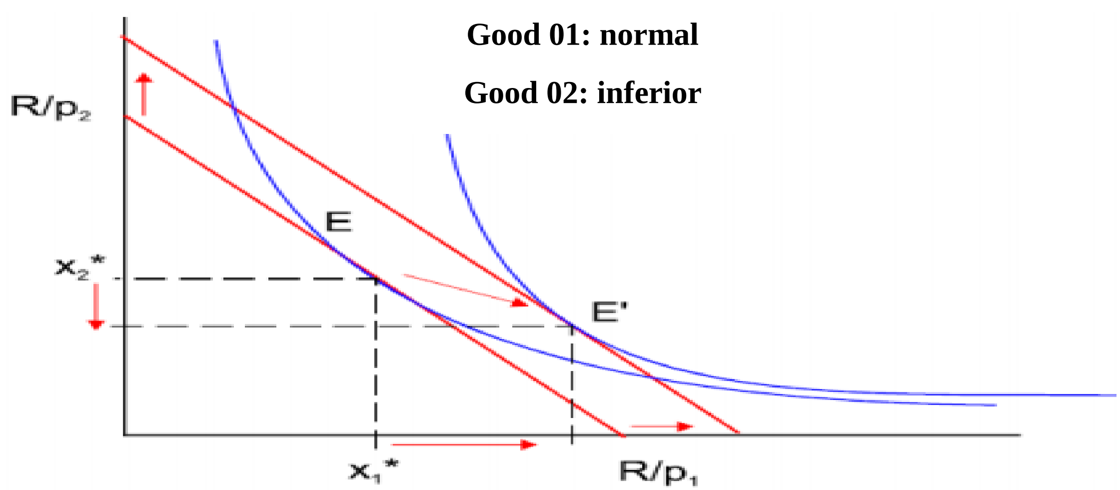
1.2 :Normal Goods and Inferior Goods

This distinction is based on how the quantity consumed reacts to changes in income, while holding the prices of goods constant.

a) **Normal Goods** If a consumer's income increases (and prices remain constant), the demand for each good increases. And if income decreases, the consumption of a normal good decreases.



b) **Inferior Goods** When income increases, the consumption of an inferior good decreases. Generally, these are goods of low quality. Consequently, these are goods whose consumption decreases as the consumer's standard of living increases: low-quality goods for which there are higher-quality substitutes that the consumer can afford if they have more income. Many food items fall into this category



2-Individual demand as a function of price

As we showed, it is possible to find the point of utility maximization if one knows a consumer's preferences, the prices of the goods, and her budget. Let us now do that, but vary the price of good 1 and see what effect that has on, q1, the quantity demanded.

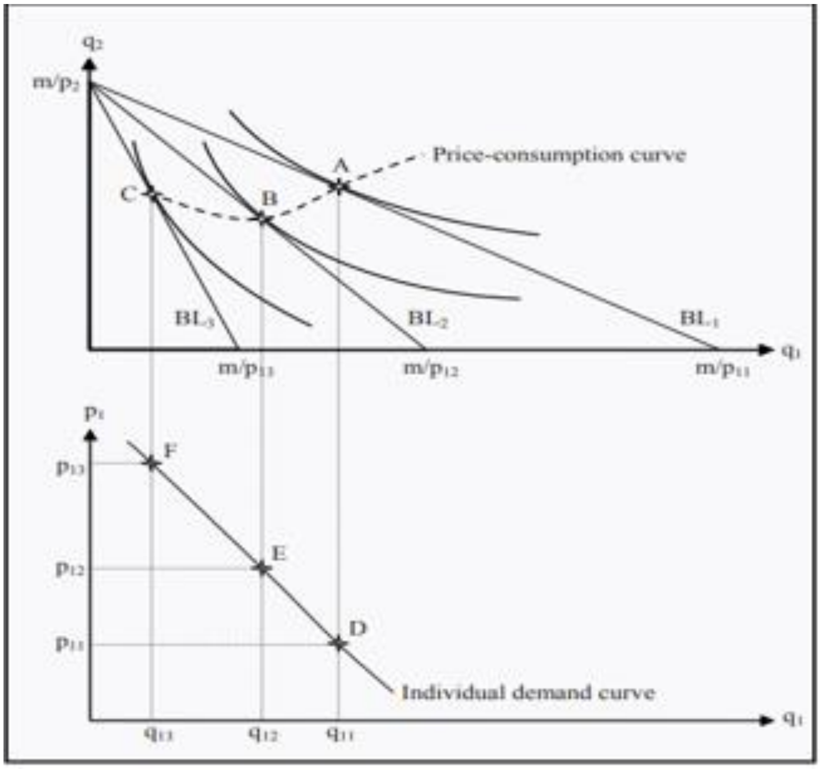
Suppose we hold the price of good **2 constant**. Then the effect of **varying** the price of good **1** will be that the budget line rotates about the intercept on the Y-axis and intersects the X-axis at different points m/p1i, where p1i is the price one has chosen for good 1.

Look at the upper part of Figure. Suppose the price of good 1 is initially p11. Then the budget line is BL1. We find the indifference curve that just touches that budget line and label the point where it does so, point A. If we would raise the price of good 1 to p12, the possible choices become limited to BL2 (that intersects the X-axis in m/p12) and then the consumer maximizes her utility in point B. If we continue to raise the price to p13, and repeat the maximization, we get point C. If we would repeat this procedure for all possible prices, we would get a curve that is called the price-consumption curve.

- ❖ When the price of a good varies, with nominal income and the prices of other goods held constant, **how does the consumer's optimal choice change**? This question can be answered in three steps: defining the price-consumption curve, defining the individual demand curve, and analyzing the income and substitution effects

2.1 the price-consumption curve It shows how the optimal choice of quantity of good 1 varies with the price of that good, given that preferences, other prices and the income are held constant.

As you can see in the figure, the consumer will usually buy less of the good when the price increases. This is, however, not necessary. To see that, imagine that the indifference curve that runs through point B had been steeper. If it had been steep enough, it would touch BL2 so far to the right that it would also be to the right of point A. Now we want to find the demand curve for good 1. To that end, we indicate the prices we used for good 1 on the Y-axis in the lower graph of the figure, i.e. p11, p12, and p13. Then we check which are the corresponding quantities demanded in the upper graph, at points A, B, and C, and indicate them on the X-axis in the lower diagram. (Note that both diagrams have q1 on the X-axis. After that, we find the points where the quantities and the corresponding prices in the lower diagram intersect, the points labeled D, E, and F. Finally, we draw a line through those points and fill in for all those numerous points for which we have not done the analysis. This curve is the individual's demand curve for good 1.



3-Income and Substitution Effects:

when we derived the individual demand curve, we saw how the quantity demanded changed when the price changed. We will now use consumer theory to perform slightly more complicated analysis of a price change. Suppose that we have a consumer, with a certain income, who has to choose between different quantities of good 1 and good 2 in such a way that she maximize her utility. If the price of good 1 falls, we get two different effects: x Since the price of good 1 falls, that good becomes cheaper relative the other good. This means that the marginal rate of transformation (MRT; the slope of the budget line) changes. Say that the prices of both goods initially are 1. The relative price is then 1/1 = 1. If the price of good 1 falls to 0.50, the relative price becomes 0.50/1 = 0.50. The consumer can now exchange one unit of good 2 for two units of good 1, and therefore good 1 becomes more attractive to her. As a result, she consumes more of the good. This effect is called the substitution effect.

The purchasing power of the consumer becomes larger because of the drop in the price. She can now buy as much as she did before the price changed, and still have money left. That extra money she can spend on both good 1 and on good 2. This is called the income effect.

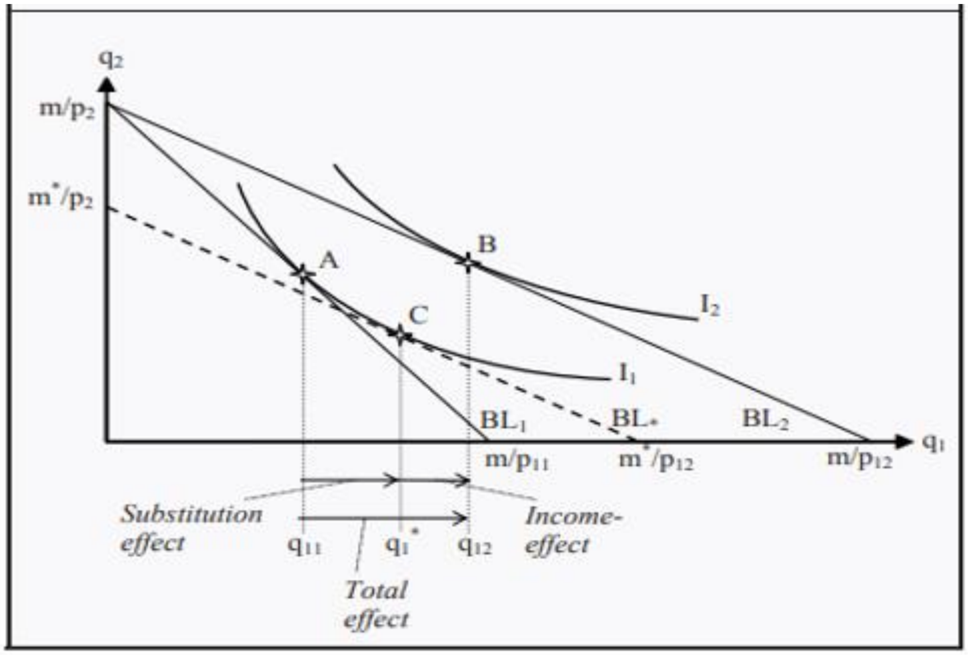
Analyzed by the British economist **R.H. Hicks**, they allow for a more refined classification of normal goods and inferior goods.

3.1 Normal Good:

Assume we have the same case as we did earlier: A consumer chooses between good 1 and good 2. Giver her income, m, the prices of the goods, p11 and p2, and her preferences, she chooses that basket of goods that maximizes her utility. In Figure, this means that she initially chooses point A.

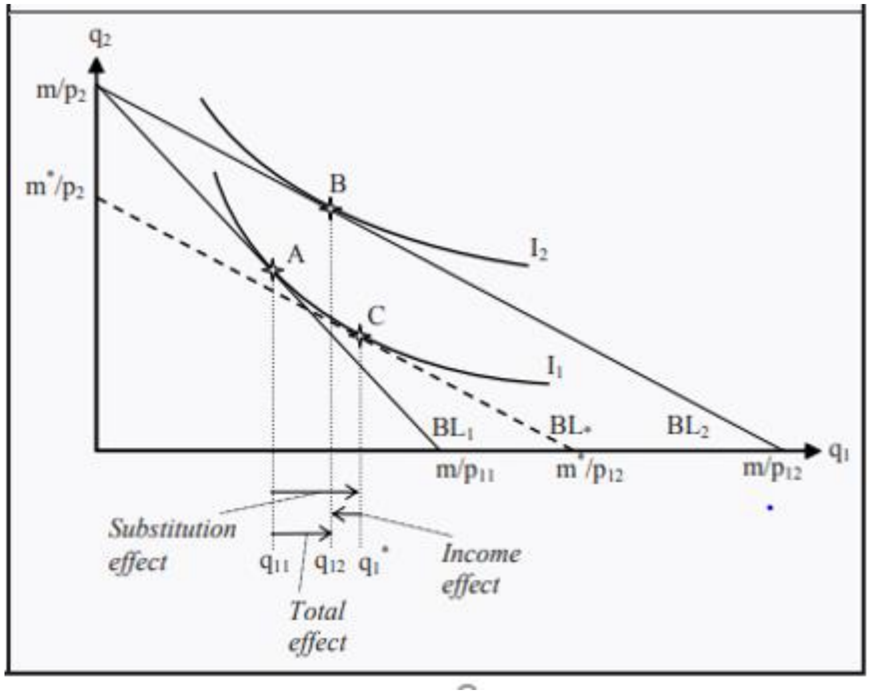
If the price of good 1 falls from p11 to p12, the budget line rotates outwards from BL1 to BL2. When the consumer chooses a new basket, she ends up in point B. Her consumption of good 1 has consequently increased from q11 to q12, which is **the total effect**. We now ask ourselves how much of the change in quantity from q11 to q12 that depends on the income effect (i.e. on the increase in purchasing power) and how much that depends on the substitution effect (i.e. on the change in the slope of the budget line).

If the relative prices change, the slope of the budget line changes. All budget lines that have the same relative prices as BL2 must also have the same slopes as that budget line. Furthermore, for the consumer to have the same utility as before, she must consume on the same indifference curve as she did before, i.e. on I1. We therefore construct an imaginary budget line, BL*, that has the same slope as BL2 and that, just as BL1, is a tangent to I1. (However, since it has a different slope than BL1, it must touch I1 at different point than that budget line does.) If this had been the real situation, the consumer would have chosen point C. She had then increased her consumption of good 1 from q11 to q1* . At the same time, she would have decreased her consumption of good 2. This substitution from good 1 to good 2 depends on the change in the relative price, but it does not result in any change in the level of utility. This part is **the substitution effect**. Th remaining change, from q1* to q12, is the part that depends on the increase in the consumer's purchasing power. As she moves to a higher indifference curve, from I1 to I2, she increases her utility. This part is **the income effect**.



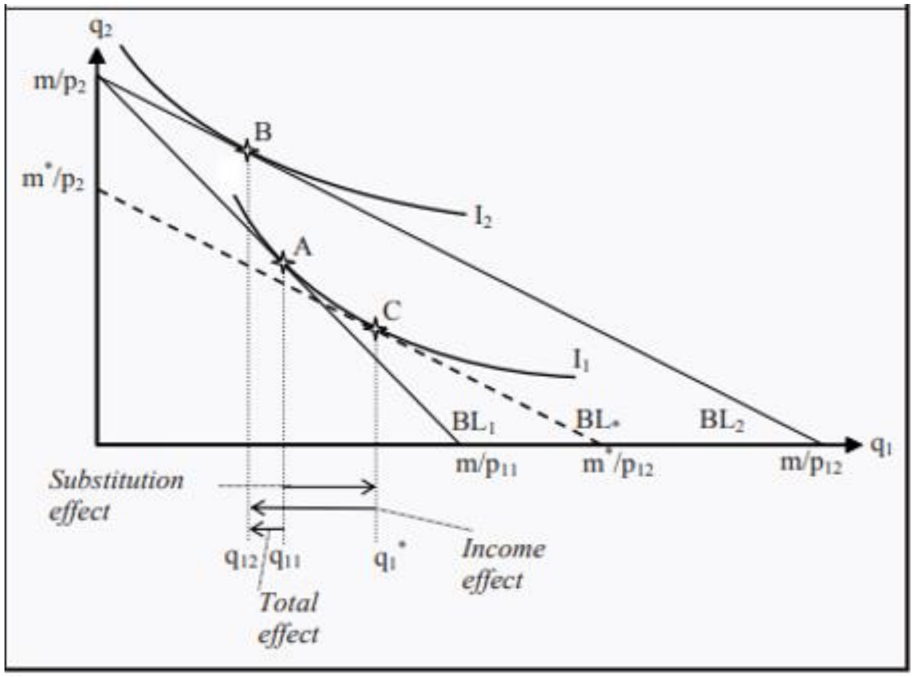
3.2 Inferior Good

As previously mentioned, an inferior good is a good one buys less of if one's income increases. The underlying reason for that is to be found in the preferences. As one becomes wealthier, one can afford to buy something of higher quality instead. This preference will have an effect on the shape of the indifference curves. This time, when we split up the total effect into a substitution effect and an income effect, the income effect for the inferior good is **negative**. The substitution effect is always **positive**, which means that we get two cases depending on whether the negative income effect is smaller or larger in magnitude than the always-positive substitution effect. Goods that belong to the latter case are called **Giffen goods**, and these are a very rare kind of goods. Their distinguishing feature is that one buys more of them if the price rises. In Section 2.1.1, we said that the demand curve almost always slopes downwards. Giffen goods are consequently an exception from that rule.



In Figure, we have almost the same situation as in previous figure. The difference is that the consumer's indifference curve I2 has been changed so that it touches the budget line BL2 at a point between points A and C. This change makes the income effect **negative** and the total effect is smaller than before.

In Figure, the indifference curve I2 has been changed again, so that it touches BL2 at a point to the left of point A. The income effect now becomes very **negative**, so negative that it dominates over the substitution effect. The total effect thereby also becomes negative and we have a **Giffen good**. Note, however, that the consumer does increase her utility. This can seem strange, as the total effect is that she consumes less of the good analyzed (and we have assumed that more is always better). The drop in the price of the Giffen good means that the consumer can afford to buy more of other goods. Furthermore, these other goods function as substitutes for the Giffen good. Hence, the increase in utility. The increase in consumption of good 2 can be read off as the distance between A and B on the Y-axis.



Example:

Consider a consumer with an income of 20 DA who buys two normal goods X and Y on the market at the same price PX = Py = 2 DA. The following table gives the coordinates of the points that allow us to draw the consumer's two indifference curves:

IC(1)	X	3	4	5	6	7	8
	Y	10	07	5	4.2	3.5	3.2
IC(2)	X	5	6	7	8	9	10
	Y	12	6	7	6.2	5.5	5.2

1- Graphically determine the consumer's equilibrium.
2- If the price of good X falls to 1 DA, what would be the total effect of this decrease on the consumer's equilibrium?
3- Decompose this total effect into substitution and income effects.

Solution:

1. Initial Equilibrium

- The consumer's initial equilibrium is at point A, where the budget constraint is **tangent** to indifference curve IC(1).
- The optimal quantities consumed are X = 5 units and Y = 5 units. (X* ; Y*) = (5 ; 5)

2. Price Decrease and New Equilibrium

- When the price of good X decreases, the budget constraint shifts outward, allowing the consumer to afford more of both goods.
- The new equilibrium is at point C, where the consumer chooses to consume $X'^{*} = 9$ units and Y'* = 5.5 units. (X'* ; Y'*) = (9 ; 5,5)

3. Decomposing the Total Effect

To separate the substitution effect from the income effect, an imaginary budget constraint is drawn parallel to the new budget constraint but tangent to the original indifference curve at point B.

Substitution effect: The movement from A to B represents the change in consumption due solely to the change in relative prices. In this case, the consumer substitutes good X for good Y, consuming less of Y (3.5 units instead of 5).

Income effect: The movement from B to C represents the change in consumption due solely to the increase in real income. In this case, the consumer consumes more of both goods (X increases from 7 to 9, and Y increases from 3.5 to 5.5).

The table summarizes the changes in the quantities demanded of goods X and Y due to both the substitution and income effects.

goods	Initial situation	Substitution effect	Intermediate situation	Income effect	Final situation	Total effects
X	X*	X''*-X*	X''*	X'* - X''*	X'*	X'*- X*
Y	Y*	Y''*-Y*	Y''*	Y'* - Y''*	Y'*	Y'* - Y*
X	5	2	7	2	9	04
Y	5	-1.5	3.5	2	5.5	0.5

Algebraic Method for Decomposing Total Effect into Substitution and Income Effects

Step 1: Determining the Initial Equilibrium

- The consumer's initial optimal consumption bundle (X*, Y*) is found by maximizing their utility function subject to their budget constraint.
- This involves solving a constrained optimization problem, typically using the method of Lagrange multipliers or substitution.

Step 2: Determining the Final Equilibrium

- The new optimal consumption bundle (X', Y') is determined by solving the same optimization problem but with the new price for good X.
- Alternatively, the new quantities can be found by substituting the new price into the demand functions derived in step 1.

Step 3: Determining the Intermediate Equilibrium

- The intermediate equilibrium (X", Y'') is found by solving a system of two equations:
  - The first equation represents the tangency condition between the indifference curve and the budget constraint, indicating that the marginal rate of substitution (MRS) equals the price ratio.
  - The second equation states that the utility level at the intermediate point is the same as the utility level at the initial equilibrium.

Step 4: Decomposition of the Total Effect

The total effect is equal to the sum of the substitution effect and the income effect

(TE = SE + RE).

Course 07

Chapter 04: Supply, Demand, and Market Equilibrium

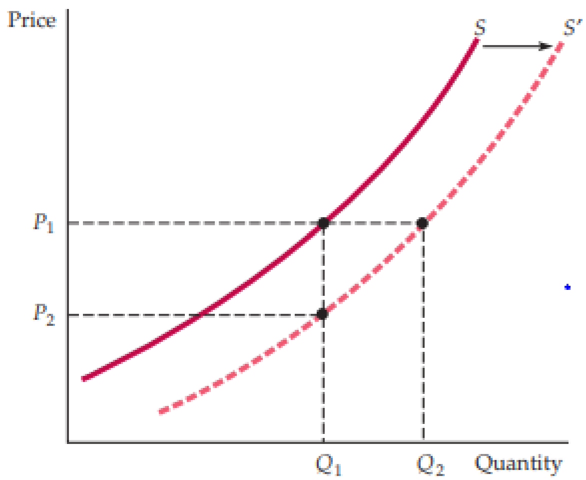
1-The supply curve;

The supply curve shows the quantity of a good that producers are willing to sell at a given price, holding constant any other factors that might affect the quantity supplied. The curve labeled S in Figure illustrates this. The vertical axis of the graph shows the price of a good, P, measured in dollars per unit. This is the price that sellers receive for a given quantity supplied. The horizontal axis shows the total quantity supplied, Q, measured in the number of units per period.
The supply curve is thus a relationship between the quantity supplied and the price. We can write this relationship as an equation:

$$Q_S = Q_S(P)$$

Note that the supply curve in Figure slopes upward. In other words, the higher the price, the more that firms are able and willing to produce and sell.

Figure (1)The Supply Curve



The supply curve, labeled S in the figure, shows how the quantity of a good offered for sale changes as the price of the good changes. The supply curve is upward sloping: The higher the price, the more firms are able and willing to produce and sell.

If production costs fall, firms can produce the same quantity at a lower price or a larger quantity at the same price. The supply curve then shifts to the right (from S to S′).

Other variables that affect Supply: The quantity supplied can depend on other variables besides price. For example, the quantity that producers are willing to sell depends not only on the price they receive but also on their production costs, including wages, interest charges, and the costs of raw materials. The supply curve labeled S in Figure was drawn for particular values of these other variables. A change in the values of one or more of these variables translates into a shift in the supply curve.

2- The Demand Curve

A demand curve is only valid if all other relevant factors are held constant (**ceteris paribus**: with other things the same). The most important other factors that can affect demand are:The buyers' income, price of complementary goods and substitute goods, Preferences.

Demand function is showing relationship between the quantity demanded of a commodity and the factors influencing demand. Dx = f (Px, Py, T, Y, A, Pp, Ep, U)

In the above equation,

Dx = Quantity demanded of a commodity

Px = Price of the commodity

Py = Price of related goods

T = Tastes and preferences of consumer

Y = Income level

A = Advertising and promotional activities

Pp = Population (Size of the market)

Ep = Consumer's expectations about future prices

U = Specific factors affecting demand for a commodity such as seasonal changes, taxation policy, availability of credit facilities, etc

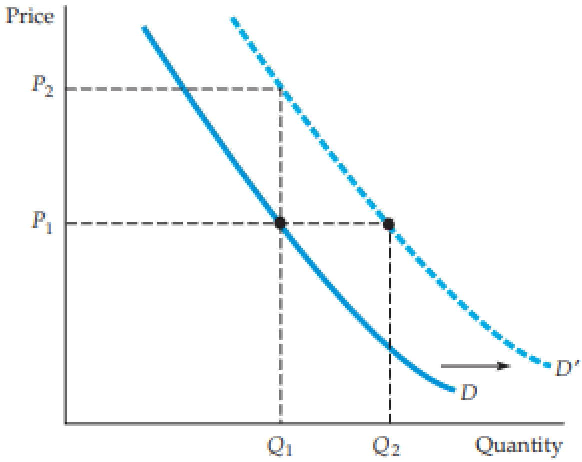
Demand may arise from individuals, household and market. When goods are demanded by individuals, it is called as individual demand. Goods demanded by household constitute household demand. Demand for a commodity by all individuals/households in the market in total constitutes market demand. **The Market demand** is the horizontal summation of individual demand .

The demand curve shows how much of a good consumers are willing to buy as the price per unit changes. We can write this relationship between quantity demanded and price as an equation:

$$Q_D = Q_D(P)$$

The demand curve, labeled D, shows how the quantity of a good demanded by consumers depends on its price. The demand curve is downward sloping; holding other things equal, consumers will want to purchase more of a good as its price goes down. The quantity demanded may also depend on other variables, such as income, the weather, and the prices of other goods. For most products, the quantity demanded increases when income rises. A higher income level shifts the demand curve to the right (from D to D′).

Figure (2)**The demand Curve**



We attributed the shift to the right of the demand curve in Figure 5.2 to an increase in income. However, this shift could also have resulted from either an increase in the price of a substitute good or a decrease in the price of a complementary good

Substitute and Complementary goods: Changes in the prices of related goods also affect demand. Goods are substitutes when an increase in the price of one leads to an increase in the quantity demanded of the other.

Goods are complements: when an increase in the price of one leads to a decrease in the quantity demanded of the other.

Law of demand:

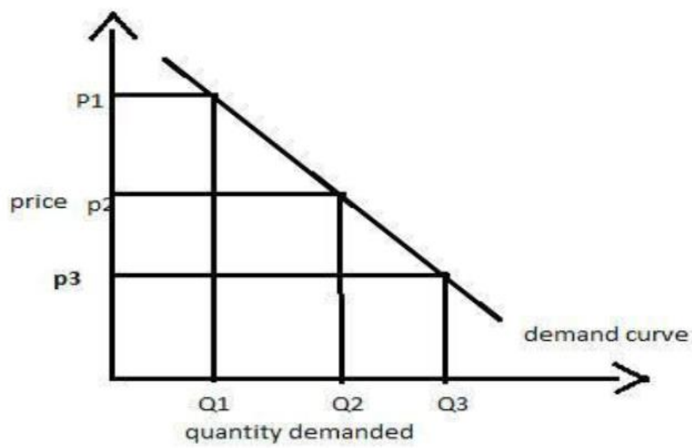
The law of demand states that there is an **inverse** relationship between quantity demanded of acommodity and its price, other factors being constant. In other words, higher the price, lower the demand and vice versa, other things remaining constant.

The demand by Buyers A, B, C and D are individual demands. Total demand by the four buyers is market demand. Therefore, the total market demand is derived by summing up the quantity demanded of a commodity by all buyers at each price

price	A	B	C	D	MD(market d)
10	1	0	3	0	4

9	3	1	6	4	14
8	7	2	9	7	25
7	11	4	12	10	37
6	13	06	14	12	45

Demand curve is a diagrammatic representation of demand schedule. It is a graphical representation of price- quantity relationship.



Demand curve has a negative slope, i.e, it slopes downwards from left to right depicting that with increase in price, quantity demanded falls and vice versa.

The reasons for a downward sloping demand curve can be explained as follows :

- ✓ Income effect
- ✓ Substitution effect
- ✓ Law of diminishing marginal utility

3 The Market Mechanism

The market mechanism is the tendency in a free market for the price to change until the market clears— i.e., until the quantity supplied and the quantity demanded are equal

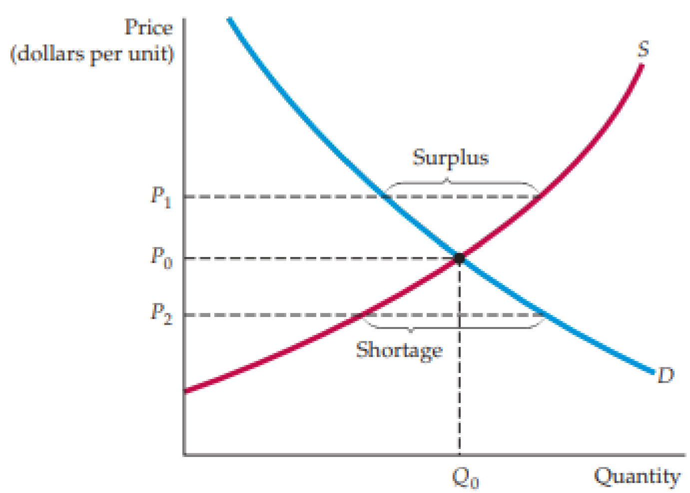
Equilibrium:

At this price (P_0 in Figure 3), the quantity supplied and the quantity demanded are just equal (to Q_0).

To understand why markets tend to clear, suppose the price were initially above the market-clearing level—say, P_1 in Figure 3. Producers will try to produce and sell more than consumers are willing to buy. A surplus—a situation in which the quantity supplied exceeds the quantity demanded—will result. To sell this surplus—or at least to prevent it from growing—producers would begin to lower prices. Eventually, as price fell, quantity demanded would increase, and quantity supplied would decrease until the equilibrium price P_0 was reached.

The opposite would happen if the price were initially below P_0—say, at P_2. A shortage—a situation in which the quantity demanded exceeds the quantity supplied—would develop, and consumers would be unable to purchase all they would like. This would put upward pressure on price as consumers tried to outbid one another for existing supplies and producers reacted by increasing price and expanding output. Again, the price would eventually reach P_0.

Figure 3 Supply and Demand



How to Find the Equilibrium Point Mathematically?

$$\begin{cases} Q_S = 85 + 30p \\ Q_D = 185 - 20p. \end{cases}$$

We now want to find the price, p*, that makes QD = QS. If the left-hand sides above are equal,

the right-hand sides must also be so. Therefore, substitute p* for p and set the right-hand sides equal to each other:

$$85 + 30p^* = 185 - 20p^*$$

To get p^* alone on the left-hand side, we add 20 p^* on both sides and subtract 85 from both sides. Then we have that

$$50p^* = 100.$$

Dividing by 50 on both sides yields the result that

$$p^* = 2.$$

If we then want to know the equilibrium quantity, Q*, we substitute the result we got for p* into either the supply or the demand function above. (Note that they must yield the same quantity, since p*, by definition, is the price that makes QD = QS.)

$$Q^* = \begin{cases} Q_S^* = 85 + 30p^* = 85 + 30 * 2 = 145 \\ Q_D^* = 185 - 20p^* = 185 - 20 * 2 = 145 \end{cases}$$

Consequently, we have the equilibrium price, p*= 2, and the equilibrium quantity, Q*= 145.

Course 08

Chapter 05: **elasticities**

Elasticities OF DEMAND:

We have seen that the demand for a good depends not only on its price, but also on consumer income and on the prices of other goods. Likewise, supply depends both on price and on variables that affect production cost. For example, if the price of coffee increases, the quantity demanded will fall and the quantity supplied will rise. Often, however, we want to know how much the quantity supplied or demanded will rise or fall. How sensitive is the demand for coffee to its price? If price increases by 10 percent, how much will the quantity demanded change? How much will it change if income rises by 5 percent? We use elasticities to answer questions like these.

An elasticity measures the sensitivity of one variable to another. Specifically, it is a number that tells us the percentage change that will occur in one variable inresponse to a 1-percent increase in another variable. For example, the price elasticity of demand measures the sensitivity of quantity demanded to price changes. It tells us what the percentage change in the quantity demanded for a good will be following a percent increase in the price of that good.

1-price elasticity of demand: Let's look at this in more detail. We write the

price elasticity of demand, Ep, as $E_p = (\%\Delta Q)/(\%\Delta P)$

Thus we can also write the price elasticity of demand as follows:

$$E_p = \frac{\Delta Q/Q}{\Delta P/P} = \frac{P\Delta Q}{Q\Delta P}$$

-Point elasticity of demand

$$e_p = \left(\frac{dQ}{Q}\right) / \left(\frac{dP}{P}\right)$$

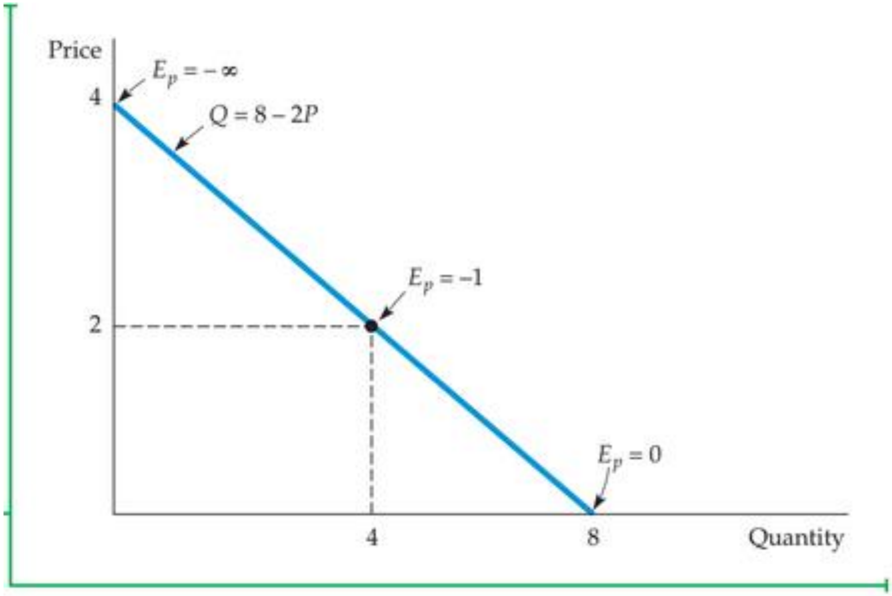
or

$$e_p = \left(\frac{dQ}{dP}\right) / \left(\frac{P}{Q}\right)$$

If the demand curve is linear : $Q = b_0 - b_1P$

Its slope is $dQ/dP = -b_1$. Substituting in the elasticity formula, we get

$$e_p = -b_1 . \frac{P}{Q}$$



The price elasticity of demand depends not only on the slope of the demand curve but also on the price and quantity. The elasticity, therefore, varies along the curve as price and quantity change. Slope is constant for this linear demand curve. Near the top, because price is high and quantity is small, the elasticity is large in magnitude. The elasticity becomes smaller as we move down the curve.

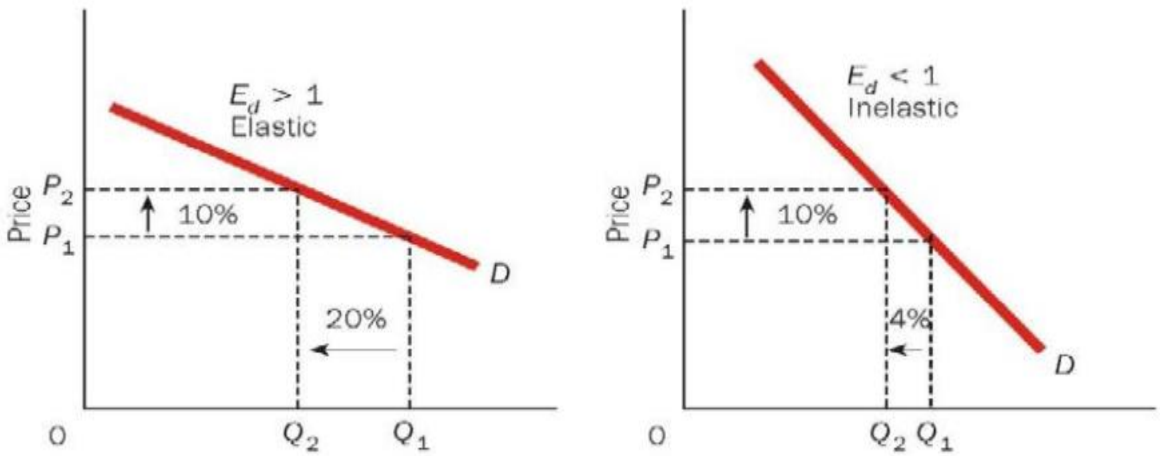
This principle is easiest to see for a linear demand curve—that is, a demand curve of the form

$Q = a - bP$

As an example, consider the demand curve $Q = 8 - 2P$

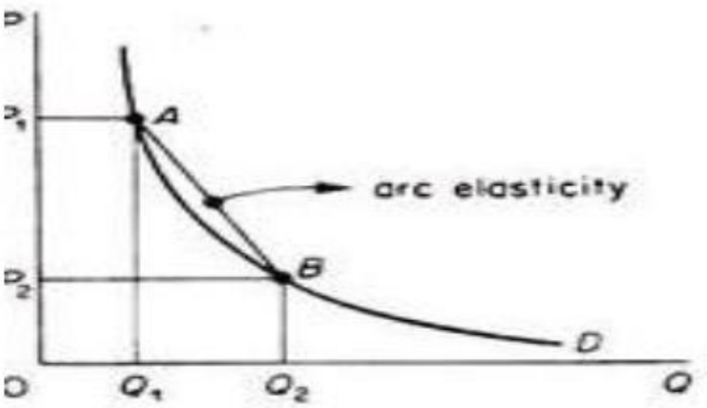
For this curve, $\Delta Q/\Delta P$ is constant and equal to -2 (a ΔP of 1 results in a ΔQ of -2). However, the curve does not have a constant elasticity. Observe from Figure 2.11 that as we move down the curve, the ratio P/Q falls; the elasticity therefore decreases in magnitude. Near the intersection of the curve with the price axis, Q is very small, so $E_p = -2(P/Q)$ is large in magnitude. When P = 2 and Q = 4, $E_p = -1$. At the intersection with the quantity axis, P = 0 so $E_p = 0$.

(a) infiniTely elaSTiC DemanD (b) COmpleTely inelaSTiC DemanD



-ARC elasticity of demand : the elasticity calculated over a range of prices. Rather than choose either the initial or the final price, we use an average of the two, P; for the quantity demanded, we use Q. Thus the arc elasticity of demand is given by

$$e_p = \frac{\Delta Q}{\Delta P} \cdot \left[\frac{\frac{P_1+P_2}{2}}{\frac{Q_1+Q_2}{2}}\right] = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1+P_2}{Q_1+Q_2}$$



-Determinants of Price Elasticity of Demand

(1) The availability of substitutes; the demand for a commodity is more elastic if there are close substitutes for it.

(2) The nature of the need that the commodity satisfies. In general, luxury goods are price elastic,while necessities are price inelastic.

(3) The time period. Demand is more elastic in the long run.

(4) The number of uses to which a commodity can be put. The more the possible uses of a commodity the greater its price elasticity will be.

(5) The proportion of income spent on the particular commodity

2-Income elasticity of demand is the percentage change in the quantity demanded, Q, resulting from a 1-percent increase in income m:

$$e_m = \frac{\Delta Q / Q}{\Delta m / m}$$

Here, em is income elasticity, and m and 'm are income and change in income, respectively. Similarly to price elasticity, goods are grouped depending on their income elasticity:

$e_m < 0$	Inferior goods
$0 < e_m$	Normal goods

$1 < e_m$	Luxury goods
$0 < e_m < 1$	Necessary goods

3-Cross-Price Elasticity of demand:

Cross-price elasticity is defined as the percentage change in demand on a good if the price of another good changes with one percent:

$$e_{12} = \frac{\Delta Q_1 / Q_1}{\Delta p_2 / p_2}$$

Here, e12 is the cross-price elasticity between good 1 and good 2; Q1 and 'Q1 are quantity demanded and quantity change for good 1, whereas p2 and 'p2 are price and price change on good 2. Again, goods are grouped depending on their cross-price elasticity

$e_{12} < 0$	Complementary goods
$e_{12} = 0$	Independent goods
$0 < e_{12}$	Substitute goods

Elasticity and revenue:

Price elasticity of demand is very important to an understanding of business decisions, especially because of the link with revenue. If the demand for a product is price elastic, a business should lower the price of the product because more products will be bought and this will produce a higher total revenue. If the demand for a product is price inelastic, a business should increase the price of the product because even though fewer items will be bought, the increased revenue from each product sold will offset this.

In general, the relationship between elasticity and total revenue can be summarized as follows:

- Elastic demand leads to a decrease in total revenue when price increases.

- Inelastic demand leads to an increase in total revenue when price increases.

Total revenue:

Total revenue is the total income generated by a company from selling its products or services. It is calculated as the product of price and quantity. In mathematical terms, total revenue is given by the following equation:

TR = P * Q

Where:

- TR is total revenue
- P is price
- Q is quantity

Average revenue:

$$RM = \frac{RT}{x} = \frac{P_x .\ x}{x} = P_x$$

Marginal revenue:

$$Rmg = \frac{\Delta RT}{\Delta x} \qquad Rmg = \lim_{\Delta x \to 0} \frac{\Delta RT}{\Delta x} = \frac{dRT}{dx}$$

The relationship between elasticity and total revenue:

$$RT = P_x . x \Rightarrow \frac{dRT}{dx} = \frac{dP_x}{dx} . x + \frac{dx}{dx} . P_x$$

$$dRT = x. \frac{dP_x}{dx} dx + \frac{dx}{dx} . P_x dx \Rightarrow dRT = x dP_x + P_x\, dx \ldots (1)$$

$$E = \frac{\mathrm{dx}}{\mathrm{d}P_{x_x}} . \frac{P_x}{\mathrm{x}} \Rightarrow P_x dx = E\ x\ \mathrm{d}P_x = -|E| x\ \mathrm{d}P_x \ldots (2)$$

From(1) and (2), we can conclude the following:

$$dRT = x dP_x + - |E| x\ \mathrm{d}P_x \Rightarrow dRT = (1 - |E|)\ x\ \mathrm{d}P_x$$

$\lvert E \rvert > 1$	$\lvert E \rvert < 1$	$\lvert E \rvert = 1$	E / dP
dRT < 0	dRT > 0	dRT = 0	dP > 0
dRT > 0	dRT < 0	dRT = 0	dP < 0

<u>Course 09</u>

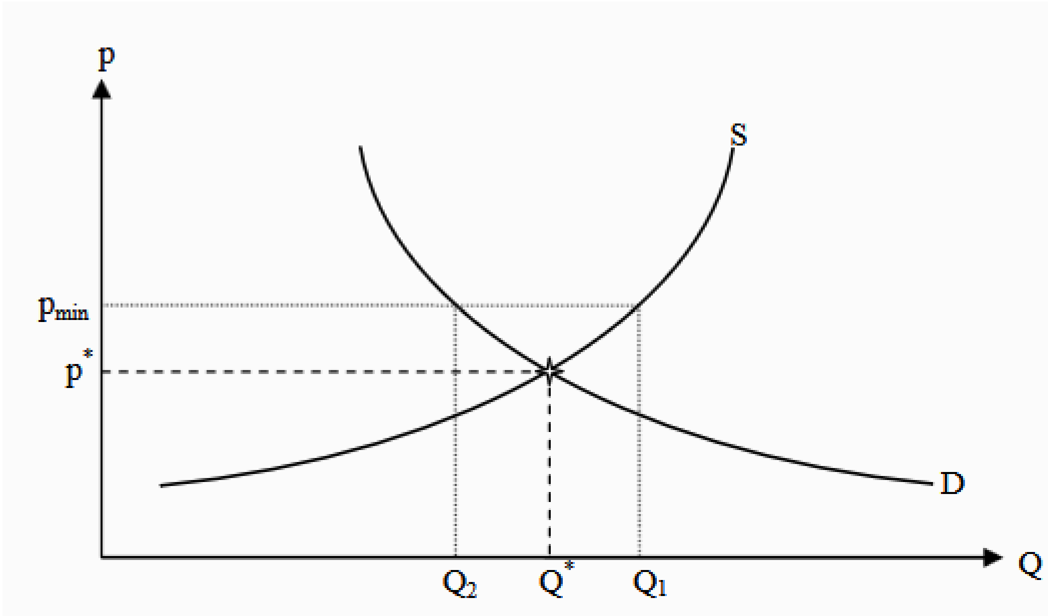
Chapter sixe: **Government regulation of the market**

1-Minimum and Maximum Prices regulation

Many markets are, for a number of reasons, regulated. The government could for instance decide about prices that the market is not allowed to go above or below, or about maximum quantities.

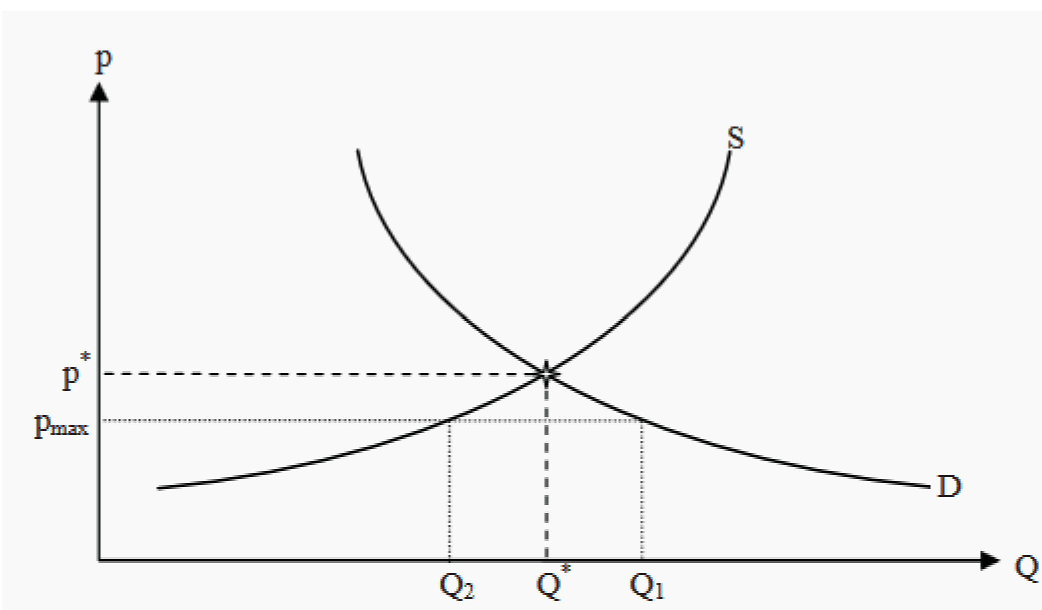
- **Minimum prices**

Minimum prices (also called price floors) are often used for wages (the price of labor) and for certain types of goods such as agricultural goods. The minimum price is usually chosen above the equilibrium price, as in the opposite case it would not have any effect. (The market participants would then choose p * instead.) Consumers and producers are consequently prevented from reaching the equilibrium price p *.



- **Maximum prices**

Maximum prices (also called price ceilings) are in several countries used for apartment rentals. For a maximum price to have any effect, it has to be below the equilibrium price, and the effects are the opposite to those of a minimum price. In Figure 2.5, p max is the maximum price. It causes the consumers to demand the quantity Q 1 whereas the producers only want to supply Q 2, and, consequently, there is a shortage. A typical consequence of a maximum price is that the search time to find an appropriate good is increased since the supply is too small to meet the demand.



2-Quantity Regulations

The effects of quantity regulations are similar to those of price regulations. Assume for instance that there is a restriction stating that one may only import the quantity Q max of a certain good, say Asian textiles.

Producers would have been willing to supply the quantity Q_{max} at a price of p_S, whereas the consumers would have been willing to buy that quantity at a price of p_D. Since the quantity is not allowed to increase, there is excess demand at all prices other than p D. When there is excess demand, consumers are likely to bid up the price, so the price that this market is likely to arrive at is p_D.

Note that at the price p_D, producers are willing to supply a much larger quantity, Q_1, but that they are prevented from doing so by the regulation. The consumers have to pay a price that is larger than the equilibrium price (p_D instead of p^*) and they get fewer units of the good, so they typically are made worse off by a quantity regulation.

Course 10

Chapter seven: **Production behavior:Production**

The theory of the firm is a branch of microeconomics that studies how firms make decisions about production, pricing, and investment. It is based on the assumption that firms are profit-maximizing entities that seek to minimize costs and maximize revenue.

theory of the firm: Explanation of how a firm makes cost-minimizing production decisions and how its cost varies with its output.

The theory of the firm can be divided into two main parts: production theory and cost theory. Production theory studies how firms transform inputs into outputs. Cost theory studies the relationship between costs and output.
Applications of the Theory of the Firm:
The theory of the firm has a wide range of applications in economics. It is used to study the behavior of firms in competitive markets, monopolistic markets, and oligopolistic markets. It is also used to study the behavior of firms in the long run and the short run. The production decisions of firms can likewise be understood in three steps:

- **1.Production Technology: (such as labor, capital, and raw materials)**
- **2. Cost Constraints: the prices of labor, capital, and other inputs.**
- **3. Input Choices: Given its production technology and the prices of labor, capital, and other inputs, the firm must choose how much of each input to use in producing its output**

Production is the process of converting inputs (such as labor, capital, and raw materials) into outputs (such as goods and services).

A production function:

indicates the maximum output Q that a firm can produce for every specified combination of inputs
We can then write the production function as $Q = f(K, L)$

Production functions describe what is technically feasible when the firm operates efficiently—that is, when the firm uses each combination of inputs as effectively as possible.

Production with one variable input, labor, can be usefully described in terms of the average product of labor AP (which measures output per unit of labor input) and The Average product of labor can be written as:

$$AP = Q/L$$

and the marginal product of labor MP (which measures the additional output as labor is increased by 1 unit) and The marginal product of labor can be written as:

$$MP = \Delta Q/\Delta L$$

$$MP = \delta Q/\delta L$$

the law of diminishing marginal returns:

the law of diminishing marginal returns (or the law of diminishing marginal product) is probably the most frequently cited concept from microeconomics.

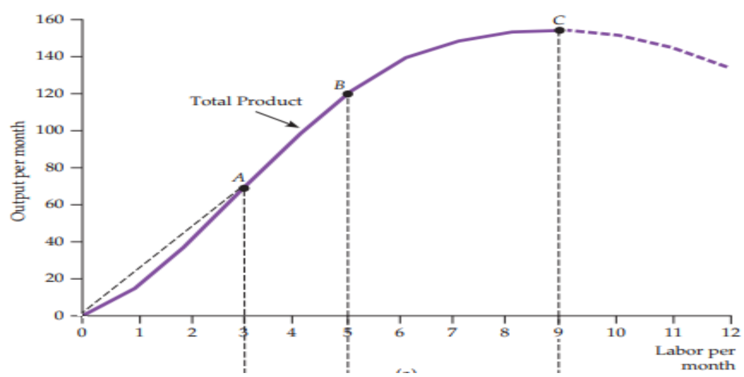
Suppose we keep everything constant, except for one single input factor, for instance L. If we increase the number of hours worked, we will probably produce more. The law of diminishing marginal returns states that the increase will eventually become smaller and smaller when the number of hours worked is large enough.

Production in the Short Run

Production in the Short Run It is common to distinguish between the short run and the long run regarding production. The short run is defined as the time during which (at least) one of the input factors is fixed, usually capital.

We will assume that in the short run, labor is variable but capital is fixed. To make it clear that the quantity of capital is fixed in the short run, the production function: $q = f(L, k_{\circ})$ or $q = f(L)$

AMOUNT OF LABOR (L)	AMOUNT OF CAPITAL (K)	TOTAL OUTPUT (q)	AVERAGE PRODUCT (q/L)	MARGINAL PRODUCT (Δq/ΔL)
0	10	0	—	—
1	10	15	15	15
2	10	40	20	25
3	10	69	23	29
4	10	96	24	27
5	10	120	24	24
6	10	138	23	18
7	10	147	21	9
8	10	152	19	5
9	10	153	17	1
10	10	150	15	−3
11	10	143	13	−7
12	10	133	11.08	−10



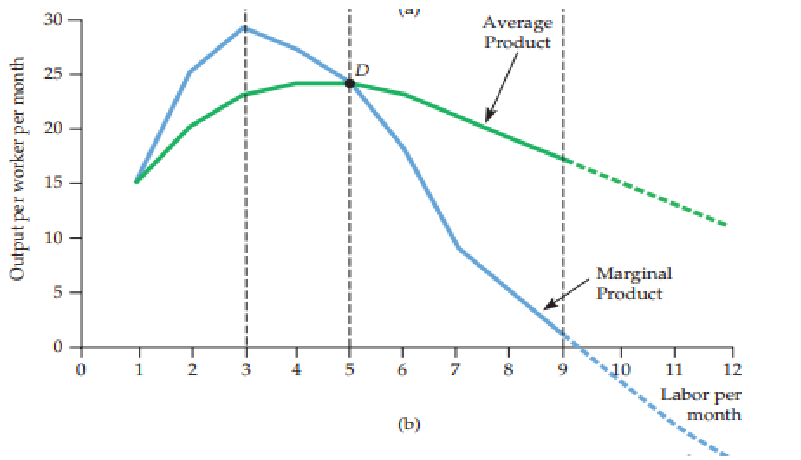
The total output curve in (a) shows the output produced for different amounts of labor input.

At point A in (a), with 3 units of labor, the marginal product is 29. The average product of labor, however,is 23, Also, the marginal product of labor reaches its maximum at this point. **At point B**, with 5 units of labor, the marginal product of labor has dropped to 24 and is equal to the average product of labor. Thus, in (b), the average and marginal product curves intersect (at point D).

Note that when the marginal product curve is above the average product, the average product is increasing.

When the labor input is greater than 5 units, the marginal product is below the average product, so the average product is falling.

Once the labor input exceeds 9 units, the marginal product becomes negative, so that total output falls as more labor is added



The relationship between TP, APʟ, and MPʟ :

Total Product (TP):
TP increases with the increase in one factor of production until it reaches its maximum point. After that, TP becomes **diminishing**.
When TP reaches its maximum, the **marginal product (MP)** becomes **zero**.
After that, MP takes **negative values**.

Marginal Product (MP):
MP reaches its maximum at the **inflection point** on the TP curve.

Average Product (AP):
AP increases initially to reach its maximum point until it intersects with MP.
Then, AP becomes **diminishing** but **positive** as long as TP is positive.

the three stages of production

0 ⟷ **AP = MP (Max AP) : Stage (1)**

AP = MP ⟷ **MP =0 (Max TP) : Stage (2)**

MP =0 → **Stage (3)**

Course 11:

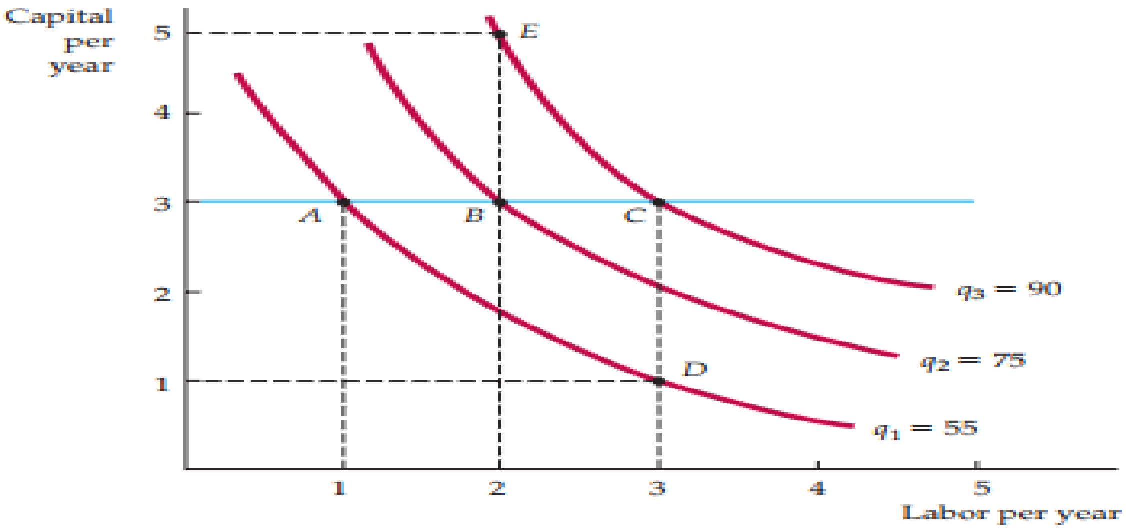
Production in the Long Run (Production with two variable Inputs)

In the long run, both labor and capital are variable inputs. That means that the quantity produced is a function of both L and K, where either of them can be changed, i.e. q= f(L,K) (and not f(L, K_0)).

Suppose that the inputs are labor and capital and that they are used to **produce food**

	LABOR INPUT				
CAPITAL INPUT	1	2	3	4	5
1	20	40	55	65	75
2	40	60	75	85	90
3	55	75	90	100	105
4	65	85	100	110	115
5	75	90	105	115	120

- ✓ **Isoquant Curve**: showing all possible combinations of inputs that yield the same output.



- ✓ **diminishing marginal returns:**

We can see why there are diminishing marginal returns to labor by drawing a horizontal line at a particular level of capital—say, 3. Reading the levels of output from each isoquant as labor is increased, we note that each additional unit of labor generates less and less additional output. For example, when labor is increased from 1 unit to 2 (from A to B), output increases by 20 (from 55 to 75). However, when labor is increased by an additional unit (from B to C), output increases by only 15 (from 75 to 90).

- ✓ **Isoquant map** Graph combining a number of isoquants, used to describe a production function.

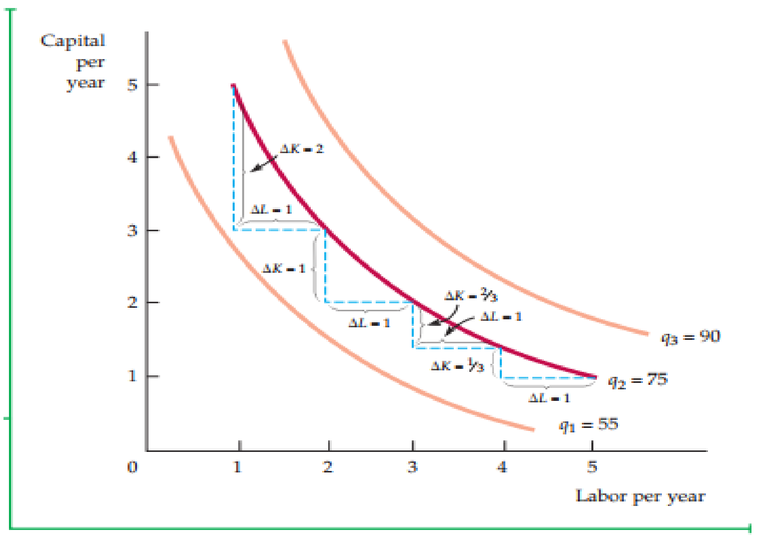
- marginal rate of technical substitution (MRTS) Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.

MRTS = -Change in capital input/change in labor input
= **-** $\Delta K/\Delta L$(for a fixed level of q)

where ΔK and ΔL are small changes in capital and labor along an isoquant

In Figure the **MRTS** is equal to 2 when labor increases from 1 unit to 2 and output is fixed at 75. However, the MRTS falls to 1 when labor is increased from 2 units to 3, and then declines to 2/3 and to 1/3. Clearly, as more and more labor replaces capital, labor becomes less productive and capital becomes relatively more productive. Therefore, we need less capital to keep output constant, and the isoquant becomes flatter.

diminishing MRTS



the **MRTS** is closely related to the marginal products of labor MP_L and capital MP_K. To see how, imagine adding some labor and reducing the amount of capital sufficient to keep output **constant**. The additional output resulting from the increased labor input is equal to the additional output per unit of additional labor (the marginal product of labor) times the number of units of additional labor:

Additional output from increased use of labor = $(MP_L)(\Delta L)$

Similarly, the decrease in output resulting from the reduction in capital is the loss of output per unit reduction in capital (the marginal product of capital) times the number of units of capital reduction:

Reduction in output from decreased use of capital = $-(\Delta K)(MP_K)$

Because we are keeping output **constant** by moving along an isoquant, the total change in output must be **zero**. Thus,

$$MP_K(K, L)dK + MP_L(K, L)dL = dq = 0 \ldots\ldots\ldots(1)$$

Now, by rearranging terms we see that

$$-dK/dL = MRTS_{LK} = MPL(K, L)/MPK(K, L) \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Course 12

Isocost line: Graph showing all possible combinations of labor and capital that can be purchased for a given total cost

To see what an isocost line looks like, recall that the total cost C of producing any particular output is given by the sum of the firm's labor cost wL and its capital cost rK:

$$C = w\,L + r\,K$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = \frac{C}{r} - \frac{w}{r}L$$

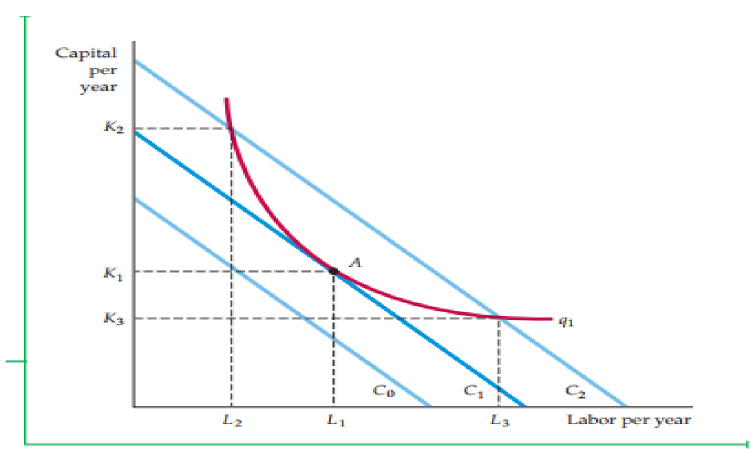
It follows that the isocost line has a slope of **$\Delta K/\Delta L = -(w/r)$,** which is the ratio of the wage rate to the rental cost of capital.

It tells us that if the firm gave up a unit of labor (and recovered w dollars in cost) to buy w/r units of capital at a cost of r dollars per unit, its total cost of production would remain the same.

For example, if the wage rate were $10 and the rental cost of capital $5, the firm could replace one unit of labor with two units of capital with no change in total cost.

Producing A given output at minimum cost:

Isocost curve C1 is tangent to isoquant q1 at A and shows that output q1 can be produced at minimum cost with labor input L1 and capital input K1. Other input combinations—L2, K2 and L3, K3—yield the same output but at higher cost



How does the isocost line relate to the firm's production process? Recall that in our analysis of production technology, we showed that the marginal rate of technical substitution of labor for capital (MRTS) is the negative of the slope of the isoquant and is equal to the ratio of the marginal products of labor and capital

$- \Delta K/\Delta L = MRTS = MP_L/MP_K$ ………………(1)

Above, we noted that the isocost line has a slope of $\Delta K/\Delta L = -w/r$ It follows that when a firm minimizes the cost of producing a particular output, the following condition holds:

$MP_L/MP_K = w/r$

We can rewrite this condition slightly as follows:

$MP_L/w = MP_K/r$

$$-\frac{w}{r} = MRTS \left[= -\frac{MP_L}{MP_K} = \frac{\Delta K}{\Delta L} \right]$$

where $MRTS_{LK}$ is the firm's marginal rate of technical substitution between labor and capital. Now, rewrite the condition :

$MP_L(K, L)/MP_K(K, L) = w/r$……………………(2)

Because the left side of (1) represents **the negative of the slope** of the isoquant, it follows that at the point of tangency of the isoquant and the isocost line, the firm's marginal rate of technical substitution (which trades off inputs while keeping **output constant**) is equal to the ratio of the input prices (which represents the slope of the firm's isocost line).

Cost Minimization:

The theory of the firm relies on the assumption that firms choose inputs to the production process that minimize the cost of producing output. If there are two inputs, capital K and labor L, the production function $F(K, L)$ describes the maximum output that can be produced for every possible combination of inputs.

We assume that each factor in the production process has positive but decreasing marginal products. Therefore, writing the marginal product of capital and labor as $MPK(K, L)$ and $MPL(K, L)$, respectively, it follows that

$$\mathrm{MP}_K(K,L) = \frac{\partial F(K,L)}{\partial K} > 0, \quad \frac{\partial^2 F(K,L)}{\partial K^2} < 0$$

$$\mathrm{MP}_L(K,L) = \frac{\partial F(K,L)}{\partial L} > 0, \quad \frac{\partial^2 F(K,L)}{\partial L^2} < 0$$

The cost-minimization problem can be written as:

Minimize $\boldsymbol{C = wL + rK}$ …………….(1)

Subject to the constraint that a fixed output q_0 be produced:

F (K, L) = q_0 ……………………….(2)

C represents the cost of producing the fixed level of output $\boldsymbol{q_0}$.
To determine the firm's demand for capital and labor inputs, we choose the values of K and L that minimize (1) subject to (2).
We can solve this constrained optimization problem in three steps :

• ***Step 1:*** Set up the **Lagrangian**, which is the sum of two components: the cost of production (to be minimized) and the Lagrange multiplier **λ** times the output constraint faced by the firm:

$\boldsymbol{\Phi = wL + rK - \lambda\,[F(K, L) - q_0]}$ **…………….(3)**

• ***Step 2:*** Differentiate the **Lagrangian** with respect to K, L, and λ. Then equate the resulting derivatives to zero to obtain the necessary conditions for a minimum.

$$\left.\begin{aligned} \frac{\partial \Phi}{\partial K} &= r - \lambda\, \mathrm{MP}K(K, L) = 0 \\ \frac{\partial \Phi}{\partial L} &= w - \lambda\, \mathrm{MP}L(K, L) = 0 \\ \frac{\partial \Phi}{\partial \lambda} &= q_0 - F(K, L) = 0 \end{aligned}\right\} \quad \text{………….(4)}$$

• ***Step 3:*** In general, these equations can be solved to obtain the optimizing values of L, K, and λ. It is particularly instructive to combine the first two conditions in (4) to obtain

$\mathrm{MP}K(K, L)/r = \mathrm{MP}L(K, L)/w$…………………….. **(5)**

Equation (5) tells us that if the firm is minimizing costs, it will choose its factor inputs to equate the ratio of the marginal product of each factor divided by its price.

Finally, we can rewrite the first two conditions of (4) to evaluate the Lagrange multiplier:

$r - \lambda MP_K(K, L) = 0 \qquad \lambda = r / MP_K(K, L)$

$w - \lambda MP_L(K, L) = 0 \qquad \lambda = w / MP_L(K, L)$

The highest production

The dual problem asks what combination of *K* and *L* will let us produce the most output at a cost of C_0. We can see the equivalence of the two approaches by solving the following problem:

Maximize $F(K, L)$ **subject to** $wL + rL = C0$**……………(1)**

.

We can solve this problem using the **Lagrangian method:**

• ***Step 1****:* We set up the Lagrangian

$$\Phi = F(K, L) - \mu(wL + rK - C0 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where μ is the **Lagrange multiplier**

• ***Step 2****:* We differentiate the Lagrangian with respect to *K*, *L*, and μ and set the resulting equation equal to zero to find the necessary conditions for a maximum:

$$\left.\begin{aligned} &\frac{\partial \Phi}{\partial K} = MPK(K, L) - \mu r = 0 \ldots\ldots\ldots\ldots(1) \\ &\frac{\partial \Phi}{\partial L} = MPL(K, L) - \mu w = 0 \ldots\ldots\ldots\ldots..(2) \\ &\frac{\partial \Phi}{\partial \mu} = wL - rK + C0 = 0 \ldots\ldots\ldots\ldots\ldots(3) \end{aligned}\right\} \ldots\ldots\ldots\ldots..(3)$$

• ***Step 3****:* Normally, we can use the equations of (2) to solve for *K* and *L*. In particular, we combine the first two equations to see that

$$\left.\begin{aligned} \mu &= MP_K(K, L) / r \\ \mu &= MP_L(K, L)/w \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

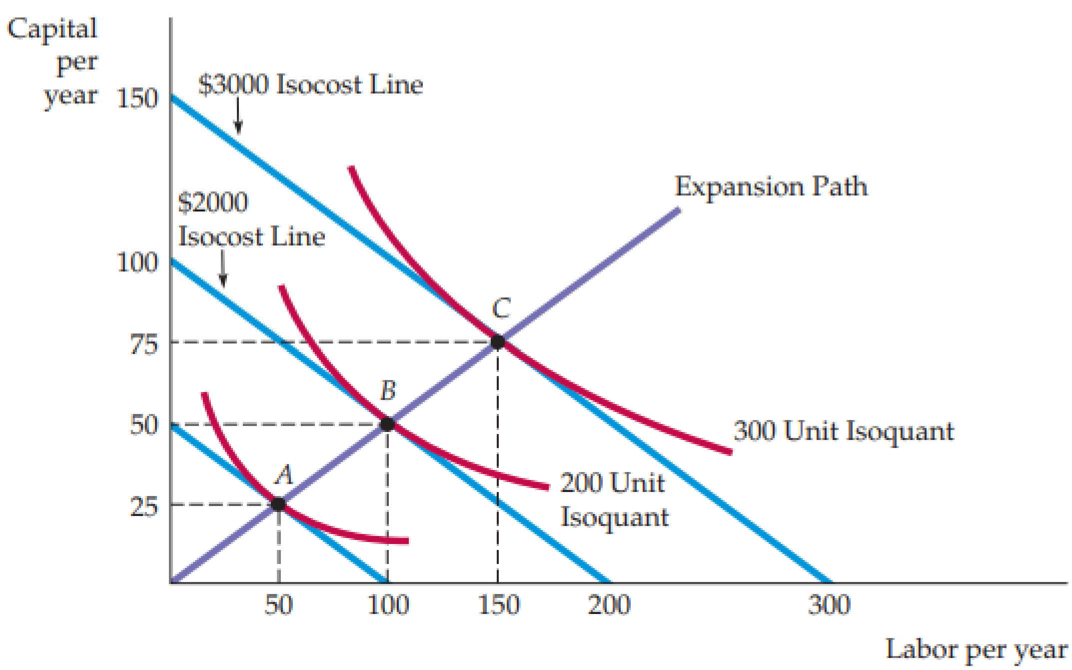
→ **MP*K*(*K*, *L*)/*r* = MP*L*(*K*, *L*)/*w*…………….(5)**
that is, the necessary condition for cost minimization.

expansion path and long Run costs

expansion path Curve passing through points of tangency between a firm's isocost lines and its isoquants.

You can see that each of the points A, B, and C in Figure (a) is a point of tangency between an isocost curve and an isoquant. Point B, for example, shows us that the lowest-cost way to produce 200 units of output is to use 100 units of labor and 50 units of capital; this combination lies on the $2000 isocost line. Similarly, the lowest-cost way to produce 100 units of output (the lowest unlabeled isoquant) is $1000 (at point A, L = 50, K = 25); the least-cost means of getting 300 units of output is $3000 (at point C, L = 150, K = 75).
The curve passing through the points of tangency between the firm's isocost lines and its isoquants is its expansion path.

The expansion path describes the combinations of labor and capital that the firm will choose to minimize costs at each output level. As long as the use of both labor and capital increases with output, the curve will be upward sloping. In this particular case we can easily calculate the slope of the line. As output increases from 100 to 200 units, capital increases from 25 to 50 units, while labor increases from 50 to 100 units. For each level of output, the firm uses half as much capital as labor. Therefore, the expansion path is a straight line with a slope equal to
$\Delta K/\Delta L = (50 - 25)/(100 - 50) = 1/2$



(a)

Returns to Scale:

Suppose that, using labor L and capital K, we produce the quantity **q** of a good. If we would double both L and K, we would probably increase the quantity produced as well, but by how much? If **q** is also doubled, we have constant returns to scale (we can think of this as that the scale is the same for (L,K) and q). If instead **q** increases by less than two times, we have decreasing returns to scale, and if it increases by more we have increasing returns to scale. More generally, we increase L and K by a factor **t**, and then check if **q** increases by more, less, or by the same factor. We can express this as:

f(t*L,t*K) < t*f(L,K)	Decreasing returns to scale
f(t*L,t*K) = t*f(L,K)	Constant returns to scale
f(t*L,t*K) > t*f(L,K)	Increasing returns to scale

Example:

$X_0 = b_0\ k^{\alpha}\ L^{\beta}$

If the factors of production change with respect to t, then we can write

$\mathbf{X^* = b_0\ (tk)^{\alpha}\ (tL)^{\beta}}$

$\mathbf{= t^r\ X_0}$

With $\boldsymbol{\alpha + \beta = r}$ (return to scale)

If $r < 1$ decreasing return to scale
If $r = 1$ constant return to scale
If $r > 1$ increasing return to scale

Course 13

Chapter 08: Costs

Production Costs in the Short Run

In the short run, not all input factors are variable. We therefore distinguish between fixed cost, FC, and variable cost, VC. Total cost, TC, is the sum of the two:

TC= VC+ FC

We also need to define a few other central concepts. Regarding average cost, we will have use for the averages of all three of the above. If we divide each of them with q, we get average total cost, ATC, average variable cost, AVC, and average fixed cost, AFC:

$$\begin{cases} ATC = \frac{TC}{q} \\ AVC = \frac{VC}{q} \\ AFC = \frac{FC}{q} \end{cases}$$

Note that the following must hold

ATC= AVC + AFC

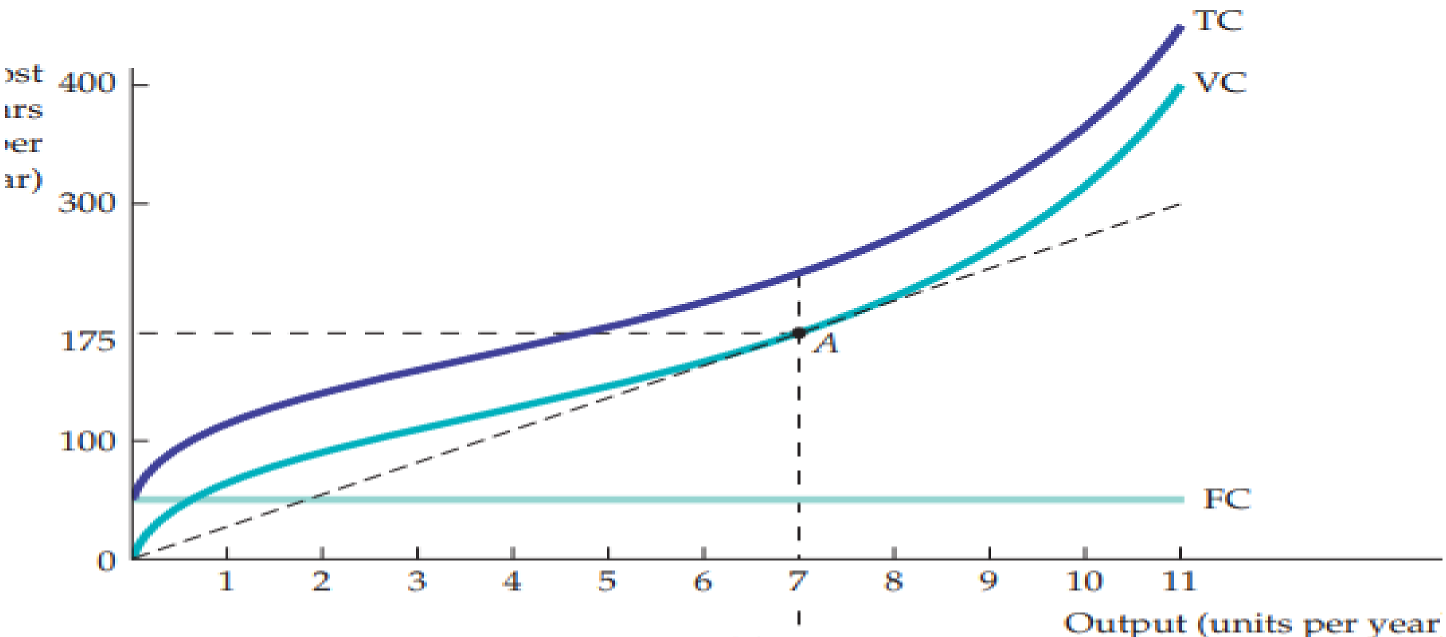
The marginal cost, MC, in turn, measures the cost of producing one more unit of the good:

$$MC = \frac{\Delta TC}{\Delta q} = \left[\frac{\Delta (VC + FC)}{\Delta q} = \right] \frac{\Delta VC}{\Delta q}$$

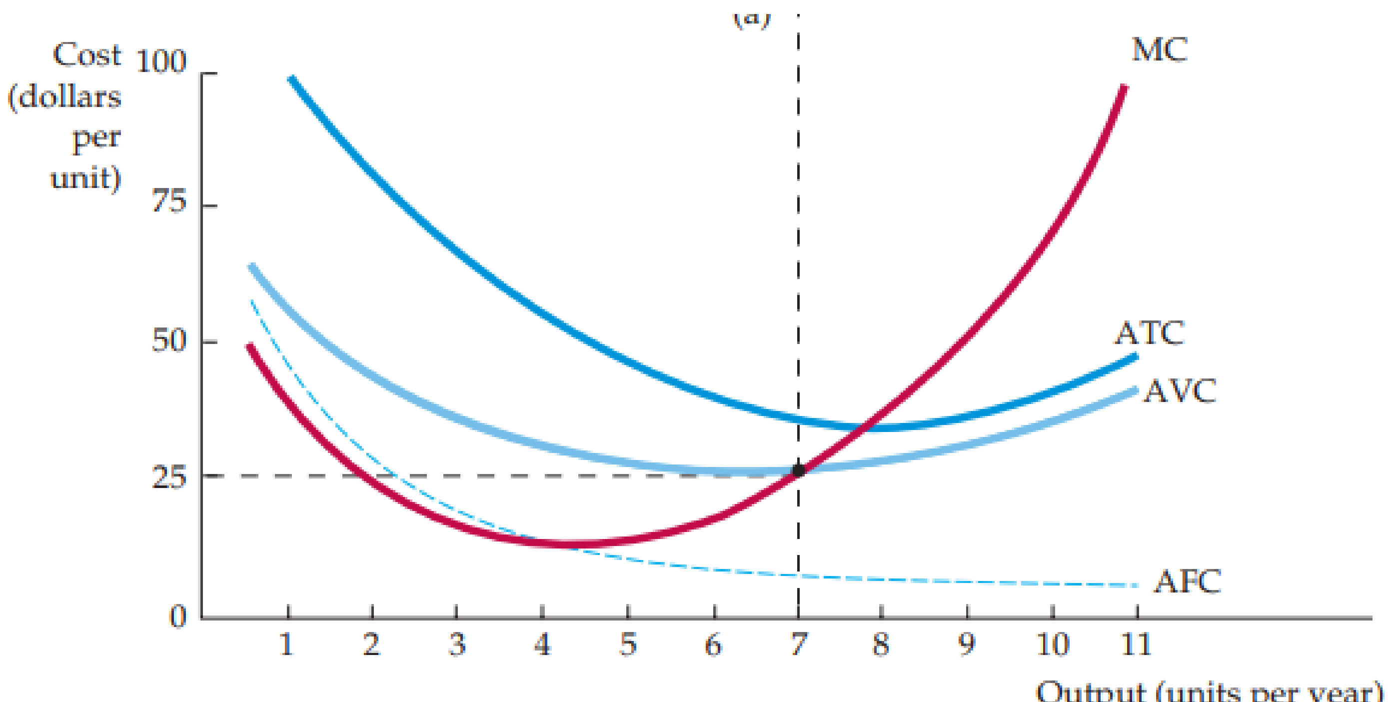
Note that we can use either the change in total cost or the change in variable cost. Both must give the same answer, since the fixed cost does not change ($\Delta FC = 0$). As before, the expression for marginal change is only an approximation. Now, we will construct a graph to illustrate these different measures of costs (see Figure 1). The fixed cost, FC, is constant, independent of how many

units we produce, so the curve illustrating FC must be a horizontal line. Total cost, TC, must always increase with production; else, the production is not efficient. Furthermore, since if we produce nothing TC must equal FC, the curve for TC must start in the same point as FC on the Y-axis. Since TC = VC + FC, the curve for variable cost, VC, must have the same shape as TC. Obviously, VC of producing nothing is zero, so the curve for VC must start at the origin.

Figure 1: The Cost Function with Average and Marginal
Costs



(a)



(b)

The retionship between production function and costs function

MC = $(d\,TC)/(d\,Q) = (d\,(FC+VC))/dQ$ ⟹ MC = $(d\,(W.\ L))/(d\,Q)$

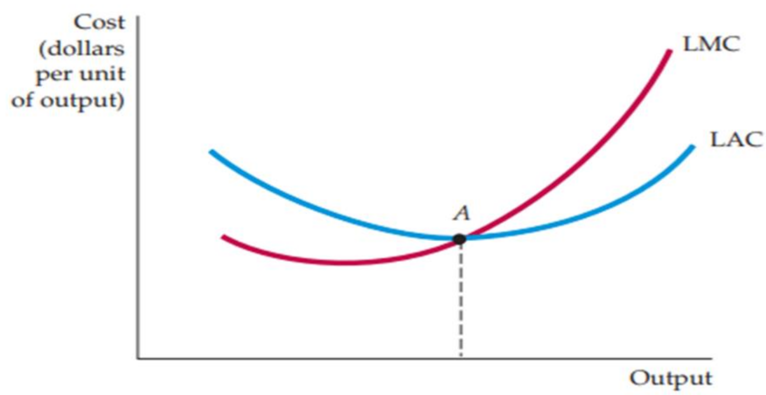
MC = w. $(d\,L)/(d\,Q)$ = w. $1/MPl$

The marginal product of labor decreases, the marginal cost of production increases, and vice versa

- Production Costs in the long run

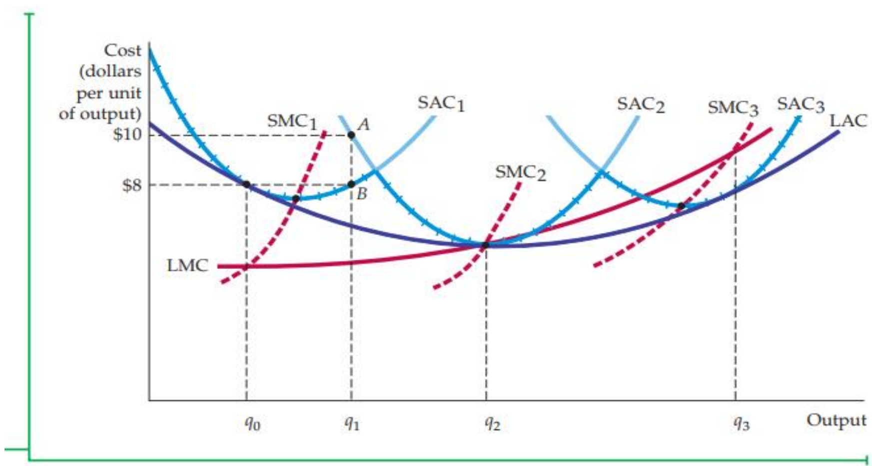
Long-run Average and Marginal Cost:

long-run average cost curve(LAC) Curve relating average cost of production to output when all inputs, including capital, are variable.
long-run marginal cost curve (LMC) Curve showing the change in long-run total cost as output is increased incrementally by 1 unit.



When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run mar ginal cost LMC is less than LAC. Conversely, when LAC is increasing, LMC is greater than LAC. The two curves intersect at *A*, where the LAC curve achieves its minimum.

the relationship between short-run and long-run cost:



The LAC curve in Figure has another interesting property. To the left of the quantity q2, LAC slopes downwards (towards q2), but to the right it slopes upwards. That means that the lowest cost per unit of the good is achieved at the quantity q2.

To the left of q2, we have economies of scale and to the right we have diseconomies of scale. Note that this can be very important if there is competition in the market. If the firm is at a point to the left of q2, it can lower its cost per unit by increasing the scale of production.
In the next step, it can undercut the price of its competitors.
Economies of scale: The more one produces, the lower is the cost per unit.

Diseconomies of scale: The more one produces, the higher is the cost per unit.

Exercises and solution

Consumer behavior

Exercise 01:

A consumer consumes sweets and enjoys total utility at every consumed volume of sweets as shown in the following table

/Quantity of sweets (Qx)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Total utility (TU)	**100**	**200**	**275**	**325**	**350**	**360**	**360**	**355**

-Calculation of marginal utility and drawing of total utility and marginal utility curves؟

Exercise 02 :

The table below illustrates the total utility data for an individual for two goods x, y, Assuming that this consumer spends his entire income on purchasing two goods at a total of 17 dollars, with a price of 2 dollars for x and a price of 1 dollar for y.

Q	1	2	3	4	5	6	7
UT_x	12	22	30	36	40	43	44
UT_γ	9	16	22	27	28.5	29.5	30

1- Find the marginal utility of the dollar spent on the two goods.

2- Determine the combinations where the dollars spent on the two goods are equal , and what is the combination where the consumer achieves equilibrium.
3- Calculate the total utility that the consumer receives at equilibrium.

Exercice03:

Utility functions for two consumers are written in the following form:

$U1=2x^2+16y2+100$

$U2=20x^{1/3}\ y^{2/3}$

Assuming that the income of the tow consumers is equal to R= 1500 and the price of the two goods x and y is"P_x=10 ,P_y =40 "

1- Calculate the optimal bundle for each consumer using **Lagrange multipliers.**
2- Calculate the maximum utility for the tow consumers.

Exercice 04:

If a consumer's utility function is written in the following form:
$Ut = 20\sqrt{XY}$, and the available income during the study period is R= 216 Assuming the price of the two goods x and y is"P_x=12 ,P_y =6 "

1- Find the individual demand functions for the two goods using **Lagrange multipliers.**
2- Determine the equilibrium quantities of two goods that achieve consumer equilibrium.
3- Do these quantities satisfy the second-degree condition?

In the event of an increase in the price of good x by four monetary units while the price of good y and available income R remain constant

1- Determine the new quantities that achieve consumer equilibrium**.**
2- Infer the demand curve for good x and what do you observe?

Exercise 05:

Let the utility function of a consumer be as follows: **$U(x, y) = x^{2/3}\ y^{1/3}$**
and the prices of goods X and Y are 2 units of currency (p_x = p_y =2).

I-In the case of a consumer choosing to spend 60 monetary units of their income to purchase goods x and y.

1- To find the equilibrium quantities using the graphical method.
2- To calculate total utility U_1
3- To derive an equation of an indifference curve.
4- To draw a budget line and indifference curve U_1 and To find the point of equilibrium.

II-In the case of a consumer increasing their income by 30 monetary units for consumption of goods x and y

1- To find the new equilibrium quantities.
2- To calculate the new total utility U_2
3- To derive an equation of an indifference curve U_2.
4- On the same previous graph, draw the new budget line and the indifference curve U_2
5- To draw the Consumption-Income Curve and To derive the Engel curve from the consumption-income curve for each commodity.
6- What isThe type of goods x and y

Exercise 06:

Let the utility function of a consumer be given by the following relationship:

$TU = 2/5\ X^2\ Y^2$

If the prices of goods x and y are given as follows:

Px = 4, Py = 3, R = 24

Required:

- Prove that (x = 3/4y)
- Calculate the equilibrium quantity of the consumer in this case
- Calculate the value of the total utility achieved
- Prove that the indifference curve is convex towards the origin
- If the price of good x becomes (Px = 6), calculate the value of the income required to obtain the same level of satisfaction as before
- If the price of good x becomes (Px = 6), calculate the new equilibrium quantity

Exercise07:

Assume a consumer buys two goods, x and y, from the market. The consumer's utility function is given by the following relationship:

TU(x, y) = 2xy + 4y

Required:

- Determine the demand functions for both (x) and (y).
- If the prices of goods x and y are given as follows: Px = 4, Py = 2, with income (R = 22), determine the consumer's equilibrium point.
- Find the equation of the indifference curve that passes through the equilibrium point. Calculate the slope at the same point. What do you conclude?

- Find the Engel curves for goods (x) and (y) at the same previous prices, and deduce the equation of the income-consumption curve.

- If the price of the good (Px = 2), calculate the new equilibrium quantity. Deduce the price-consumption curve for the good (x), and derive the demand curve for the good (x).

- Analyze the effects of substitution and income on the quantity consumed of the two goods.

- **Producer behavior**

Exercise 08:: Briefly explain what is meant by the following terms:

Production function/factors of production/total production/marginal production/average production/short run and long run/law of diminishing returns and when does it take effect.

Exercise 09: In order to grow grains Q With a limited area estimated at 05 hectares, it requires the use of the land(T) element. In addition to the labor component (L), the uses of which vary according to need, and therefore the change in total production corresponding to the change in the labor component can be clarified through the following table:

land(T)	5	5	5	5	5	5	5	5	5
labor(L)	0	1	2	3	4	5	6	7	8
Level production (Q)	0	3	8	12	15	17	17	16	13

Required:

1- Find the average and marginal production of the labor (L) element ?
2- Draw the total, average and marginal production curves on one parameter and explain these curves?
3- Where does the effect of diminishing returns begin?
4- What does it mean to have positive, negative, and zero marginal production?
5- Identify the three stages of production?

Exersice 10:

In a confectionery workshop, known for its good quality, the owner of the workshop showed us the production function of sweets, which is as follows:

$$Q = 50\,K^{0.4}\,L^{0.6}$$

A budget constraint for purchasing factors of production: TC=600 and Prices of factors of production: PL= 6 , PK= 2

Questions:

1. Find the marginal rate of technical substitution (MRTS) between labor and capital.
2. Find the optimal combination of factors of production to maximize output.
3. Determine the nature of returns to scale in the short run.
4. Determine whether the marginal product of labor is diminishing, increasing, or constant in the short run.

Exercise 11:

Assuming that the total cost function of a product operating in a perfectly competitive market is as follows :

$$TC = 4\,Q^2 + 16$$

1- Determine the VC , FC , ATC , AVC , AFC
2- Draw the ATC , MC , AFC , AVC curves on the same coordinates.

What is the level of output at the minimum point of average cost?

Solution

Consumer behavior

Solution (ex 01):

The marginal utility of a good is the change in total utility that results from consuming one additional unit of the good. It can be calculated by dividing the change in total utility by the change in the quantity consumed.

In this case, the marginal utility of sweets can be calculated as follows:

$$\mathbf{Um = (TU_{2} - TU_{1}) / (Q_{2} - Q_{1})}$$

where:

- Um is the marginal utility of sweets
- TU is the total utility of sweets
- Q is the quantity of sweets consumed

QX	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
UT	**100**	**200**	**275**	**325**	**350**	**360**	**360**	**355**
MU	**100**	**100**	**75**	**50**	**25**	**10**	**0**	**-5**

The table shows that the consumer's total utility increases with the quantity of sweets consumed, up to a point of diminishing marginal utility. After the point of diminishing marginal utility, the consumer's total utility continues to increase, but at a slower rate.

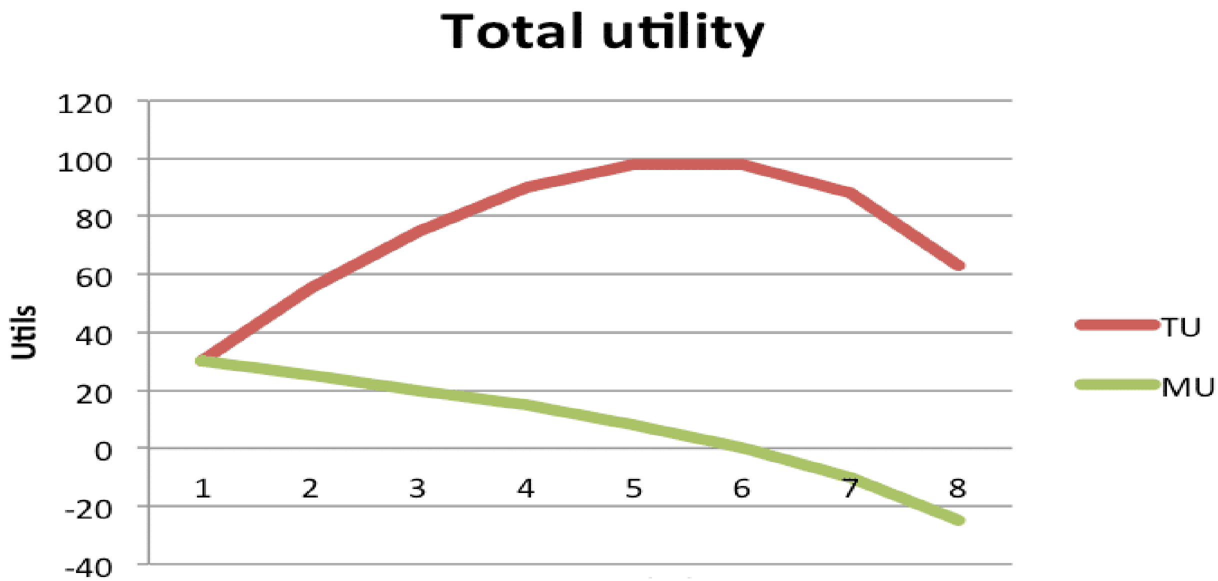
Explanation:

The consumer's total utility is the sum of the marginal utilities of each unit of sweets consumed. The marginal utility is the additional utility that the consumer receives from consuming one additional unit of sweets.

In this case, the consumer's marginal utility decreases with each additional unit of sweets consumed. This is because the consumer's taste for sweets decreases after consuming a certain amount.

The point of diminishing marginal utility is the point at which the marginal utility is zero. In this case, the point of diminishing marginal utility is at the consumption of units of 4 sweets.

After the point of diminishing marginal utility, the consumer's total utility continues to increase, but at a slower rate. This is because the consumer's marginal utility is still positive, but it is less than it was at the beginning.



Solution (ex 02)

1- To find the marginal utility of the dollar spent on the two goods, we need to calculate the marginal utility for each good, which is the additional utility gained from consuming one more unit of that good for each dollar spent.

Marginal Utility (MU_x) = Change in UT_x / Change in Q
Marginal Utility (MU_γ) = Change in UT_γ / Change in Q

calculate MU_x and MU_γ:

For x:
- $MU_x(1) = (UT_x(2) - UT_x(1)) / (2 - 1) = (22 - 12) / 1 = 10$
- $MU_x(2) = (UT_x(3) - UT_x(2)) / (3 - 2) = (30 - 22) / 1 = 8$
- $MU_x(3) = (UT_x(4) - UT_x(3)) / (4 - 3) = (36 - 30) / 1 = 6$
- $MU_x(4) = (UT_x(5) - UT_x(4)) / (5 - 4) = (40 - 36) / 1 = 4$
- $MU_x(5) = (UT_x(6) - UT_x(5)) / (6 - 5) = (43 - 40) / 1 = 3$
- $MU_x(6) = (UT_x(7) - UT_x(6)) / (7 - 6) = (44 - 43) / 1 = 1$

For y:
- $MU_\gamma(1) = (UT_\gamma(2) - UT_\gamma(1)) / (2 - 1) = (16 - 9) / 1 = 7$
- $MU_\gamma(2) = (UT_\gamma(3) - UT_\gamma(2)) / (3 - 2) = (22 - 16) / 1 = 6$
- $MU_\gamma(3) = (UT_\gamma(4) - UT_\gamma(3)) / (4 - 3) = (27 - 22) / 1 = 5$
- $MU_\gamma(4) = (UT_\gamma(5) - UT_\gamma(4)) / (5 - 4) = (28.5 - 27) / 1 = 1.5$
- $MU_\gamma(5) = (UT_\gamma(6) - UT_\gamma(5)) / (6 - 5) = (29.5 - 28.5) / 1 = 1$
- $MU_\gamma(6) = (UT_\gamma(7) - UT_\gamma(6)) / (7 - 6) = (30 - 29.5) / 1 = 0.5$

2- To determine the combinations where the dollars spent on the two goods are equal, we need to compare the prices and marginal utilities of the goods.

Price of x (Px) = $2
Price of y (Py) = $1

The consumer's equilibrium is achieved when the consumer allocates their budget in a way that the marginal utility per dollar spent on each good is equal. Therefore, we need to compare the marginal utilities per dollar:

$MU_x / Px = MU_Y / Py$ and $R = Px\ X + Py\ Y$

For the equilibrium, both sides of the equation should be equal:

Q	1	2	3	4	5	6	7
UTX	12	22	30	36	40	43	44
UTY	9	16	22	27	28.5	29.5	30
MUX	12	10	8	6	4	3	1
MUY	9	7	6	5	1.5	1	0.5
MUX/PX	6	5	4	3	2	1.5	0.5
MUY/PY	9	7	6	5	1.5	1	0.5

Q (x.y) (1.3) and (2.4) (6.5) (7.7)
so R= Px X + Py Y - 17= 2X+Y
1/ Q(1.3) = 5$ <17
2/ Q(2.4) = 8$ <17
1/ Q(6.5) = 17$ =17
1/ Q(7.7) = 21$ >17

From the above calculations, we can see that equilibrium is achieved when consumer Q(x.y) (6.5)

3- To calculate the total utility that the consumer receives at equilibrium, we can simply sum up the utility for both goods at Q = (6, 5)

Total Utility at Equilibrium = $UT_x(6) + UT_Y(5) = 43+ 28.5 = 71.5$

So, the total utility the consumer receives at equilibrium is 71.5.
Solution (ex 03)

To find the optimal bundles and maximum utility for each consumer, we will use the given utility functions, the income constraint, and the prices of goods to set up and solve the utility maximization problem with the Lagrange multiplier method.
Given utility functions:
$U1=2x^2+16y2+100$

$U2=20x^{1/3} y^{2/3}$

Income of the consumers (*R*): 1500
Prices of goods:
$P_X=10$ and $P_y=40$

Consumer 1 (Utility Function U_1)
Setting up the Lagrangian function:

The maximization problem is to maximize $U1=2x^2+16y^2+100$ subject to the same budget constraint $10x+40y=1500$.

$L =-2x^2+16y^2+100+\lambda(1500-10x-40y)$

$L_x'=4x-10\lambda = 0$ ------------------------(1)

$L_y'=32y- 40\lambda= 0$--------------------------(2)

$L'_\lambda= 1500-10x-40y =0$ ------------------ (3)

Setting the partial derivatives equal to zero to find the critical points from $L'=0$ We have
$4x=10\lambda$ ----------------------------(1)
$32y=40\lambda$--(2)
$1500-10x-40y=0$-------------------------------(3)

Calculate x y λ answer 50.25.20

So, for Consumer 2:
- **Optimal bundle: $(x,y)=(50,25)$**
- **Maximum utility: = U2=15100**

Consumer 02:

$L =20x^{1/3} y^{2/3} +\lambda(1500-10x-40y)$

$L_x'=20x^{1/3-1} y^{2/3}- 10\lambda = 0$ ------------------------(1)

$L_y' = 20x^{1/3}\ 2/3y^{2/3-1} - 40\ \lambda = 0$--------------------------(2)

$L'_\lambda = 1500 - 10x - 40y = 0$ ------------------ (3

Solution (ex04):

- **Ut = 20 √XY**
- **(R) = 216**
- **Px=12, Py=6**

Lagrange:

L = 20 √XY +λ (216 – 12 x – 6 y)

10√(Y/X) - λPx = 0---------------(1)
10√(X/Y) - λPy = 0-------------(2)
R-Px.x-Py.y=0--------------------(3)

So , we will divide *equation 1 to equation 2 :*

$$\frac{\mathbf{10\sqrt{(Y/X)}}}{\mathbf{10\sqrt{(X/Y)}}} = \frac{Px}{Py}$$

$y = \frac{x.Px}{Py}$……….*(4)*

We will remplace this function (4) in equation (3)

R- x Px. – ($\frac{x.Px}{Py}$) Py =0⟶ x= $\frac{R}{2Px}$……….the individual demand equation for good (x).

$y = \frac{R}{2Py}$………… the individual demand equation for good (y).

Determining Equilibrium Quantities

Using the demand functions derived in part 1, we can determine the equilibrium quantities of goods X and Y that achieve consumer equilibrium. This equilibrium occurs when the consumer maximizes utility within the budget constraint.

$X^* = \frac{216}{2(12)} = 9$, $Y^* = \frac{216}{2(6)} = 18$

<u>Determining the new Equilibrium Quantities</u>:(Px increase by 4 units means Px =16)

$X^* = \frac{216}{2(16)} = 6.75$

Solution (ex 05)

I. Initial Situation (Income = 60)

MRS IN equilibrium quantities MRS = UMX/UMY= PX/PY
Now, you can solve this equation to find the relationship between x and y at equilibrium
y = 1/2 x……………………(1) . Individuel demande fonction.
Replace y in budget line 2x + x = 60
X= 20 and y = 10 .
calculate total utility U1(20,10) = **(x2 y)1/3** =(4000) **1/3** = 15.87
(x2 y)1/3 =(4000) 1/3 → x2 y = 4000 → Y= 4000/ x2 derive an equation of an indifference curve

II. Income Increase (Income = 90)

y = 1/2 x

Replace y in budget line 2x + x = 90

X= 30 and y = 15

New Total Utility (U2)

U2 ≈ 11.49

Equation of New Indifference Curve (U2)

U2 = 11.49

Solve for $y = 11.49^3 / x^2$

1. **Graph (on the same graph as before):**

   Draw the new budget line (X+Y = 45). X-intercept = 45, Y-intercept = 45.

   Draw the new indifference curve (y=11.49^3/x^2). It will be higher and to the right of the first one, tangent to the new budget line at (X=30, Y=15).

2. **Consumption-Income Curve (CIC) and Engel Curves:**

   CIC: Connect the two equilibrium points (X=20, Y=10) and (X=30, Y=15) on your graph. This line shows how consumption changes as income changes.

   Engel Curve (for X): Plot income on the Y-axis and quantity of X on the X-axis. You'll have two points: (20, 60) and (30, 90). Connect them.

Engel Curve (for Y): Plot income on the Y-axis and quantity of Y on the X-axis. You'll have two points: (10, 60) and (15, 90). Connect them.

3. **Type of Goods**

Since the consumption of both X and Y increases when income increases, both goods X and Y are normal goods.

Solution (ex 06)

1- Prove that (x = 3/4y)
We have UMx/UMy =px/py and R= px X +py Y ,

UMX = 4/5 xy^2

UMY = 4/5 x^2 y

MRS = y/x = px/py , y/x = 4/3 , 3y = 4x

x = 3/4y

- Equilibrium Quantities

- 4x + 3y = 24
- 4(3/4y) + 3y = 24
- 3y + 3y = 24
- 6y = 24
- y = 4
- x = 3/4 (4) = 3
- Calculate the value of the total utility achieved

- U = 57.6

- If the price of good x becomes (Px = 6), calculate the value of the income required to obtain the same level of satisfaction as before.
  In the Same satisfaction Y = 12 /x (indifference curve Ut 1 = 57.6) with same Quantities of the equilibrium (3.4) we need an income equal
  R = 6 x + 3 Y --------- R = 30
  If the price of good x becomes (Px = 6), calculate the new equilibrium quantity
  When new Px2 = 6 → (Umx/UMy =px/py) →Y= 2X and R=6x+3Y ….X=24/12 → X= 2 and
  Y = 4 new the equilibrium quantity (x,y) = (2,4).

Solution (ex 07)

1-To find the demand functions, we need to maximize the utility function subject to the budget constraint:

Budget Constraint: Px * X + Py * Y = R

Lagrangian Function: L = 2XY + 4Y + λ(R - Px * X - Py * Y)

First-order conditions:

$\partial L/\partial X = 2Y - \lambda Px = 0$

$\partial L/\partial Y = 2X + 4 - \lambda Py = 0$

$\partial L/\partial \lambda = R - Px * X - Py * Y = 0$

From the first two equations, we get: 2Y/PX = (2X+4)/PY

Solving for Y: $y = (R/\ 2 + px)/PY$

Substituting Y back into the equation for X: $x = (R - 2px)/2Px$

2. Equilibrium Point , x=1.75,y=7.5

3. Indifference Curve and Slope

The consumer's utility function is: TU(x,y)=2xy+4y → TU(1.75,7.5)=26.25+30=56.25

Thus, the indifference curve passing through the point (1.75,7.5) is given by:

2xy+4y=56.25 → $Y = 56{,}25/2x+4$

The slope of an indifference curve is the Marginal Rate of Substitution (MRS),

Thus, the MRS is: $MRS = -\ 2y/2x+4 = -2$

The slope of the indifference curve at the equilibrium point is -2. This slope (MRS=-2) represents the consumer's rate of substitution between goods x and y.

Specifically:

• The consumer is willing to give up 2 units of y to gain 1 additional unit of x while maintaining the same level of utility.

At the equilibrium point, the slope of the indifference curve (-2) equals the slope of the budget line, which ensures utility maximization. This confirms that the consumer is in equilibrium.

From the previous solution, the demand functions for x and y are: $y = (R/\ 2 + px)PY$, $x = (R - 2px)/2Px$

so The **Engel curves** describe the relationship between R and the quantities demanded for x and y.

R=8(x+1) , R=4(y-2)

Income-Consumption Curve (ICC) .

The **income-consumption curve** shows the combinations of x and y as income (R) changes, holding prices constant.

From Engel Curve for , R=8(x+1) , R=4(y−2) ,so 8(x+1)=4(y−2) we found that the equation of the incomeconsumption curve is: y=2x+4

Interpretation: The ICC shows that as income increases, the consumer increases their consumption of both goods
x and y in a linear pattern, with y growing at twice the rate of x.

New Equilibrium Quantity
Given: Px=2 (new price of x), Py=2 , Income (R=22).
The demand function for x and y is: : $y = (R/\ 2 + px)/PY$, $x = R\text{-}2px/2Px$
so , Thus, the equilibrium quantities are: x=4.5,y=6.5
Price-Consumption Curve (PCC)
The **PCC** shows how the consumer's consumption of x and y changes as the price of xxx varies, holding income (R) and the price of y constant.

<u>Producer behavior</u>

Solution (ex 08):

- **Production Function:** A mathematical equation or relationship that shows the maximum output that a firm can produce given a set of inputs (factors of production) and technology. It summarizes how inputs are transformed into outputs.
- **Factors of Production:** The resources used to produce goods and services. These are typically categorized as:**Land (Natural Resources):** Includes land, minerals, water, etc.**Labor:** Human effort, both physical and mental, used in production.**Capital:** Man-made resources such as machinery, tools, and buildings.**Entrepreneurship:** The skill and initiative to organize and manage the other factors of production.
- **Total Production (TP):** The total quantity of output produced by a firm with a given combination of inputs.
- **Marginal Production (MP):** The change in total production resulting from using one additional unit of a variable input (keeping all other inputs constant). Mathematically, it is the derivative of the total production with respect to the variable input. For example, the marginal product of labor (MPL) is the additional output gained by adding one more worker.
- **Average Production (AP):** The total production divided by the quantity of a variable input. For example, the average product of labor (APL) is total output divided by the number of workers. APL = TP/L.
- **Short Run:** A period of time in which at least one input (usually capital) is fixed, and the firm cannot change that input. The firm can change variable inputs such as labor.
- **Long Run:** A period of time in which all inputs are variable. The firm can change all factors of production.
- **Law of Diminishing Returns:** States that as one variable input is added to a fixed input, the marginal product of the variable input will eventually decrease. Initially, adding more variable input might increase the MP and AP. However, at some point, adding more

variable input will cause MP to decrease. It takes effect in the short run because some inputs are fixed.

Solution (ex 09):

land(L)	labor(L)	Level production (Q)	Marginal Product of Labor (MPL)	Average Product of Labor (APL)
5	0	0	-	-
5	1	3	3	3
5	2	8	5	4
5	3	12	4	4
5	4	15	3	3.75
5	5	17	2	3.4
5	6	17	0	2.83
5	7	16	-1	2.28
5	8	13	-3	1.63

1. **Find the average and marginal production of the labor (L) element?**

Marginal Product of Labor (MPL): The change in output for each additional unit of labor. This is calculated as the change in Total Production divided by the change in labor.

- MPL with 1st worker = 3-0=3.
- MPL with 2nd worker = 8-3=5.
- MPL with 3rd worker = 12-8=4
- MPL with 4th worker = 15-12=3.
- MPL with 5th worker = 17-15=2.
- MPL with 6th worker = 17-17=0.
- MPL with 7th worker = 16-17=-1.
- MPL with 8th worker = 13-16=-3.

Average Product of Labor (APL): The total production (Q) divided by the number of labor.

- APL with 1 worker = 3/1=3
- APL with 2 worker = 8/2 =4
- APL with 3 worker = 12/3=4
- APL with 4 worker = 15/4=3.75.
- APL with 5 worker = 17/5 = 3.4
- APL with 6 worker = 17/6= 2.83
- APL with 7 worker = 16/7 = 2.28
- APL with 8 worker = 13/8 = 1.63

- **Total Production (TP):** The TP curve will increase initially at an increasing rate then increase at a decreasing rate and finally decrease.
- **Average Production (AP):** The AP curve will initially increase, reach a peak, and then decrease, but it will remain positive.
- **Marginal Production (MP):** The MP curve will initially increase, reach a peak, decrease, become zero, and then go into negative values.
  * **Relationship:**
  * When MP is greater than AP, AP is increasing.
  * When MP is less than AP, AP is decreasing.
  * When MP = AP, AP is at its maximum.
  * The MP curve crosses the x-axis (becomes zero) where the total production reaches a maximum.
  * MP will be negative once TP starts to decline.

1. **Where does the effect of diminishing returns begin?**

   The law of diminishing returns sets in when the marginal product starts to decline, which is between the 2nd and 3rd worker when the marginal production of labor falls from 5 to 4.

2. **What does it mean to have positive, negative, and zero marginal production?**

   Positive Marginal Product: Adding one more worker will increase total production.

   Zero Marginal Product: Adding one more worker will not change the total production. Total production reaches its maximum.

   Negative Marginal Product: Adding one more worker will decrease total production. This means that you're overcrowding the production process.

3. **Identify the three stages of production?**

   Stage 1: Increasing Returns: This stage extends from 0 until MP = AP. This occurs at a labor amount of 2. The average product of labor is increasing, and so is the marginal product of labor.

   Stage 2: Diminishing Returns: This stage extends from when MP starts decreasing until it becomes 0. This occurs at labor between 2 and 6. Both AP and MP are positive in this stage, but MP is decreasing.

Stage 3: Negative Returns: This stage extends from when MP becomes negative and includes all labor quantities beyond that. This occurs at a labor amount of 7 and beyond. In this stage, MP is negative, and TP is decreasing.

Solution (ex 10):

1-The marginal rate of technical substitution (MRTS) between labor and capital is given by the ratio of the marginal product of labor to the marginal product of capital, while holding output constant. Mathematically, it can be expressed as:MRTS $_{L,K} = \frac{MPL}{MPK} = \frac{3k}{2l}$

2-To find the optimal combination of factors of production to maximize output, we need to equate the marginal rate of technical substitution (MRTS) to the ratio of factor prices. Mathematically, this condition is known as the equality of factor price ratios to the MRTS: MRTS $_{L,K}$ = pL/pk

MRTS $_{L,K} = \frac{3k}{2l} = \frac{6}{2}$ = => K=2L

We have:

$$\begin{cases} K = 2L \\ 600 = 6L + 2K \end{cases}$$

When Calculate L;K from this equation we find L=60 K= 120

3- Returns to scale in the short run can be determined by analyzing the behavior of the production function as all inputs are increased proportionally.

$$Q = A\, L^{\alpha}\, K^{\beta}$$

$$Q_{(\lambda L.\, \lambda K)} = A(\lambda\, L)^{\alpha}\, (\lambda K)^{\beta}$$

$$Q_{(\lambda L.\, \lambda K)} = A\ \lambda^{\alpha}\, L^{\alpha}\, \lambda^{\beta} K^{\beta}$$

$$Q_{(\lambda L.\, \lambda K)} = A\ \lambda^{\alpha+\beta}\, L^{\alpha}\, K^{\beta}$$

$$Q_{(\lambda L.\, \lambda K)} = \lambda^{\alpha+\beta}(A\, L^{\alpha}\, K^{\beta}) = \lambda^{\alpha+\beta} Q$$

Therefore, there are constant returns to scale in the short run.

4- $Q = 50\, K^{0.4}\, L^{0.6}$

5- MPL= $50(0.6)\, K^{0.4}\, L^{-0.4}$ => $\frac{dMPL}{dl} = 30(-0.4)\, K^{0.4}\, L^{-1.4} = -12\, K0.4\, L{-1.4} < 0$

So, In the short run, the marginal product of labor MPL- diminishing .

Solution (ex 11):

TC = 4 Q2 + 16

1- Determine the VC, FC, ATC, AVC, AFC

• Variable Cost (VC): This is the cost that varies with the level of output (Q). It is the cost associated with the variable inputs. VC = 4 Q2

• Fixed Cost (FC): This is the cost that remains constant regardless of the level of output. It is the cost associated with fixed inputs. FC = 16

• Average Total Cost (ATC): This is the total cost per unit of output. It's calculated by dividing the total cost (TC) by the quantity of output (Q).

ATC = (4 Q2 + 16)/Q

• Average Variable Cost (AVC): This is the variable cost per unit of output. It's calculated by dividing the variable cost (VC) by the quantity of output (Q). AVC = (4 Q2)/Q = 4Q

• Average Fixed Cost (AFC): This is the fixed cost per unit of output. It's calculated by dividing the fixed cost (FC) by the quantity of output (Q).

AFC =FC/Q → 16/Q2- Draw the ATC, MC, AFC, AVC curves on the same coordinates.

Marginal Cost (MC): This is the additional cost incurred by producing one more unit of

output. It is the derivative of the total cost function with respect to quantity (Q).

MC = 8Q.

3- What is the level of output at the minimum point of average cost?

Level of output at the minimum point of average cost:

To find the minimum point of the ATC curve,

First Method: we'll set its derivative equal to zero and solve for

ATC = (4 Q2 + 16)/Q or ATC = 4Q +(16 Q-1)

so, derivative equal to zero ATC'= 0 → 4 -16 Q-2 =0

we found this ¼= Q-2 or Q2 = 4

Q = 2 or (-2) …we take Q=2

So, the level of output at the minimum point of average cost is when Q=2

Second Method

From the shape of the curve, we notice that the level of production at the lowest point of

average cost is at the point where curve CM intersects curve ATC (ATC=CM)

So, ATC=CM → 4Q +(16 Q-1) = 8Q we fond Q2 = 4 we take Q=2

So, the level of output at the minimum point of average cost is when Q=2

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